Lecture Notes 10/18/04

Mergers

• Start with Whinston chapter on horizontal mergers:

• Tools for government bureaucrats

• Williamson Trade-off
Farrell and Shapiro: helpful tips

- Take a static homogenous products $N$-firm Cournot oligopoly, where firm cost functions may differ.

- Firms 1 and 2 contemplate a merger. Assume w.l.o.g. that $x_1 \geq x_2$ in premerger case.

- Premerger FONC ($X$ is premerger output)

  \[
  P'(X)x_1 + P(X) - c'_1(x_1) = 0 \\
  P'(X)x_2 + P(X) - c'_2(x_2) = 0
  \]

  which implies

  \[
  P'(X)(x_1 + x_2) + 2P(X) - c'_1(x_1) - c'_2(x_2) = 0
  \]
• Assume merged firm’s profit is concave in output. Optimal output (given rival’s premerger output) is greater than the sum of the two firms premerger output iff

\[ P'(X)(x_1 + x_2) + P(X) - c'_M(x_1 + x_2) > 0 \]

or

\[ c'_2(x_2) - c'_M(x_1 + x_2) > P(X) - c'_1(x_1) \]

which can happen only if (since \( x_1 \geq x_2 \))

\[ c'_M(x_1 + x_2) < c'_1(x_1). \]

• Stringent requirement, reallocation not enough.
So maybe price goes up. But any tips for when welfare might increase anyway?

- Sufficient to show sum of welfare of non-participants and consumers goes up.

\[ E = \int_{P(X)}^{\infty} x(s)ds + \sum_{i \notin I} [P(X)x_i - c_i(x_i)] \]

- So what happens if the merger decreases output a little, \(dX_I < 0\)? Then

\[ dX = dX_I + \sum_{i \notin I} x_i < 0 \]

- Effect on \(E\) is

\[ -XP'(X)dX + \sum_{i \notin I} x_iP'(X)dX + \sum_{i \notin I} \left[P(X) - c_i'(x_i)\right] dx_i \]
\[-X_I P'(X) dX + \sum_{i \notin I} \left[ -P'(X) x_i \right] dx_i \]
\[-P'(X) dX \left[ X_I + \sum_{i \notin I} x_i \left( \frac{dx_i}{dX} \right) \right] \]
\[-P'(X) X dX \left[ s_I + \sum_{i \notin I} s_i \left( \frac{dx_i}{dX} \right) \right] \]

so \( dE > 0 \) iff

\[s_I < -\sum_{i \notin I} s_i \left( \frac{dx_i}{dX} \right)\]

So

\[
\frac{dx_i}{dX} = \frac{\sum\frac{dx_i}{dX_I}}{1 + \sum_{j \notin I} \frac{dx_j}{dX_I}}
\]

Special case: Linear demand, constant marginal cost.

\[(A - X_i - nx_i) x_i - cx_i\]
\( FONC \quad : \quad (A - X_I - nx_i) - x_i - c = 0 \)

\[
    x_i = \frac{A - X_I - c}{n + 1}
\]

\[
    \frac{dx_i}{dX_I} = -\frac{1}{n + 1}
\]

substitute into the above

\[
    \frac{dx_i}{dX} = \frac{dx_i}{dX_I} \cdot 1 + \sum_{j \notin I} \frac{dx_j}{dX_I}
\]

\[
    = -\frac{1}{n+1} \cdot \frac{1}{1 - \frac{n}{n+1}} = -\frac{1}{n+1} \cdot \frac{n+1}{n+1} = -1
\]

So condition is:

\[
    s_I < \sum_{i \notin I} s_i
\]

- But note here that unless cost decreases for the merger, two firms with less than half the market share would not
want to merge (Salant Switzer, Reynolds). See in example:

**Cournot quantity**: \[ q = \frac{A - c}{n + 1} \]

**Cournot Profit**: \[ \pi(n) = (A - c) \left( \frac{1}{n + 1} \right)^2 \]

Now

\[ \pi(1) > 2\pi(2) \]
\[ \pi(n) < 2\pi(n + 1), \ n \geq 2 \]

- Suppose costs are \( c(x, k) = \frac{1}{2} \left( \frac{x^2}{k} \right) \), so merger means something more than moves in a Cournot game. Suppose \( P(X) = A - X \) as above. Then external effect is positive if

\[
s_I = \left( \frac{1}{\varepsilon} \right) \sum_{i \notin I} s_i^2
\]
where $\varepsilon$ is the elasticity of demand calculated at premerger price. Cute result because it has a Hirfindahl index in it.
Do a little more work: Econometric Approaches

- Estimate a demand curve, e.g.

\[
\ln(x_i) = \beta_0 + \beta_1 \ln(p_i) + \beta_2 \ln(q_i) + \beta_3 \ln(y_i) + \varepsilon_i
\]

or BLP type

\[
u_{ij} = \beta x_j - \alpha p_j + \xi_j + \varepsilon_{ij}
\]

- Estimate costs. Directly, or nowaways, indirectly (take demand and back out cost from)

\[
p_i - c'_i(x_i(p)) \frac{\partial x_i(p_i, p_{-i})}{\partial p_i} + x_i(p) = 0
\]

Pins down \( c'_i(x_i(p)) \).
• Now simulate the effect of the merger.
Recent Example, Pesendorfer "Horizontal Mergers in the Paper Industry"
Mergers as Reallocation, Jovanovic and Rousseau

• Two margins of reallocation, liquidation and mergers
  – Merger waves accompanied by rise in exits
  – Exits lead mergers
  – Mergers have grown relative to exits
Model

- Macro $Ak$ model with preferences

\[ U = \frac{1}{1 - \sigma} \int_0^\infty e^{-\rho t} c_t^{1-\sigma} dt \]

- One-technology version of the model

\[
\begin{align*}
y &= zk \\
\dot{k} &= -\delta k + x \\
y &= c + x
\end{align*}
\]

get

\[
\begin{align*}
\dot{y} &= \dot{c} = z - \delta - \rho \\
y &= c \\
x &= \dot{k} + \delta = z - \delta(1 - \sigma) - \rho \\
k &= \dot{k} + \delta = z - \delta(1 - \sigma) - \rho
\end{align*}
\]
• Two Technology version of the model

  – New technology $z_2$ appears. At the outset all existing capital embodies old technology $z_1$.

  – How get old technology over to new? Say takes $T$ periods, the reallocation period.

  – Three ways: De Novo investment, mergers, exits
Mergers

- Owners of $k_2$ buy capital from owners of $k_1$ at a cost $c$ that varies

- CDF of cost $c$ is $F(c)$, density $f(c)$.

- Let $mk_2$ be the number of units acquired, then cost of last unit $r$ is

  $m = F(r)$

- Total conversion costs are $\phi(m)k_2$,

  $\phi(m) = \int_0^r c dF(c)$
the unit cost of adapting the acquired capital. Note $\phi(0) = 0$, $\phi' = r(m) > 0$, So increasing and convex.

- Example: if $c$ is uniform on $[0, c^m]$, then

$$
\phi(m) = \left(\frac{c^m}{2}\right) m^2
$$
Exits

- Seller with $k_1$ units of inefficient capital faces the cost $c^\sim G(c)$ of adapting his capital for sale.

- If all $k_1$ units are sold total cost is

  $$k_1 \int_0^\infty cdG(c)$$

- If sell a fraction $\varepsilon$, sell least costly first, those where $c \leq R(\varepsilon)$, where $R(\varepsilon)$ solves

  $$\varepsilon = G(R)$$
Cost is $\psi(\varepsilon) k_1$,

$$\psi(\varepsilon) = \int_0^R c dG(c)$$

increasing and convex. If $c$ is uniform on $[0, c^\varepsilon]$, then

$$\psi(\varepsilon) = \left( \frac{c^\varepsilon}{2} \right) \varepsilon^2.$$
• Output and the evolution of $k_1$ and $k_2$

\[
\begin{align*}
    y &= (z_1 - \psi(\varepsilon)) k_1 + (z_2 - \phi(m)) k_2 \\
    c &= y - x_1 - x_2 \\
    \dot{k}_1 &= -\delta k_1 + x_1 - \varepsilon k_1 - mk_2 \\
    \dot{k}_2 &= -\delta k_2 + x_2 + \varepsilon k_1 + mk_2
\end{align*}
\]
• Equilibrium: $m, \varepsilon, x_1, x_2$ such that firms maximize profits and representative agent consumes optimally.

• initial conditions $k_{1,0} = 1, \ k_{2,0} = 0$.

• Upgrading: $q$ prices of $k_1, Q$, price of $k_2$.

$$\psi'(\varepsilon) = Q - q$$
$$\phi'(m) = Q - q$$

• Investment. $x_2 > 0$ implies

$$Q = 1$$

• It will turn out that $q < 1$ for $t \in [t, T]$, therefore $x_1 = 0$. 
• Output and upgrading rents

\[
\pi^\varepsilon(q) = \max_{\varepsilon} \{ \varepsilon - (q\varepsilon + \psi(\varepsilon)) \}
\]
\[
\pi^m(q) = \max_{m} \{ m - (qm + \phi(m)) \}
\]

• If solve for \( q \), can infer \( \varepsilon, m, \pi^\varepsilon(q), \pi^m(q) \), and \( \dot{c}/c \) and rate of interest

Price of \( k_1 \) must be such that

\[
z_1 + \pi^\varepsilon(q) = (r + \delta)q - \dot{q}
\]

And

\[
z_2 + \pi^m(q) = r + \delta
\]

combining and eliminating \( r \) yields

\[
\frac{\dot{q}}{q} = z_2 + \pi^m(q) - \frac{z_1 + \pi^\varepsilon(q)}{q}
\]
• Argument that $q = 1$ at $T$ and $\dot{q} > 0$ throughout the transition.
1. At \( t = 0 \), output falls from \( z_1 k_1 \) to \( (z_1 - \psi [z_0]) k_1 \) and then starts to rise monotonically.

2. The value of capital also falls from 1 to \( q_0 \). Wealth falls from \( k_{1,0} \) to \( q_0 k_{1,0} \).

3. Thereafter, \( q_t \) rises monotonically to 1, and \( k_1 \) falls monotonically to zero at date \( T \), as do \( \varepsilon \) and \( m \).

4. Total exits, \( q \in k_1 \), decline monotonically, whereas total acquisitions, \( q m k_2 \), start and end at zero and are essentially inverted U-shaped during the transition.

5. The rate of interest jumps from \( z_1 - \delta \) to \( z_2 - \delta + \pi^m (q_0) \) and then declines monotonically to \( z_2 - \delta \) where it remains thereafter.

6. Consumption falls at date zero. After that consumption growth declines monotonically. More precisely,

\[
g_c = \begin{cases} 
\frac{z_1 - \delta - \rho}{z_2 + \pi^m (q_0) - \delta - \rho} & \text{for } t < 0 \\
\frac{z_2 - \delta - \rho}{\sigma} & \text{for } t \in (0, T) \\
\frac{z_2 - \delta - \rho}{\sigma} & \text{for } t \geq T.
\end{cases}
\]  
(15)

7. Investment \( x_1 = 0 \) throughout, \( x_2 > 0 \) and using (2)

\[
\lim_{t \to T} \left( \frac{x_{2,t}}{k_{2,t}} \right) = \frac{z_2 - \delta (1 - \sigma) - \rho}{\sigma}.
\]  
(16)
1. At $t = 0$, output falls from $z_1k_1$ to $(z_1 - \psi[e_0])k_1$ and then starts to rise monotonically.

2. The value of capital also falls from 1 to $q_0$. Wealth falls from $k_1,0$ to $q_0k_1,0$.

3. Thereafter, $q_t$ rises monotonically to 1, and $k_1$ falls monotonically to zero at date $T$, as do $\varepsilon$ and $m$.

4. Total exits, $q\varepsilon k_1$, decline monotonically, whereas total acquisitions, $qmk_2$, start and end at zero and are essentially inverted U-shaped during the transition.

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\frac{z_2 - \delta - \rho}{\sigma} & \text{for } t \in (0, T) \\
\frac{z_2 - \delta - \rho}{\sigma} & \text{for } t \geq T.
\end{cases}$$

(15)
Model settings:
$z_1 = 0.130, z_2 = 0.145, c^e = 0.63, c^m = 0.51, \rho = 0.05, \sigma = 1, \delta = 0.08.$

Figure 3. Transitional dynamics I.
Figure 4. The values of exiting firms and merger targets in two technological epochs.

Figure 5. Prices of the two types of capital in the IT transition.
Figure 6. Normalized cumulative distributions of exits, mergers, and total reallocation.
Model settings:
\[ z_1 = 0.140, \; z_2 = 0.145, \; c^o = 0.63, \; c^m = \text{infinity}, \; \rho = 0.05, \; \sigma = 1, \; \delta = 0.08. \]

Figure 7. Transitional dynamics II.
Figure 9. Target values vs. changes in GPT shares over 10-year periods by sector.
$y (c^m=0.51) / y (c^m=\infty)$