Econ 8601—Fall 2006—Take-Home Final

First, take as long as you need to read the exam. Then allow yourself 4 hours to comlete the exam. Do 2 out of 3 questions. Feel free to use your notes or read any references. But don't work with other students.

Question 1

Consider an industry with a final good and two intermediate inputs. Let z denote units of the final good and let y_1 and y_2 denote levels of interemediate 1 and intermediate 2. Assume one unit of final good production requires one unit each of intermediate 1 and interemediate 2.

There are two inputs to the industry, input 1 and input 2, plus entrepreneurship input. Let x_1 and x_2 denote units of inputs 1 and 2. Input 1 is used for intermediate 1 and input 2 is used for intermediate 2. Entrepreneurs vary in productivity parameter θ. An entrepreneur of ability θ hiring inputs x_1 and x_2 produces $y_1 = \theta x_1$ and $y_2 = \theta x_2$ of the intermediate goods. So input 1 is used to make intermediate 1 and input 2 is used to make intermediate 2. Entrepreneurs face a *management constraint* that

$$x_1 + x_2 \leq 2.$$

An entrepreneur of type θ who enters the industry must choose whether to be

- *vertically integrated producer.* $x_1 = x_2 = x$. This firm produces $y = \theta x$ of each intermediate input and combines them to produce θx units of final good.
- *intermediate 1 specialist.* $x_1 > 0$ and $x_2 = 0$. This firm produces $y_1 = \theta x_1$ of intermediate 1 and sells this into the open market.
- *outsourcer.* $x_1 = 0$ and $x_2 > 0$. This firm produces $y_2 = \theta x_2$ units of intermediate input 2 on its own and buys (or outsources) $y_1 = \theta x_2$ units of intermediate 1 on the open market to produce θx_2 units of final good.

The distribution of entrepreneurial productivity θ is given by $F(\cdot)$ with continuous density $f(\theta)$ on the support $\theta \in [0, \overline{\theta}]$. Assume the outside option of an entrepreneur is zero. Assume the industry faces a perfectly elastic demand at a price p. The industry faces perfectly elastic supplies of factor 1 and factor 2 at the same input price w.

Without the additional complication added below, it is straightforward to solve for the competitive equilibrium of this industry. Given the symmetry of the two inputs, we can assume that the firms in the industry are vertically integrated final good producers. Any type θ firm that enters sets $x_1 = x_2 = 1$ to satisfy the management constraint that $x_1 + x_2 \leq 2$. With one unit of each input, the firm producers θ units of final good. Given the exogenous output and input prices, total revenue is $p\theta$ and total costs are $2w$. The critical level of θ for entry into the industry is where $p\theta = 2w$ or $\hat{\theta} = 2w/p$. So $\theta > \hat{\theta}$ enter, $\theta < \hat{\theta}$ do not and industry supply is

$$Q = \int_{\hat{\theta}}^{\overline{\theta}} f(\theta)\theta d\theta$$

Suppose an open market were to exist in intermediate input 1 and let m_1 denote the price per unit. Obviously, $m_1 = p/2$. At this price, entrepreneurs would be indifferent to the three organization forms.

Now the complication. There is an imperfection in the market for input 1. The suppliers of factor 1 have the ability to grab a fraction $\gamma > 0$ of the *profit* of a firm. Anticipating that workers will grab output, a firm hiring input 1 can offer a lower wage $\tilde{w}_1(\theta) < w$ that can depend upon type θ. For a vertically integrated firm of type θ, total payments to input 1 will be $\tilde{w}_1(\theta) + \gamma\left(p\theta - \tilde{w}_1(\theta) - w\right)$ (The firm is hiring 1 unit of input 1 and 1 unit of input 2). For an intermediate 1 specialist of type θ, total payments to input 1 will be $2\tilde{w}_1(\theta) + \gamma\left(m_1 2\theta - 2\tilde{w}_1(\theta)\right)$. (The firm is hiring 2 units of input 1, and producing 2θ units of intermediate 1 which it sells for m_1 per unit). So total payment to input one per unit hired is $\tilde{w}_1(\theta) + \gamma\left(m_1\theta - \tilde{w}_1(\theta)\right)$.

(a) Suppose it is feasible for $\tilde{w}_1(\theta)$ to be negative. Then the equilibrium described earlier where all firms are vertically integrated is still an equilibrium with suitable choice of $\tilde{w}_1(\theta)$. What does $\tilde{w}_1(\theta)$ have to be in this case?

(b) Suppose that $\tilde{w}_1(\theta) \geq 0$ must always hold. Determine a critical value $\hat{\gamma}$, such that if $\gamma < \hat{\gamma}$, the complete vertical integration equilibrium in (a) is still an equilibrium but if $\gamma > \hat{\gamma}$ it is not an equilibrium.

(c) Assume $\gamma = \hat{\gamma} + \varepsilon$, for *small* $\varepsilon > 0$. Construct an equilibrium in which some firms are specialized and some firms are vertically integrated depending upon θ. Derive the equations determining which type θ does what.

(d) The parameter γ governs the amount of "holdup" in this industry. Compare the relationship between hold up and vertical integration in this model with the relationship in standard IO theories of vertical integration.

Question 2

Consider the following competitive industry with three stages. In stage 1, a measure of firms N enters the industry. Each entering firm pays a fixed cost ϕ and draws a productivity parameter $\theta \in [0, \overline{\theta}]$ from a common continuous distribution $F(\theta)$ with positive density $f(\theta)$ on the support $[0, \overline{\theta}]$. There is an unlimited supply of potential entrants to this industry, all *ex ante* identical.

In stage 2, after observing its own initial productivity draw θ°, each firm chooses whether to pay $\kappa > 0$ dollars to increase productivity to $\theta = \lambda\theta^{\circ}$, where $\lambda > 1$, or to not incur this cost in which case productivity remains at the initial level θ°.

In stage 3, there is a competive market for the industry. The production function of a firm with productivity θ is $q = \theta g(x)$, where x is the amount of labor employed by the firm and q is the output and $g(x)$ increasing and strictly concave. The demand curve for the industry is $Q^D = D(p)$, where p is the industry price. Assume that $D(p) > 0$, $D'(p) < 0$ for all p, and $\lim_{p\to\infty} D(p) = 0$. Assume the supply of labor to this industry is perfectly elastic at wage w. Let y denote the fraction of firms entering in stage 1 that invest to increase productivity in stage 2.

(a) Assume the cost κ is small enough that in equilibrium, the fraction y investing to increase productivity is strictly positive. Characterize the competitive equilibrium in this economy.

(b) Suppose demand doubles to $\tilde{D}(p) = 2D(p)$. How does this change the competitive equilibrium?

(c) Suppose you interested in estimating the parameters of this industry. Suppose you directly observe: p and w. Suppose $g(x)$ has the form $g(x) = x^{\alpha}$ for $\alpha < 1$. Suppose for each firm in the industry you observe employment x and output q. How would you estimate α, λ, κ, and the distribution of initial productivity draws $F(\cdot)$?

Question 3:

(Note: This is an example of a Williamson type holdup model discussed in class. We did part (a) and (b) in lecture, so (c) is really the new part.)

There are two kinds of individuals, type A and type B. The measure of type A individuals is N_A and the measure of type B individuals is N_B, where $N_A < .5N_B$.

There are two periods, $t = \{1, 2\}$. Each individual is endowed with a single labor unit in each period. The discount factor is $\beta = 1$.

There are two technologies for producing the final good, the regular technology and the special technology. With the regular technology, a type j individual can produce q_j units of the final good with a single unit of time. Assume that $q_A > q_B$ so that a type A individual has an absolute advantage in production with the regular technology.

The special technology works as follows. In period 0, a type A individual builds a factory (type B individuals cannot build factories). Factories vary in quality i. To build a factory of quality i requires i units of time in period 0. A type A individual building a factory of quality i uses the balance $1 - i$ of his or her time endowment in period 0 to make the final good (and thus produce $(1 - i)q_A$ units in period 0).

A factory produces no final good in period 0. A factory will produce output in period 1 if an individual uses his or her unit time endowment to manage the factory in period 1. When a factory is build in period 0, it is customized to be managed in period 1 by a *particular* individual. If this particular individual manages the factory in period 1, the output in period 1 is $f(i) + q_S$, where $f(0) = 0$, $f'(0) > 0$, and $f''(0) < 0$. It does not matter whether this particular individual is a type A or type B person; the output is the same in either case. It also makes no difference whether the particular individual is the type A person who built the plant or whether the particular individual is someone else.

But there is a difference if the person who manages the factory is different from the person the factory was customized for. In this case, the output is q_S instead of $f(i) + q_S$; i.e., the investment i is wasted.

(a) Suppose that contracts are complete and can specify a publicly observable investment level i. Show how the equilibrium allocation in this economy is determined. Under what conditions is the special technology used?

(b) Consider the possibility where in period 0 a type-A person named x customized a factory to be managed by another person y in period 1. Suppose no binding contracts can be written at date 0 (the Williamson case) Use the Nash bargaining solution to model the bargaining process at date 1. Assume that the weight on agent x is α and the weight on agent y is $1-\alpha$. Find the conditions that determine whether either (1) *specialization* occurs where a type A person customizes a factory for a type B person or (2) *integration* occurs where a type A person customizes a factory for himself to later manage or (3) the special technology is not used.

(c) Now make the following changes in the model. Suppose a type A person can customize a factory for *two* different B people (but only one can use it), say B_1 and B_2. One of these two B people will turn out to be the *efficient* producer with productivity $f(i) + q_S$ from operating the factor. The other will be the *inefficient* producer with productivity $f(i)+q_S-\mu$ for $\mu \geq 0$. Suppose the probability is .5 that B_1 is the efficient producer and B_2 is the inefficient producer. The probability things are reversed is .5. Suppose that rather than Nash Bargaining, the two B people simultaneously submit sealed bids to the the type A person for the factory. Show that if $\mu = 0$, the first-best investment level obtains in equilibrium. Next determine the equilibrium for μ close to zero.