Econ 8601–Graduate Industrial Organization (Fall 1997)

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Paper: Hugo Hopenhayn, "Entry, Exit, and Firms Dynamics in Long Run Equilibrium," *Econometrica 60*, Sept. 1992., 1127-1150.

1. Model

Partial equilibrium model of an industry

- $P(Q)$ inverse demand function
- Production function $q = \phi h(n)$, $\phi \in [0,1]$ productivity parameter, n employment. Assume $h' > 0$, $h'' < 0$, $\lim_{n \to 0} h'(n) = \infty$.
- ϕ follows a Markov process

  $$\phi_{t+1} \text{ distributed } F(\cdot, \phi_t)$$

  where $\frac{\partial F}{\partial \phi} < 0$
- Assume that for each $\varepsilon > 0$ and ϕ_t there exists an n such that $F^n(\varepsilon|\phi_t) > 0$, where $F^n(\varepsilon|\phi_t)$ is what the distribution of ϕ_{t+n} would be if exit were infeasible.
- The exists a fixed cost $c_f > 0$ to remain in the market
- There is a cost of entry $c_e > 0$. Entrants draw from a distribution G.

2. Timing

			Exit and get 0
Incumbent	Observes ϕ_t	Pays fixed cost c_f	or
		Sets q to max π	Stay in and draw ϕ_{t+1}
New entrant	pays c_e	sames as incumbent	

3. Stationary Equilibrium

Set of objects:

- Price p
- μ measure of types ϕ of incumbents at the beginning of the period
- M measure of new entrant to enter in the period

  That satisfy

- Supply equals demand in the output market
- Firms maximize profits in output decisions and exit decisions
- Entry condition holds(return to entry is zero of $M > 0$ and otherwise nonpositive).
- The exit and entry behavior implies the invariant measure μ.

4. Individual Behavior

(1) Production decision:

$$\max_n p\phi h(n) - wn - c_f$$

The FONC is

$$p\phi h'(n) - w = 0$$

Let $n(\phi, p)$ solve this problem. Let $q(\phi, n) = p\phi h(n(\phi, n))$ be the optimal quantity and let $\pi(\phi, p)$ be the maximized profit.

(2) Exit decision

$$v(\phi, p) = \pi(\phi, p) + \max\left\{0, \beta \int_0^1 v(\phi', p) f(\phi'|\phi) d\phi'\right\}$$

Standard dynamic programing arguments show a solution $v(\phi, p)$ exists and is strictly increasing in ϕ and p. (Note: this claim uses the fact that an increase in ϕ shifts the distribution of ϕ' in a first-order stochastic dominance fashion.) Let $E(\phi, p)$ be the expected return to staying,

$$E(\phi, p) = \beta \int_0^1 v(\phi', p) f(\phi'|\phi) d\phi'$$

This is strictly increasing in p and ϕ. Suppose that $E(1,p) > 0$ and $E(0,p) < 0$. Then let $x(p)$ be the unique point in $(0,1)$ satisfying

$$E(x(p),p) = 0$$

This is the value of ϕ where the individual is just indifferent to staying or leaving. If $E(1,p) \leq 0$, then let $x(p) = 1$ and if $E(0,p) > 0$ let $x(p) = 0$. It the cutoff $x(p)$ is not at a corner it is strictly increasing in p.

(3) Entry Decision. The return to entry is

$$\int_0^1 v(\phi,p)g(\phi)d\phi - c_e$$

The first term is plotted in figure 1. Let p^* be the unique price where the above is zero.

5. The Stationary Distribution

Focus on case where $x^* = x(p^*) > 0$. (If $x(p^*) < 0$ there exist equilibria with no entry or exit. Equilibrium will depend upon the initial stock of firms) In the case where $x(p^*) > 0$ there is a unique stationary equilibirum. The stationary price is p^* and the quantity is $Q^* = D(p^*)$.

What is the stationary distribution of firms?

- Let μ_t be the distribution of types at time t.
- γ the distribution of entrants given a unit measure of entry.
- $M\gamma$ distribution of entrant given a mass M of entry.
- $\hat{P}_x$ mapping that first truncates all $\phi < x$ and then runs it through F

The equilibrium distribution of firms must satisty the stationarity condition:

$$\mu^* = \hat{P}_{x^*}\mu^* + M^*\gamma$$

Or, rewriting, it solves:

$$\left[\hat{P}_{x^*} - I\right]\mu^* = M^*\gamma$$

or

$$\mu^* = \left[\hat{P}_{x^*} - I\right]^{-1} M^* \gamma$$

It also must satisfy the product market equilibrium condition

$$p^e(\mu^*) = p^*$$

where $p^e(\mu)$ is defined as the price solving

$$\int_0^1 q(p, \phi)\mu(\phi)d\phi = D(p)$$

In summary, to solve for the equilibrium do the following: (1) Take p^* as the price solving the free-entry condition. Then find the flow of entrants M^* so that the following holds:

$$p^e(M^* \left[\hat{P}_{x^*} - I\right]^{-1} \gamma) = p^*$$

6. Example

Suppose two types $\phi_1 = 0$, $\phi_2 = 1$. Suppose the distribution function satisfies

$$\begin{pmatrix} f_{11} & f_{12} \\ f_{21} & f_{22} \end{pmatrix} = \begin{pmatrix} 1 & 1 - f_{22} \\ 0 & f_{22} \end{pmatrix}$$

In this example, type 1 always exits.

$$v_1(p) = \pi_1(p) = -c_f$$

Assume that demand is strong enough so that in equilibrium type 2 stays in and there is positive entry each period.

$$v_2 = \pi_2 + \beta(1 - f_{22})v_1 + \beta f_{22} v_2$$

Or

$$v_2 = \frac{1}{1 - \beta f_{22}}\pi_2 + \frac{\beta(1 - f_{22})}{1 - \beta f_{22}}(-c_f)$$

The equilbrium p^* can be found from figure 3

For this special case, $\hat{P}_{x^*}$ mapping is

$$\begin{aligned} \hat{P}_{x^*} &= \begin{pmatrix} f_{11} & f_{12} \\ f_{21} & f_{22} \end{pmatrix} \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \\ &= \begin{pmatrix} 1 & 1 - f_{22} \\ 0 & f_{22} \end{pmatrix} \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 1 - f_{22} \\ 0 & f_{22} \end{pmatrix} \end{aligned}$$

Recall there are two parts of this mapping. The first part is the selection part. Firms with $\phi = \phi_1$ are shut down. This is accounts for the second term above. The second part is the firm goes through the F processing mapping states this period to states next period. This is the first term above.

7. Applications of the Model

A. Firm Dynamics

Fact: Examine a cohort of entering firms and follow survivors. The average size of the survivors increases. The probability of discontinuance decreases.

Model: Look at special case.

Period	Measure in state		Prob survive
	ϕ_1	ϕ_2	
1	$M\gamma_1$	$M\gamma_2$	γ_2
2	$(1-f_{22})M\gamma_2$	$f_{22}M\gamma_2$	f_{22}

To be consistent with the empirical literature need $f_{22} > \gamma_2$. This also implies average size increases.

In the general model analogous mechanical conditions are needed. The distribution of new entrants can't be too good compared with the transition function F.

B. A Cross Section of Industries

Study effects of changes in c_e and c_f on equilibrium variables:

Variable	Definition	$\Delta c_e > 0$	$\Delta c_f > 0$
price	p	+	+
cutoff	x	-	? (+ under condition)
average firm size	$\frac{\int_0^1 q(p,\phi)\mu(\phi)d\phi}{\int_0^1 \mu(\phi)d\phi}$		+ (under condition)
k concentration	$\frac{\int_{\phi_k}^1 q(p,\phi)\mu(\phi)d\phi}{\int_0^1 q(p,\phi)\mu(\phi)d\phi}$ where ϕ_k defined by $k = \frac{\int_{\phi_k}^1 \mu(\phi)d\phi}{\int_0^1 \mu(\phi)d\phi}$		
profit	$\frac{\int_0^1 \pi(p,\phi)\mu(\phi)d\phi}{\int_0^1 \mu(\phi)d\phi}$		
Tobin's q	$\frac{\int_0^1 v(p,\phi)\mu(\phi)d\phi}{c_e \int_0^1 \mu(\phi)d\phi}$		

Condition referred to above: Condition U.2 The profit function is separable $\pi(p,\phi) = y(\phi)z(p)$.

Figure 1

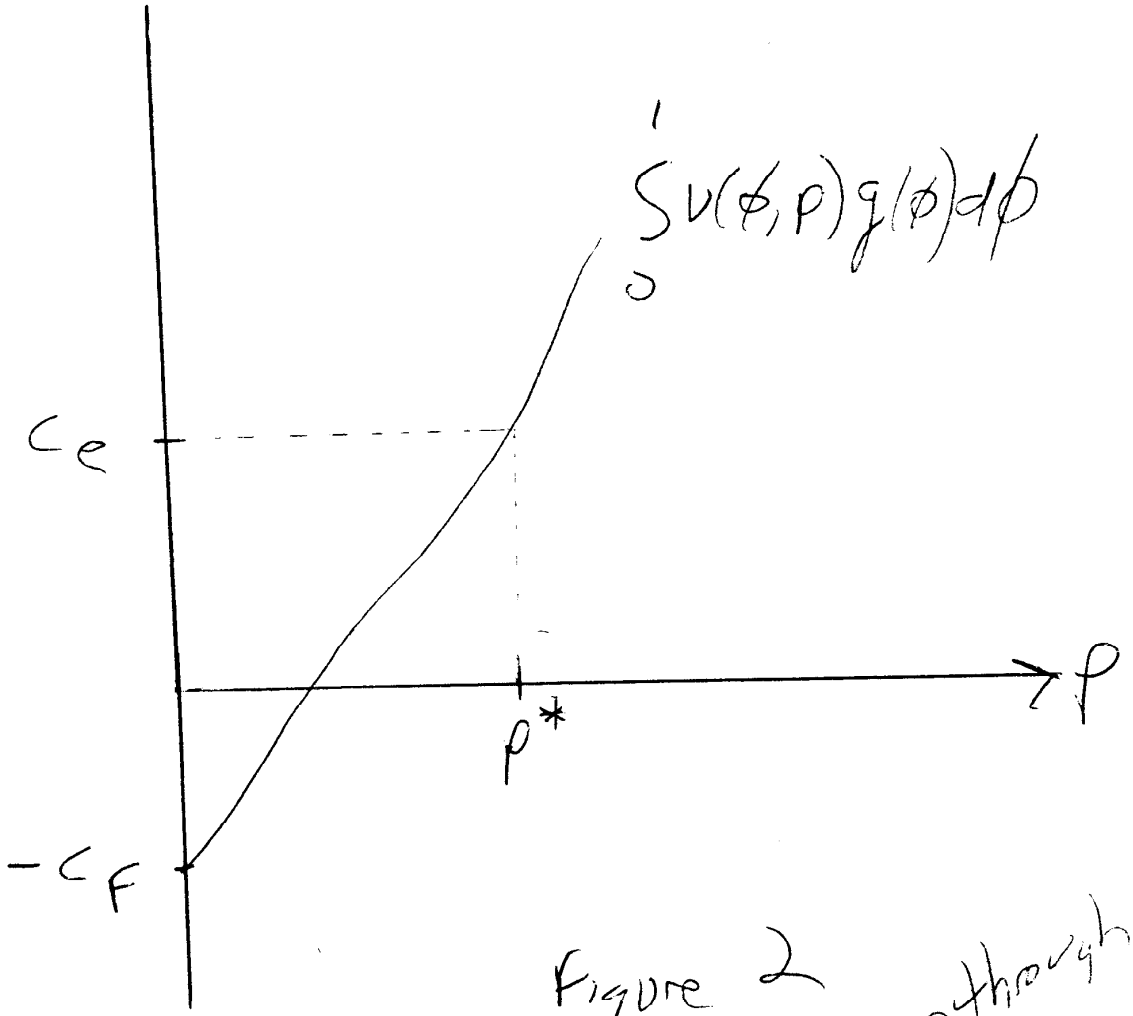

Figure 2

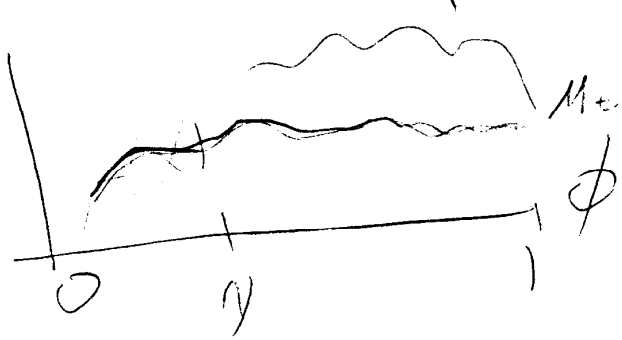

Figure 3
Special case