This is an open-book, open-note exam. You can take up to four hours to take this exam. Any time you spend reading the notes, the text, or journal articles does not count against this time.

**Question 1**

Consider the following model of consumer choice. Let \( y_i = 1 \) indicate that consumer \( i \) buys a car and \( y_i = 0 \) consumer \( i \) does not buy a car. Consumer \( i \) has income \( z_i \). Suppose the utility to consumer \( i \) of a car is

\[
u_{i1} = \beta_0 + \beta_1 z_i + \varepsilon_{i1}\]

and the utility of no car is

\[
u_{i0} = \varepsilon_{i0}.
\]

Assume that \((\varepsilon_{i0}, \varepsilon_{i1})\) is distributed i.i.d. type I extreme value. Assume that income is distributed in the population according to the exponential distribution \( F(z, \lambda) = 1 - e^{-\lambda z} \) with density \( f(z, \lambda) = e^{-\lambda z} \) and mean \( E_z = \frac{1}{\lambda} \).

(a) Suppose the unknown parameter vector is \( \theta = (\beta_0, \beta_1, \lambda) \). Suppose the data is a sample \( \{(z_i, y_i), i = 1, ... N\} \) uniformly drawn from the population. Write down the log likelihood likelihood function.

(b) Let

\[
s_1 = E[y]
\]
where expectations are taken over \((z_i, \varepsilon_i0, \varepsilon_i1)\). Suppose that \(s_1\) is known. So our data is a sample \(\{(z_i, y_i), i = 1, ...N\}\) as well as \(s_1\). Construct three moments that you could use to estimate the parameter vector \(\theta = (\beta_0, \beta_1, \lambda)\) with GMM.

(c) Now suppose the data comes in a different form. You have two data sets. One is a random sample of individuals who bought a car, \(y_i = 1\) for all \(i\). You observe \(z_i^1\) for all these individuals. The superscript \(1\) denotes this is from data set 1. The second data is random sample from the population and from this we get \(z_i^2\) for all \(i\) in data set 2. So the data sets are \(\{z_i^1, i = 1, ..N_1\}\) and \(\{z_i^2, i = 1, ..N_2\}\). Finally, assume you also know \(s_1\). Explain a procedure to consistently estimate \(\theta = (\beta_0, \beta_1, \lambda)\).

\[P(y = 1|x) = \Phi(\beta_0 + \beta_1 x + \beta_2 x^2).\]

The partial effect of \(x\) on the response probability at \(x^\circ\) is

\[\frac{\partial P(y = 1|x^\circ)}{\partial x_2} = \phi(\beta_0 + \beta_1 x^\circ + \beta_2 x^\circ^2) (\beta_1 + 2\beta_2 x^\circ),\]

where \(\Phi\) is the standard normal c.d.f and \(\phi\) is the density.

(a) How would you estimate the partial effect of \(x\) at \(x^\circ\)?

(b) How would you estimate the standard error of the estimated partial effect? (Don’t use a bootstrap. Rather appeal to asymptotic results)

Question 3

This question is a very simple example of an estimation strategy proposed by Bajari, Benkard, and Levin (2004). Time is continuous. The interest rate is \(r\). There are exactly 2
firms in an industry. Let the integers \( n_1 \) and \( n_2 \) denote the states of firm 1 and 2, respectively. Suppose a firm is either in state 0, 1, or 2 and furthermore \( n_1 + n_2 = 2 \). Hence knowing the state of one of the firms is sufficient for knowing the state of the industry. If firm \( i \) is in position \( n_i < 2 \) and it spends on investment at rate \( C(I_i) \), it has a Poisson hazard \( I_i \) of moving to state \( n_i + 1 \). (Hence, the other firm transits from state \( 2 - n_i \) to state \( 2 - n_i - 1 \).) Denote the profit flow (gross of investment) of firm 1 by \( \pi(n_1) \) and denote its value function by \( V(n_1) \).

Let \( \bar{I}(n) \) the equilibrium policy of firm 1 given state \( n \) in a Markov-perfect equilibrium. Restrict attention to symmetric MPE so that the policy of firm 2 is \( \bar{I}_2(n) = \bar{I}(1-n) \). One can readily verify that in a Markov-perfect equilibrium, the policy function and value functions for firm 1 satisfy

\[
rV(0) = \max_I \{-C(I) + I[V(1) - V(0)]\}.
\]

\[
rV(1) = \max_I \{\pi(1) - C(I) + I[V(2) - V(1)] - \bar{I}(1)[V(1) - V(0)]\}.
\]

\[
rV(2) = \pi(2) - \bar{I}(0)[V(2) - V(1)]
\]

(a) Explain how to use observations on the evolution of industry structure over time to infer the decision rule at each value of the state: \( \bar{I}(0) \) and \( \bar{I}(1) \). From gross profit data for the industry you can infer \( \pi(1) \) and \( \pi(2) \). Together, these are the stage I estimates.

(b) Suppose the cost of investment function is \( C(I) = \alpha_0 + \alpha_1 I + \alpha_2 I^2 \), where \( \alpha_0 \), \( \alpha_1 \), and \( \alpha_2 \) are unknown parameters. Show that these parameters are not identified just using observations on \( I \) and investment spending \( C \). Instead, show how to use your stage I estimates, data on investment spending \( C \), and the structure of the problem to infer \( \alpha_0 \), \( \alpha_1 \), and \( \alpha_2 \).