A Model of Mode Choice

• Assume consumers choose modes of transit \( i \in \{1, 2, \ldots N\} \).
  (Example \( i = 1 \) automobile, \( i = 2 \) bus, etc.)

• Individuals face different choice sets:
  —Let \( x_{ij} \) be the time cost of mode \( i \) for consumer \( j \).
  —Let \( c_{ij} \) be the monetary cost of mode \( i \) for consumer \( j \)
• Individuals have same preferences.

—$\alpha$ is value of time

—$\beta_i$ is psychic cost of mode $i$

• Individual $j$ picks mode $i$ to minimize cost

$$\min_i \{ \alpha x_{ij} + c_{ij} + \beta_i \}$$

• The demand for mode $i = \# \{ \text{of } j \text{ where } i \text{ solves above problem} \}$
Example

- Two modes. \( i = 1 \) car, \( i = 2 \) bus

- Note that only \( \beta_2 - \beta_1 \) matters, so set \( \beta_1 = 0 \) and let \( \beta = \beta_2 \).

- An individual takes the bus if and only if
  \[
  \alpha x_1 + c_1 \geq \alpha x_2 + c_2 + \beta
  \]
  or
  \[
  -[c_2 - c_1] \geq \alpha (x_2 - x_1) + \beta
  \]
  \[
  -\Delta c \geq \alpha \Delta x + \beta
  \]

- Plot data on \( \Delta c_j \) and \( \Delta x_j \) for individuals.
• Estimate $\alpha$ and $\beta$.

• Then can use this to consider experiments. Suppose add a subsidy of $\sigma > 0$. Then

$$\Delta c_j' = \Delta c_j^o - \sigma$$

What is effect on demand?
Generalizations

• Unobserved consumer heterogeneity

—Logit error terms $\varepsilon_{ij}$

—Random coefficients

—See Nobel prize winning work of Daniel McFadden

• Richer model of demand (time cost varying by mode, etc.)

• Selection issues?

• Short-run vs. Long run?
Economics of Public Transit

• Natural Monopoly?

—Graphical discussion of what natural monopoly is

—Optimal pricing under natural monopoly

(i) case of break-even constraint for government (second best)

(ii) no break-even constraint. \( P = MC \)
A Richer Model with Endogenous Quality

• Let $x \geq 0$ denote product quality (frequency of service is key for public transit)

• Let $p$ be price of the produce. Consumers care about $n = p - x$, price net of value

• Let $D(p - x)$ be demand per person at the net price. Assume $D' < 0$.

• Assume $m$ individuals to $mD(p - x)$ is total demand at net price $p - x$. 
Cost Structure

- Let $f(x)$ be the fixed cost to provide quality $x$. Assume $f(0) > 0$, $f'(x) > 0$.

- Let $c$ be marginal cost.

- So if $x$ is constrained to be 0 then problem is simple case covered earlier.
Problem with no break-even constraint

\[
\max_{p,x} \text{Consumer Surplus - Total Cost} = \max_{p,x} \left[ \text{area under demand} \right] - mD(p - x)c - f(x)
\]

- Clear that given \(x\), set \(p = c\) (price equal marginal cost)

- Now pick \(x\). Graphical argument

- Clear optimal \(x\) is increasing in market size \(m\)
Problem with break-even constraint

- Clear in optimum firm breaks even

- So problem boils down to maximizing consumer welfare or

\[ \min_{p,x} p - x \]

subject to \( mD(p - x)(p - c) \geq f(x) \)

- Clear optimal \( x \) is increasing in \( m \).
• Logic here is that subsidies are contributing to fixed cost. In the analysis above consumers pay at least marginal cost.

• Subsidize marginal cost?

—Some economic justification if congestion externality for driving and can’t set congestion tax.

—Always more efficient to use congestion tax if can do it.

• Fare box ratios (percentage of operating cost covered by fares)

<table>
<thead>
<tr>
<th>Mode</th>
<th>Percentage</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bus</td>
<td>32%</td>
</tr>
<tr>
<td>Heavy Rail</td>
<td>57%</td>
</tr>
<tr>
<td>Light Rail</td>
<td>34%</td>
</tr>
<tr>
<td>Commute Rail</td>
<td>49%</td>
</tr>
<tr>
<td>All Public Transit</td>
<td>38%</td>
</tr>
</tbody>
</table>
Hiawatha Light Rail

- Construction Funding:

  Federal Transit Administration - $334.3
  State of Minnesota - $100
  Metropolitan Airports Commission - $87
  Hennepin County Regional Rail Authority - $84.2
  Federal Grant for Congestion Mitigation & Air Quality - $49.8
  Transit capital grant - $39.9
Minnesota Department of Transportation - $20.1

TOTAL $715.3

- Ridership

24,600 a day by 2020 (optimistic)

12,300 round trips

Construction cost per round trip=$60,000

- Public Transit in Twin Cities metro area
—250,000 trips a day

—10,000,000 total trips, 2.5% share. (no surprise few notice the strike)