A General Equilibrium Model of the Cost of Monopoly
Class Notes—Econ 8601, Fall 2003
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Suppose the set of goods in an economy is fixed at \([0, 1]\) and let \(x \in [0, 1]\) denote a particular good. Suppose the utility function for the representative consumer is CES,

\[
U = \left( \int_0^1 q(x)^{\frac{1}{\mu}} \, dx \right)^\mu
\]

\[
\sigma = \frac{\mu}{\mu - 1}
\]

for \(\mu > 1\). The representative consumer has a unit endowment of time. Production of each good is constant average cost of one unit of labor per unit of output.

Let labor time be numeraire, \(w = 1\). So production cost per unit is \(c = 1\).

Suppose that goods \(x \in [0, \lambda]\) are controlled by a monopolist and goods \(x \in (\lambda, 1]\) are perfectly competitive.

The representative consumer owns shares in all the firms. Assume the monopoly firms that produce goods \(x \in [0, \lambda]\) maximize profit and turn this profit over to the representative consumer. Let \(\pi_M\) be the equilibrium monopoly profit in one such industry. Then the income of the representative consumer is

\[
I = 1 + \lambda \pi_M.
\]

The price in competitive industries is \(p_C = 1\). Given the constant elasticity of demand, the price in monopoly industries is \(p_M = \mu\).

Let \(q_M\) and \(q_C\) be quantities in the equilibrium of this economy. From consumer demand, we must have

\[
\frac{q_M}{q_C} = \left( \frac{p_M}{p_C} \right)^{-\sigma}
\]
But \( p_C = c \) and \( p_M = \mu \), so

\[
q_M = q_C \mu^{-\sigma}
\]

To determine the equilibrium, we can examine the resource constraint for labor,

\[
\lambda q_M + (1 - \lambda) q_C = 1
\]

\[
\lambda q_C \mu^{-\sigma} + (1 - \lambda) q_C =
\]

\[
q_C = \frac{1}{(1 - \lambda + \lambda \mu^{-\sigma})}
\]

Consider the gains from an antitrust policy that eliminates monopoly in the economy (decreases \( \lambda \) to zero). Let \( v \) be the compensating variation, the change in income at the new prices so the representative consumer is indifferent to old system. With new prices, \( p = 1 \) everywhere. Let \( Y = 1 - v \) be income. With perfect competition everywhere, it is immediate that utility equals income, \( U = Y \). So the compensating variation equals

\[
1 - v = \left( \lambda \left( \frac{1}{q_M} \right)^{\frac{1}{\mu}} + (1 - \lambda) q_C^\mu \right)^{\mu}
\]

\[
= \left( \lambda \left( q_C \mu^{-\sigma} \right)^{\frac{1}{\mu}} + (1 - \lambda) \frac{1}{q_C} \right)^{\mu}
\]

\[
= q_C \left( \lambda \mu^{-\sigma} + 1 - \lambda \right)^{\mu}
\]

\[
= \frac{(\lambda \mu^{-\sigma} + 1 - \lambda)^{\mu}}{(1 - \lambda + \lambda \mu^{-\sigma})}
\]

or

\[
v = 1 - \frac{(1 - \lambda + \lambda \mu^{-\frac{1}{\mu-1}})^{\mu}}{(1 - \lambda + \lambda \mu^{-\frac{\mu}{\mu-1}})}
\]

What happens as \( \mu \) goes to 1? Observe that

\[
\lim_{\mu \to 1} \mu^{-\frac{1}{\mu-1}} = \lim_{\mu \to 1} \mu^{-\frac{\mu}{\mu-1}} = .3679
\]

Thus as \( \mu \) goes to 1 (which means the elasticity of substitution goes to infinity), \( v \) goes to zero, so the welfare cost of monopoly is zero.
Note the counter intuitive fact that \( v = 0 \) at \( \lambda = 1 \), complete monopoly. When all sectors are monopolized, the monopoly price does not lead to distortions. Labor is equally allocated across all sectors, and the efficient allocation is obtained.

The following table calculates the welfare cost of monopoly for various levels of \( \mu \) and \( \lambda \).

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