Econ 8601–Graduate Industrial Organization (Fall 1997)
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Class notes for Sept. 30, 1997
Paper: Hugo Hopenhayn, “Entry, Exit, and Firms Dynamics in Long Run Equilibrium,”

1. Model
Partial equilibrium model of an industry

- $P(Q)$ inverse demand function
- Production function $q = \phi h(n)$, $\phi \in [0,1]$ productivity parameter, $n$ employment.
  Assume $h' > 0$, $h'' < 0$, $\lim_{n \to 0} h'(n) = \infty$.
- $\phi$ follows a Markov process

\[
\phi_{t+1} \text{ distributed } F(\cdot; \phi_t)
\]

where $\frac{\partial F}{\partial \phi} < 0$

- Assume that for each $\varepsilon > 0$ and $\phi_t$ there exists an $n$ such that $F^n(\varepsilon|\phi_t) > 0$, where $F^n(\varepsilon|\phi_t)$ is what the distribution of $\phi_{t+n}$ would be if exit were infeasible.
- The exists a fixed cost $c_f > 0$ to remain in the market
- There is a cost of entry $c_e > 0$. Entrants draw from a distribution $G$.

2. Timing

<table>
<thead>
<tr>
<th>Incumbent</th>
<th>Observes $\phi_t$</th>
<th>Pays fixed cost $c_f$ or</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Sets $q$ to max $\pi$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Stay in and draw $\phi_{t+1}$</td>
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</tbody>
</table>

Exit and get 0

New entrant pays $c_e$ same as incumbent
3. Stationary Equilibrium

Set of objects:

- Price $p$
- $\mu$ measure of types $\phi$ of incumbents at the beginning of the period
- $M$ measure of new entrant to enter in the period

That satisfy

- Supply equals demand in the output market
- Firms maximize profits in output decisions and exit decisions
- Entry condition holds (return to entry is zero of $M > 0$ and otherwise nonpositive).
- The exit and entry behavior implies the invariant measure $\mu$.

4. Individual Behavior

(1) Production decision:

$$\max_n p\phi h(n) - wn - c_f$$

The FONC is

$$p\phi h'(n) - w = 0$$

Let $n(\phi, p)$ solve this problem. Let $q(\phi, n) = p\phi h(n(\phi, n))$ be the optimal quantity and let $\pi(\phi, p)$ be the maximized profit.

(2) Exit decision

$$v(\phi, p) = \pi(\phi, p) + \max \left\{ 0, \beta \int_0^1 v(\phi', p) f(\phi' | \phi) d\phi' \right\}$$

Standard dynamic programing arguments show a solution $v(\phi, p)$ exists and is strictly increasing in $\phi$ and $p$. (Note: this claim uses the fact that an increase in $\phi$ shifts the distribution of $\phi'$ in a first-order stochastic dominance fashion.) Let $E(\phi, p)$ be the expected return to staying,

$$E(\phi, p) = \beta \int_0^1 v(\phi', p) f(\phi' | \phi) d\phi'$$
This is strictly increasing in $p$ and $\phi$. Suppose that $E(1, p) > 0$ and $E(0, p) < 0$. Then let $x(p)$ be the unique point in $(0, 1)$ satisfying

$$E(x(p), p) = 0$$

This is the value of $\phi$ where the individual is just indifferent to staying or leaving. If $E(1, p) \leq 0$, then let $x(p) = 1$ and if $E(0, p) > 0$ let $x(p) = 0$. It the cutoff $x(p)$ is not at a corner it is strictly increasing in $p$.

(3) Entry Decision. The return to entry is

$$\int_0^1 v(\phi, p)g(\phi)d\phi - c_e$$

The first term is plotted in figure 1. Let $p^*$ be the unique price where the above is zero.

5. The Stationary Distribution

Focus on case where $x^* = x(p^*) > 0$. (If $x(p^*) < 0$ there exist equilibria with no entry or exit. Equilibrium will depend upon the initial stock of firms) In the case where $x(p^*) > 0$ there is a unique stationary equilibrium. The stationary price is $p^*$ and the quantity is $Q^* = D(p^*)$.

What is the stationary distribution of firms?

- Let $\mu_t$ be the distribution of types at time $t$.
- $\gamma$ the distribution of entrants given a unit measure of entry.
- $M\gamma$ distribution of entrant given a mass $M$ of entry.
- $\hat{P}_x$ mapping that first truncates all $\phi < x$ and then runs it through $F$

The equilibrium distribution of firms must satisfy the stationarity condition:

$$\mu^* = \hat{P}_x^*\mu^* + M^*\gamma$$

Or, rewriting, it solves:

$$[\hat{P}_x^* - I]\mu^* = M^*\gamma$$
or

\[ \mu^* = \left[ \hat{P}_x^* - I \right]^{-1} M^* \gamma \]

It also must satisfy the product market equilibrium condition

\[ p^e(\mu^*) = p^* \]

where \( p^e(\mu) \) is defined as the price solving

\[ \int_0^1 q(p, \phi)\mu(\phi)d\phi = D(p) \]

In summary, to solve for the equilibrium do the following: (1) Take \( p^* \) as the price solving the free-entry condition. Then find the flow of entrants \( M^* \) so that the following holds:

\[ p^e(M^* \left[ \hat{P}_x^* - I \right]^{-1} \gamma) = p^* \]
6. Example

Suppose two types $\phi_1 = 0$, $\phi_2 = 1$. Suppose the distribution function satisfies

$$\begin{pmatrix} f_{11} & f_{12} \\ f_{21} & f_{22} \end{pmatrix} = \begin{pmatrix} 1 & 1 - f_{22} \\ 0 & f_{22} \end{pmatrix}$$

In this example, type 1 always exits.

$$v_1(p) = \pi_1(p) = -c_f$$

Assume that demand is strong enough so that in equilibrium type 2 stays in and there is positive entry each period.

$$v_2 = \pi_2 + \beta(1 - f_{22})v_1 + \beta f_{22}v_2$$

Or

$$v_2 = \frac{1}{1 - \beta f_{22}}\pi_2 + \frac{\beta(1 - f_{22})}{1 - \beta f_{22}}(-c_f)$$

The equilibrium $p^*$ can be found from figure 3.

For this special case, $\hat{P}_x^*$ mapping is

$$\hat{P}_x^* = \begin{pmatrix} f_{11} & f_{12} \\ f_{21} & f_{22} \end{pmatrix} \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 1 - f_{22} \\ 0 & f_{22} \end{pmatrix} \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 1 - f_{22} \\ 0 & f_{22} \end{pmatrix}$$

Recall there are two parts of this mapping. The first part is the selection part. Firms with $\phi = \phi_1$ are shut down. This is accounts for the second term above. The second part is the firm goes through the $F$ processing mapping states this period to states next period. This is the first term above.
7. Applications of the Model
A. Firm Dynamics

Fact: Examine a cohort of entering firms and follow survivors. The average size of the survivors increases. The probability of discontinuance decreases.

Model: Look at special case.

<table>
<thead>
<tr>
<th>Period</th>
<th>Measure in state</th>
<th>Prob survive</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$M\gamma_1$</td>
<td>$M\gamma_2$</td>
</tr>
<tr>
<td>2</td>
<td>$(1 - f_{22})M\gamma_2$</td>
<td>$f_{22}M\gamma_2$</td>
</tr>
</tbody>
</table>

To be consistent with the empirical literature need $f_{22} > \gamma_2$. This also implies average size increases.

In the general model analogous mechanical conditions are needed. The distribution of new entrants can’t be too good compared with the transition function $F$.

B. A Cross Section of Industries

Study effects of changes in $c_e$ and $c_f$ on equilibrium variables:

<table>
<thead>
<tr>
<th>Variable</th>
<th>Definition</th>
<th>$\Delta c_e &gt; 0$</th>
<th>$\Delta c_f &gt; 0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>price</td>
<td>$p$</td>
<td>+</td>
<td>+</td>
</tr>
<tr>
<td>cutoff</td>
<td>$x$</td>
<td>-</td>
<td>? (+ under condition)</td>
</tr>
<tr>
<td>average firm size</td>
<td>$\frac{\int_0^1 q(p,\phi)\mu(\phi)d\phi}{\int_0^1 \mu(\phi)d\phi}$</td>
<td>+ (under condition)</td>
<td></td>
</tr>
<tr>
<td>$k$ concentration</td>
<td>$\frac{\int_0^1 q(p,\phi)\mu(\phi)d\phi}{\int_0^1 q(p,\phi)\mu(\phi)d\phi}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>where $\phi_k$ defined by $k = \frac{\int_0^1 \mu(\phi)d\phi}{\int_0^1 \mu(\phi)d\phi}$</td>
<td></td>
</tr>
<tr>
<td>profit</td>
<td>$\frac{\int_0^1 \pi(p,\phi)\mu(\phi)d\phi}{\int_0^1 \mu(\phi)d\phi}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Tobin’s q</td>
<td>$\frac{\int_0^1 v(p,\phi)\mu(\phi)d\phi}{c_e \int_0^1 \mu(\phi)d\phi}$</td>
<td></td>
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</tr>
</tbody>
</table>

Condition referred to above: Condition U.2 The profit function is separable $\pi(p, \phi) = y(\phi)z(p)$. 
Figure 1

\[ \sum \Phi \pi \rho \phi \]  

Figure 2

Run through $F$
\[
\text{Example:}\ [\text{Function}]
\]

\[
\text{Graph:}
\]

\[\text{Analysis:}
\]

\[\text{Conclusion:}
\]

\[\text{References:}
\]