Question 1.

Consider the durable goods monopoly problem discussed in class. There are two periods, $t \in \{1, 2\}$. The inverse demand for lightbulbs in each period is

$$p = 2 - \frac{1}{50}Q$$

where $p$ is the rental rate for one bulb in the period and $Q$ is the total number of bulbs used in the period.

The monopolist has zero marginal cost to produce bulbs. Bulbs sold in period 1 last through period 2.

Suppose the monopolist can only sell bulbs, not rent them. Furthermore, the monopolist cannot in period 1 commit to its behavior in period 2.

Let $\tilde{P}_t$ denote the purchase price of a bulb in period $t$. As discussed in class, this equals the discounted value of the rental rates,

$$\tilde{P}_1 = p_1 + \beta p_2$$
$$\tilde{P}_2 = p_2.$$

(The sale price in period 2 is equivalent to the rental rate because it's the terminal period.) Let $q_t$ denote the number of lightbulbs sold in period $t$ so that the total quantity $Q_t$ in each period is

$$Q_1 = q_1$$
$$Q_2 = q_1 + q_2.$$
In class we worked out the case with a 10 percent interest rate or $\beta = \frac{1}{1 + .10} = .909$. Your assignment is to work out the case with a 0 percent interest rate or $\beta = 1$. Specifically

(a) Solve for the equilibrium sequence of rental rates $(p_1, p_2)$ and the equilibrium purchase price $\tilde{P}_1$ in period 1, and the equilibrium values of $q_1$, $q_2$ and $Q_1$ and $Q_2$.

(b) Suppose bulbs only last one period, their nondurable. Then as discussed in class, the optimal policy for the monopolist is to sell 50 bulbs in each period for a price of $\tilde{P}_1 = \tilde{P}_2 = 1$. Compare this allocation with the durable goods case from part (a). Specifically, calculate the difference in consumer surplus in each period and as well as the difference in consumer surplus over both periods. Compare total profits in the to cases. Define total surplus as consumer surplus plus profit. Calculate the difference in total surplus between the two cases.

(c) Give an example of a product that you think is or has been made intentionally made less durable by a firm in order to increase the firm’s profit.
Question 2.

There are two firms in an industry. Let $q_1$ and $q_2$ be the output of firm 1 and firm 2 and $Q = q_1 + q_2$ be total output. The inverse demand in the industry is $P^D(Q) = 30 - Q$. The cost function for each firm is $c(q) = 6q$.

Assume the oligopolistic competition between the two firms is Stackelberg. Firm 1 is the Stackelberg leader moving first to set $q_1$. Then firm 2 sets $q_2$ after seeing firm 1’s move.

(a) Calculate firm 2’s reaction function $q_2 = R(q_1)$ to the output choice of firm 1.

(b) Write down the firm 1’s profit as a function of $q_1$, taking into account firm 2’s reaction.

(c) Solve firm 1’s problem for the optimal $q_1$. What is the equilibrium sequence of outputs $(q_1^s, q_2^s)$ to the Stackelberg game? What is the profit of each firm?

(d) Suppose instead the competition is Cournot. Calculate the Cournot output levels $(q_1^{\text{Cournot}}, q_2^{\text{Cournot}})$ and compare the profit of each firm under Stackelberg and Cournot.

(e) Which do consumers like better, Cournot or Stackelberg?