

Econ 8601, Fall 2006 (due Tuesday, Sept. 26)

Homework 1

Question 1 (old prelim question). Consider the following model of differentiated products industry. Suppose differentiated goods are indexed by x and let $q(x)$ be a quantity of good x . There is a composite industry good that is CES in the differentiated products,

$$Q = \left[\int_0^\infty q(x)^{\frac{1}{2}} dx \right]^2,$$

i.e., the elasticity of substitution is 2. Suppose consumers have an inelastic demand for \bar{Q} units of the composite good.

There is a fixed cost $\phi > 0$ dollars of entering the industry to produce a particular differentiated product. The marginal cost is one dollar.

(a) Suppose the government sets a price floor $\lambda > 2$, such that the price of any differentiated product x in this industry is required to satisfy $p(x) \geq \lambda$. Determine the equilibrium under this policy. How does this policy affect the welfare of consumers and firms?

(b) In the above, we assume that the fixed cost ϕ is constant for all products. Now assume that the fixed cost $\phi(x)$ depends upon x and that $\phi(x)$ is strictly increasing in x . Redo parts (a) and (b) under this alternative assumption.

Question 2

Consider the following duopoly model. There are two firms, 1 and 2, and time is discrete, $t \in \{0, 1, 2, \dots\}$. Let $q_{1,t}$ and $q_{2,t}$ be the output of each firm in period t and let $Q_t = q_{1,t} + q_{2,t}$ be total output. Suppose the inverse demand is constant over time and linear, $P = A - Q$.

The average production cost in period t is constant in current production but depends upon *industry* production in the previous period. Let $X_t = Q_{t-1}$ denote total industry output from the previous period. Specifically, the marginal cost (and average cost) in period t of each firm is $c(X_t)$, where $c(0) \equiv \bar{c} < A$, $c' < 0$, $c'' > 0$, and $\lim_{X \rightarrow \infty} c(X) \equiv \underline{c} > 0$. So given last period industry output X , if firm i produces q_i units in the current period, its total cost in the period is $q_i c(X)$. Note the learning by doing here occurs at the industry

level. There is a *knowledge spillover* here since firm 2's production cost next period are smaller when firm 1 produces more in the current period.

Suppose the discount factor is $\beta < 1$.

Assume in each period t the two firms simultaneously choose output levels $q_{1,t}$ and $q_{2,t}$ in a Cournot fashion.

(a) Define a Markov-perfect equilibrium in this model. Define a stationary equilibrium.

(b) Suppose $\beta = 0$. Determine the transition equation mapping last period's industry output X to this period's industry output Q . Under what condition does there exist a unique stationary equilibrium?

(c) Consider a two-period version of the model, $t \in \{1, 2\}$ and let $X_1 = 0$ be the initial state. If is possible that an equilibrium path for $\beta > 0$ would be the same as the equilibrium path when $\beta = 0$?

Question 3

Take the dynamic industry model discussed in class. Assume the parameterization

$$\begin{aligned} c(q) &= \frac{q^2}{2} \\ D(p) &= p^{-\varepsilon_D} = p^{-2} \\ P(Q) &= Q^{-\frac{1}{2}} \end{aligned}$$

where ε_D is the elasticity of demand. Assume $\beta = .5$ and $\sigma = 1 - \delta = .5$. Following the class notes:

$$\begin{aligned} q^* &= \frac{1}{\sigma} = 2 \\ p_C^* &= (1 - \beta)c'(q^*) + \beta\sigma c(q^*) \\ &= .5q^* + .25\frac{q^{*2}}{2} = 1.5 \\ p_M^* &= \frac{\varepsilon_D}{\varepsilon_D - 1}p_C^* = 3 \\ Q_C^* &= p_C^{*-2} \\ Q_M^* &= p_M^{*-2} \\ K_C^* &= \sigma Q_C^* \\ K_M^* &= \sigma Q_M^* \end{aligned}$$

(a) Use value function iteration and Chebyshev approximation (page 223 in Judd) to calculate the equilibrium value function for the monopoly problem.

Use $n = 5$ (the order of the polynomials) and $m = 10$ (the number of grid points). Let $a = .5K_M^*$ and $b = 1.5K_M^*$; these are the endpoints of the grid using Judd's notation.

Iterate on the vector (a_0, a_1, \dots, a_n) which is the vector determining the approximation of $w(K)$ (Sorry for the awkward notation where a denotes two things; this is Judd's fault). Start with $a_i = 0$ for all i and stop when

$$\max_{i \in \{0, n\}} |a_i^{t+1} - a_i^t| < .000001$$

where t denotes a particular iteration.

After the value function converges approximate the policy function $q(K)$. Let the initial capital level be $K_0 = a = .5K_M^*$ and calculate for periods 1-25 the following variables: K_t , q_t , P_t and $w_t(K_t)$. Compare with K_M^* , q^* , P_M^* and w_M^* , the stationary monopoly levels.

(b) Let (a_0, \dots, a_n) be the coefficient vector for the value function $v_1(K_1, K_2)$ approximation and (b_0, \dots, b_n) the coefficient vector for the policy function $q_1(K_1, K_2)$ approximation. Use Judd's techniques for approximation in R^2 (page 238) to approximate the Markov perfect equilibrium. Note you need to iterate on q_1 as well as v_1 since firm 1 takes firm 2's action as given in the problem (and $q_2(x, y) = q_1(y, x)$).

Let $a = .25K_M^*$ and $b = K_C^*$ be the end points of the grid.

Solve for the equilibrium path for the first 25 periods starting at $K_{1,0} = b$ and $K_{2,0} = a$. Print out $q_{1,t}$, $q_{2,t}$, $K_{1,t}$, $K_{2,t}$, and P_t . What happens to market share over time?