Question 1: Third Degree Price Discrimination

Suppose there are two kinds of consumers, $L$ and $H$. The inverse demand curves for the two markets are $p^D_L = 16 - q_L$ and $p^D_H = 20 - q_L$. The seller in the market is a monopolist with cost function $c(q) = 2q$. 

(a) Suppose the firm can discriminate between the two markets and practice third-degree price discrimination. Suppose in market $i$, the firm offers a uniform price $p_i$. Solve the the optimal price and quantity in each market. Calculate consumer surplus in each market and calculate total profit in each market.

(b) Suppose the firm cannot discriminate. It offers a uniform price $p_U$ that is the same in both markets. Solve for $p_U$, the quantity in each market, consumer surplus and profit in each market. How does the ability to price discriminate change consumer surplus for each type, compared to the uniform price case?

(c) Suppose that in addition to the variable cost of 2 per unit, there is a fixed cost of 129. Suppose the monopolist will not operate if it cannot cover its fixed cost. Compare consumer surplus of each type with and without the ability to discriminate.

Question 2: Optimal Pricing with Self-Selection Constraints

There are two types of consumers, $L$ and $H$ for home entertainment systems. Let $q$ denote the number of features in a system. The utility function of each consumer type for features $q$ and spending $y$ on other goods is

\[
U_L(q, y) = 16q - \frac{q^2}{2} + y
\]

\[
U_H(q, y) = 20q - \frac{q^2}{2} + y
\]
There is a monopolist on systems that faces a marginal cost of $2q$ to make a system with $q$ features. Hence the net total surplus of selling a system with $q$ features to a low or high type consumer is

$$N_L(q) = 16q - \frac{q^2}{2} - 2q$$

$$N_H(q) = 20q - \frac{q^2}{2} - 2q.$$ 

Let $(q, a)$ denote a product bundle specifying a quantity of features and a total markup $a$ over cost. So a bundle with $q = 6$ and $a = 10$ will have a total cost to the consumer of $10 + 2 \times 6 = 22$ since the variable cost of the bundle is 12.

(a) Plot the net surplus of each type on the same graph as a function of $q$. (Please use graph paper.)

(b) Suppose the monopolist can observe customer type and make an offer of a bundle contingent on customer type (i.e., the self-selection constraints are not imposed). What is the profit maximizing set of bundles $(q^*_L, a^*_L), (q^*_H, a^*_H)$? What is the firm’s profit on each. Illustrate the bundles in your graph.

(c) Suppose that the number of customers of each type is unity, $N_L = 1$ and $N_H = 1$. Suppose the monopolist cannot observe customer type so that the set of offers must satisfy the self-selection constraints that each type chose the bundle intended for his or her type. Write down the monopolist’s problem. Solve for the optimal bundles $(q^*_L, a^*_L), (q^*_H, a^*_H)$ that satisfy the self-selection constraint. Plot them on your graph. Draw the indifference curve of the type $H$ individual through $(q^*_H, a^*_H)$. Calculate total profit and total surplus. Compare this with total profit and total surplus from part (b).