1. Introduction

In the 1920s, Henry Ford famously built a factory in which the raw materials for steel came in on one end and finished automobiles came out the other. But extreme vertical integration such as this is not the fashion today. Ford Motor has recently spun off a significant portion of its parts making operations as a separate company and General Motors has done the same. While General Motors has not attempted to spin off final assembly plants, they are currently trying to outsource to outside contractors the janitorial services within an assembly plant as well as forklift operations. Northwest Airlines has recently begun outsourcing the cleaning of airplanes to outside contractors.

The mantra underlying much of the discussion about outsourcing by business consultants is that companies need to focus on their core competencies and they should consider outsourcing the rest. This idea of specializing according to comparative advantage would be very familiar to Ricardo. Surely comparative advantage based on production efficiency is a major force behind outsourcing. But it is not the only force and it has little to do with why General Motors wants to outsource janitorial services. GM’s motivation is no secret and is not disputed: Janitors who work for General Motors are members of the United Auto Workers and they get a union salary of $28 an hour; janitors through an outside contractor would receive approximately $12. Analogously, Northwest Airlines is hiring outside contractors to clean airplanes because the contractors’ wages are lower than the union wages Northwest would be paying if they did the job themselves.

This paper develops a theory of outsourcing in which the circumstances under which factors of production can grab rents play the leading role. One factor has some monopoly

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power (call this *labor*) while a second factor does not (call this *capital*). There are two
stages of production, a labor intensive task and a capital intensive task. All firms are the
same in *core competencies* and a transaction friction is incurred when the two tasks are
not integrated in the same firm. So in the absence of any monopoly power by labor, all
firms are completely integrated doing the labor intensive and capital intensive tasks in the
same facility. But if labor has sufficient monopoly power, there is vertical disintegration
with some firms specializing in the labor intensive task and other firms specializing in the
capital intensive task. The firms doing only the labor-intensive task are more sensitive to
wage increases than a vertically-integrated firm would be, weakening the bargaining power
of labor. Now firms that choose to specialize in just the capital intensive task pay higher
wages than they would if vertically integrated. But in the end, it is cheaper for firms doing
the capital intensive task to outsource the labor intensive task rather than do it themselves.

In what we have described so far, all firms are ex ante the same. But if firms differ in ways
that relate to rent extraction by labor, outsourcing is all the more likely to occur. There
is sorting in a vertical equilibrium with those most vulnerable to rent extraction avoiding
labor by specializing in the the capital intensive task. In this environment, outsourcing is
bad for labor—if the transaction friction were large enough to preclude outsourcing, labor
would be able to extract more rents.

There is a growing recent literature on outsourcing in the context of international trade
where it is called offshoring. Papers such as Antras, Garican, and Rossi-Hansberg (2006) and
Helpman and Rossi-Hansberg (2006) highlight specialization based on comparative advan-
tage. Another recent literature examines how information flows affect integration decisions
(Alonso, Dessein and Matouschek (2006), Friebel and Raith (2006)).

The largest literature on the subject focuses on how incomplete contracts and relationship-
specific investment affect the organization of the firm. The early literature (Williamson
(1979)) focused on how the choice of the organization of the firm determines possibilities for
ex post opportunistic behavior. A supplier making an investment specific to an upstream
producer needs to worry about being “held up” after the fact. Klein, Crawford, and Alchian
(1978) made famous the 1926 merger of General Motors with Fisher Body as an example
of what they argued was a vertical merger to eliminate holdup. Our paper follows this
literature in that how firms are organized affects the opportunities for rent extraction. The difference here is that integration doesn’t necessarily eliminate opportunistic behavior, it can potentially increase it. Certain factors of production may be in a better position to grab rents within the boundaries of the firm than outside the boundaries. On this point, it is worth noting the argument by Freeland (2000) that after GM acquired Fisher Body, it put the Fisher brothers who were now employees of GM in a better position to grab rents from GM that they exploited.

The subsequent literature on incomplete contracts focused on how the allocation of property rights affects ex ante investment incentives (Grossman and Hart (1986), Hart and Moore (1990)). In our analysis, we can think of the choice to be vertically disintegrated as an investment. If all residual rights of control can be allocated to the labor union before decisions are made about vertical disintegration then the first best is obtained where there is no outsourcing and no transaction friction incurred. Put in another way, if the union owned the firm, nothing interesting would happen in our model. We abstract from this possibility and we appeal to institutional features of labor law for justification. We argue that if one union took ownership of a firm and began to operate it like a profit-maximizing enterprise, it would get voted out by workers. Essential in our analysis is a commitment problem. There is no way for workers to commit in advance not to try to extract rents. Outsourcing is a way to address the commitment problem.

While more work needs to be done, the existing empirical evidence suggests that limiting rent extraction is a significant factor behind important components of outsourcing activity. The contract cleaning industry mentioned above in the context of General Motors and Northwest Airlines is a good place to start looking for this factor. Abraham (1990) shows that building service employees employed in the business service sector (e.g. contract cleaning firms) receive substantially lower wages and benefits than the employees doing the same jobs employed by manufacturing firms. And Abraham and Taylor (1996) show that it is the higher wage firms who are more likely to contract out cleaning services. They don’t find a connection between unionization and tendency to outsource, but they note that a union might be powerful enough to prevent a firm from doing the outsourcing they otherwise would want to do. The fact that General Motors is currently paying workers $28 an hour to do
janitorial work is evidence on this point. In contrast to Abraham and Taylor, Autor (2003) does find a positive connection between outsourcing and unionism. He finds that states with slower declines in unionism have had more rapid growth in temporary help services. There is also evidence for specific industries. Forbes and Lederman (2005) discuss how airlines spin off short routes to regional airlines because the pilots of these airlines are less able to extract rents. Doellgast and Greer (2007) provide a case study of the German automobile industry for how outsourcing parts has cut rents. We don’t know of such a study for the U.S. automobile industry but since General Motors has spun off its Delphi parts operations, Delphi is trying to cut “wages from as much as $30 an hour to as little as $10 an hour.”

The empirical evidence that this factor matters is strong enough that in motivating the paper we view our main job is not to convince the reader this factor matters. It does. Rather we need to address the following two issues. (1) Since this seems like such a first-order issue, hasn’t this paper already been written? and (2) If firms can bust unions by outsourcing do we really need a theory paper to show they will do this? In addressing the first issue we cannot find a theory paper that treats outsourcing like we do. To the best of our knowledge the literature has focused on holdup by producers but has ignored employees. In addressing the second point, we believe there to be many subtleties in our analysis. One is that while the fragmentation of production lowers the wage paid to workers doing the labor intensive task, it actually raises the wages paid to workers to the capital intensive task. So there are offsetting effects and it is not obvious a priori how these settle out. Working this out requires working through the industry equilibrium; this is not a decision theoretic analysis. A second subtlety comes up regarding how a firm’s decision to specialize depends upon a measure of the firm’s rents. We find that there is sorting but it is not always a strict montonic relationship. For some parameter regions there may be pooling where firms over a certain range of rents are indifferent about what task they engage in.

\[2\] The quote is from the New York Times, Nov. 19, 2005, "For a G.M Family, the American Dream Vanished," by Danny Hakim.
2. Model

We model an industry in which a final good is made in two steps, one producing a labor intensive intermediate good and the other producing a capital intensive intermediate good. A firm may be vertically integrated and execute both steps itself, or it may be specialized in the production of either intermediate good. Each intermediate good \( q_i \), for \( i = 1, 2 \), is made with the following technology

\[
q_i = y_i (y_1 + y_2)^{(1-\gamma)}
\]

where \( \gamma \in (0, 1) \) gives diminishing returns to scale which apply to the sum of the firm’s production of the two intermediate goods. This ensures that there is no motive for vertical disintegration owing solely to the decreasing returns to scale. In turn, \( y_i \) is produced from capital \( k \) and labor \( l \) as follows

\[
y_i = f_i (k, l) = \frac{1}{\alpha_i^{\alpha_i} (1 - \alpha_i)^{1-\alpha_i} \rho_i k^{1-\alpha_i}}.
\]

We assume that task 1 is labor intensive, while task 2 is capital intensive, and that the intensities are symmetric (although the latter is not a crucial assumption):

\[
\alpha_1 \equiv \alpha > 1 - \alpha \equiv \alpha_2
\]

When it is clear which intermediate good we are referring to, we will suppress the \( i \) subscript. The final good results from combining the two intermediate goods, according to the technology

\[
q = \min \{ q_1, q_2 \}.
\]

The price of the final good is normalized to 1, and the price of intermediate good \( i \) is \( p_i \), with \( p_1 + p_2 = 1 \). Firms choose amounts of labor and capital to maximize profits. Maximized profits are denoted \( \pi_i (w) \), where \( w \) is the wage. Each firm also receives an additive amount of rents, \( \theta \), which is distributed with density \( h (\theta) \) on support \( [0, \theta] \). Firms observe their rents \( \theta \) before choosing their organization structure.

There is a labor union located at each firm which observes the firm’s type \( \theta \) and organization structure (vertically integrated, specialized in task 1, or specialized in task 2), and
offers a wage. The union is able to go on strike, and in doing so, dissipate a share $\delta \in [0,1]$ of firm profits.

Facing a wage $w$, a firm can either accept and receive

$$\pi_i (w) + \theta$$

or the firm can break the union and receive

$$(1 - \delta) (\pi^o_i + \theta)$$

where $\pi^o_i$ are the profits earned at the outside wage $w^o$.

Each labor union is a monopolist supplier of labor to the firm at which it is located. Workers not employed by the union have the option of working at an outside wage $w^o$. The union takes this wage as the opportunity cost of labor, and so its problem is

$$\max_w (w - w^o) l(w)$$

s.t. $$(1 - \delta) (\pi^o_i + \theta) \leq \pi (w) + \theta.$$  

Here $l(w)$ represent the firm’s labor demand as a function of the wage $w$.

The unconstrained solution to the union’s problem is to pick a wage $w^*$ which satisfied the inverse elasticity pricing rule:

$$\frac{w^* - w^o}{w^*} = \frac{1}{\varepsilon}$$

where $\varepsilon$ is the elasticity of labor demand. In the case of the specialized firm, we have $\varepsilon = \frac{1-(1-\alpha)\gamma}{1-\gamma}$, which yields

$$w^* = \left(1 + \frac{1-\gamma}{\gamma\alpha}\right) w^o.$$  

The constrained solution to the union’s problem is to choose a wage $\overline{w}(\theta, \delta)$ which satisfies

$$(1 - \delta) (\pi^o + \theta) = \pi (\overline{w}(\theta, \delta)) + \theta.$$  

In other words, the union never asks for more than it can get. If the firm would rather withstand a strike than accept the unconstrained wage, then the union will instead offer the constrained wage, which makes the firm indifferent to a strike.
Firms anticipate the wage that they will be charged by the union when making their organization decision. A firm choosing to vertically integrate receives

\[ v_{VI} = \pi_{VI} + \theta. \]  

(11)

A firm choosing to specialize in step \( i \) receives

\[ v_i = \pi_i + \theta - \tau \]

(12)

where \( \tau \in [0, \infty] \) is a transaction cost paid by non-vertically integrated firms. Firms compare profits across the three organization structures and choose the structure which gives the highest profits.

The timing is as follows. First, each firm learns the amount of its rents \( \theta \). Firms then choose an organization structure. The union sees the amount of rents and the organization structure, and uses this information to offer a wage \( w \). The firm either accepts the wage or rejects it, losing a fraction \( \delta \) of profits, but it may then employ labor at the (lower) outside wage \( w^o \). The firm then hires labor and capital and produces the good.

### 2.1 Analysis of a Specialized Firm

Conditional on having specialized in step \( i \), the firm’s problem is straightforward. The firm receives the wage offer from the union and decides whether to accept it or reject it and withstand a strike. The firm then hires labor and capital and produces. However, as mentioned previously, the union never offers a wage so high that the firm would reject it. Whether the firm will be offered the unconstrained wage or a constrained wage depends on the union’s strength \( \delta \) and the firm’s rents \( \theta \). In particular, the firm will be charged the constrained wage if

\[ (1 - \delta) (\pi^o + \theta) \geq \pi^* + \theta \]

where \( \pi^* \) denote profits at the constrained wage. Equivalently, the firm will be charged the constrained wage if its rents \( \theta \) are such that

\[ \theta \leq \tilde{\theta} (\delta) \equiv \delta^{-1} [(1 - \delta) \pi^o - \pi^*]. \]

(13)

Profits at the constrained wage are

\[ \pi (\overline{w}(\theta, \delta)) = (1 - \delta) \pi^o - \delta \theta \]
Maximized profits at the unconstrained wage are

$$\pi^* = (1 - \gamma) \left( \frac{\gamma \gamma^p}{(w^u)^{\alpha \gamma} r^{(1-\alpha)\gamma}} \right)^{\frac{1}{1-\gamma}}$$

$$= (1 - \gamma) \left( \frac{\gamma^p}{\left(1 + \frac{1-\gamma}{\gamma \alpha} (w^o)^{\alpha \gamma} r^{(1-\alpha)\gamma} \right)^{\frac{1}{1-\gamma}}} \right).$$

Notice that if $w^o \geq r$ and $p_1 = p_2$, then $\pi^*$ is strictly decreasing in $\alpha$. To see this observe that we can rewrite $\pi^*$ as

$$\pi^* = (1 - \gamma) (\gamma^p)^{\frac{1}{1-\gamma}} \left(1 + \frac{1-\gamma}{\gamma \alpha}\right)^{-\frac{\alpha \gamma}{1-\gamma}} [(w^o)^{\alpha \gamma} r^{-(1-\alpha)\gamma}]^{\frac{1}{1-\gamma}}.$$ 

The first two terms are positive constants. The third term is strictly decreasing in $\alpha$ if and only if

$$\left(1 + \frac{1}{x}\right)^{-x}$$

is strictly decreasing in $x$. This can be verified numerically. Finally, the fourth term is decreasing in $\alpha$ if and only if

$$(w^o)^{-\alpha} r^{-(1-\alpha)}$$

if decreasing in $\alpha$. Differentiating with respect to $\alpha$, we see that this will be the case if and only if

$$w^{-\alpha} r^{-(1-\alpha)} (\ln r - \ln w^o) \leq 0$$

which holds if and only if

$$r \leq w^o.$$ 

This shows that at the unconstrained wage, the higher a firm’s labor intensity, the more the union will be able to dent its profits.

Maximized profits at the outside wage are

$$\pi^o = (1 - \gamma) \left( \frac{\gamma \gamma^p}{(w^o)^{\alpha \gamma} r^{(1-\alpha)\gamma}} \right)^{\frac{1}{1-\gamma}}$$

Obviously $\pi^*_i$ and $\pi^o_i$ depend on the relative prices of the two intermediate goods, which are endogenous to the model and also depend on $\theta$ and $\delta$. 

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Hence we may define the value function of a firm specialized in task $i$ as follows:

$$v_i(\theta, \delta) = \begin{cases} 
\pi_i^* + \theta - \tau & \text{for } \theta \in [0, \bar{\theta}(\delta)], \\
(1 - \delta) (\pi_i^* + \theta) - \tau & \text{for } \theta \in [\bar{\theta}(\delta), \bar{\theta}] 
\end{cases}$$

### 2.2 Analysis of an Integrated Firm

When examining the case of a vertically integrated firm, we encounter the difficulty that the integrated firm’s problem is in general not analytically solvable. Hence we focus on the special case with $\alpha = 1$ for which we do obtain analytic solutions, and rely on numerical calculations to fill out the analysis.

The vertically integrated firm’s problem is analogous to that of the specialized. $\bar{\theta}(\delta)$ is defined in the same way. Using $\alpha = 1$ to solve the vertical firm’s problem analytically, we obtain

$$w_{VI}^* = \gamma^{-1} [(1 - \gamma) r + w^o]$$

$$\pi_{VI}^* = \frac{1}{2} (1 - \gamma) \left( \frac{\gamma}{w^o + r} \right)^{\frac{1}{1 - \gamma}}$$

$$\pi_{VI}^o = \frac{1}{2} (1 - \gamma) \left( \frac{\gamma^2}{w^o + r} \right)^{\frac{1}{1 - \gamma}}$$

Its value function is given by

$$v_{VI}(\theta, \delta) = \begin{cases} 
\pi_{VI}^* + \theta & \text{for } \theta \in [0, \bar{\theta}(\delta)], \\
(1 - \delta) (\pi_{VI}^o + \theta) & \text{for } \theta \in [\bar{\theta}(\delta), \bar{\theta}] 
\end{cases}$$

Notice that the vertically integrated firm avoids the transaction costs.

### 2.3 Equilibrium

Equilibrium in the industry requires an equal supply of each of the two intermediate goods. Formally, an equilibrium consists of prices $\{p_1, p_2\}$, union wages $w_i(\theta, \delta)$, firm integration decisions (given by cutoff rules in $\theta$ and $\delta$), and firm input demands $l_i(w), k_i(w)$, such that
firms maximize profits, unions maximize profits, and there is an equal supply of each of the two intermediate goods.

3. Outsourcing to Limit Rent Extraction

A key section showing that outsourcing takes place.

(1) Look at limiting case: all same $\theta$ all same $\delta$. Let $w_0 \geq r$ be the outside wage. (Wage after a strike). Assume all firms have same $w_0$. Firms can vary in $\tau$ but assume $\tau \sim [0, \bar{\tau}]$ with continuous density and that there is positive density as $\tau$ goes to zero. Show that for small enough $\tau$, firms disintegrate. This should be a theorem for the case of $\alpha_1 = 0$ and $\alpha_2 = 1$. This is Proposition 1 of the paper. We should use computer analysis to check the case of $\alpha_1 \in (0, \frac{1}{2})$ to see if we can extend the proposition in this way.

(2) Next assume heterogeneity in $\theta$. Start with heterogeneity in $\theta$, for fixed $\delta$. Characterize the equilibrium as proposition 2. Next show that increasing the variance in $\theta$ increases VI. Assume all have same $\theta$, call it $\bar{\theta}$. Now suppose half have $\theta_L = \bar{\theta} - \Delta$ and half have $\theta_H = \bar{\theta} + \Delta$ (or something like that). Show that for a given $\tau$ (maybe all have the same), the outsourcing more likely. In other words. Assume all have the same $\tau$. Let $\hat{\tau}$ be the cutoff such that if $\tau \leq \hat{\tau}$ then firms specialize. Let $\hat{\tau}(\Delta)$. Conjecture: $\hat{\tau}$ increases in variance.

(3) Next assume heterogeneity in $\delta$. Assume half have $\delta = 0$ and half have $\delta = 2\bar{\delta}$ so the mean is $\bar{\delta}$. Easy to see that transactions friction $\hat{\tau}$ is bigger here than if all firms have the same $\bar{\delta}$. this is very easy to work out and can be done in a paragraph. If half of the firms have $\delta = 0$, then these half specialize in the labor task and pay $w_0$. So effect is same as no firms having union power.

In summary: without heterogeneity in vulnerability to rent extraction, we get a vertical equilibrium with outsourcing if the transactions friction is low enough. With more heterogeneity, get even more outsourcing.

In this section we show in a variety of environment that firms will opt to vertically disintegrate to limit rent extraction by unions.

To begin, suppose that all firms have the same rents $\theta$ and that all firms face the same union strength $\delta$. Suppose that $w^\alpha \geq r$, and that $w^\alpha$ is the same for all firms. Assume that firms vary in the transaction costs that they face. Specifically, assume that $\tau$ is distributed
on $[0, \tau]$ with continuous density $g(\tau)$ such that $\lim_{\tau \to 0} g(\tau) > 0$.

**Proposition 1:** Suppose the above assumptions hold, that $\alpha = 1$, and that $w^o \geq r$. There exists a $\tilde{\tau}(\delta) \geq 0$ such that any firm with $\tau < \tilde{\tau}(\delta)$ will vertically disintegrate. Moreover, if $\delta$ is high enough, then $\tilde{\tau}(\delta)$ is strictly positive.

**Proof:** Firms will vertically disintegrate if they make higher profits in that fashion than they do vertically integrated. Notice that in equilibrium we cannot have only one task being done by vertically disintegrated firms. Equilibrium intermediate goods prices must adjust so that disintegrated firms are indifferent between the two tasks. This ensures that there we can obtain an equal supply of the two intermediate goods, as required by the Leontief final good technology.

Since $\theta$ is homogenous, we will have that all firms of the same organization structure face either the constrained wage or the unconstrained wage. This will depend on $\delta$, and will be determined by 13. We can invert 13 for each organization structure to obtain a cutoff level of $\delta$ as a function of $\theta$. Specifically, we obtain

$$\tilde{\delta}_i(\theta) = \frac{\pi^o_i - \pi^*_i}{\pi^o_i + \theta}$$

If $\delta \geq \tilde{\delta}_i(\theta)$ for $i = 1, 2, VI$, then a type $i$ firm will face the unconstrained union wage. If $\delta < \tilde{\delta}_i(\theta)$, then the union is weaker and the firm will face the constrained union wage.

If $\delta \geq \max\{\tilde{\delta}_1(\theta), \tilde{\delta}_2(\theta), \tilde{\delta}_{VI}(\theta)\}$, then firms of all three organization types will face the unconstrained union wage. Let $p^*_1$ be the price which sets $\pi^*_1(p^*_1) = \pi^*_2(1 - p^*_1)$, and let $p^*_2 \equiv 1 - p^*_1$. (We need $\pi^*_1 = \pi^*_2$ so that if any firms disintegrate in equilibrium, there will be an equal supply of the two intermediate goods.) Using

$$\lim_{x \to 0} \left[1 + \frac{1}{x}\right]^{-x} = 1$$

(14) to obtain $\pi^*_2$ at $1 - \alpha = 0$, it is easy to calculate that

$$p^*_1 = \frac{(w^o)^{\gamma}}{(w^o)^{\gamma} + \gamma r^\gamma},$$

and

$$p^*_2 = \frac{\gamma r^\gamma}{(w^o)^{\gamma} + \gamma r^\gamma}.$$
A firm will be indifferent between vertical integration and specialization when
\[ v_{VI}(\theta, \delta) = v_1(\theta, \delta) = v_2(\theta, \delta). \]

Notice that the indifference condition can be stated this way for any value of \( \theta \) or \( \delta \) since we require \( v_1(\theta, \delta) = v_2(\theta, \delta) \) so that any specialized firms will create an equal supply of the two intermediate goods. In the high \( \delta \) case, this means that we need
\[ \pi^*_{VI} + \theta = \pi^*_1 + \theta - \tau \]

So define the \( \tau \) to be the level of transaction costs that makes a firm indifferent between integration and specialization:
\[
\tilde{\tau} = \pi^*_1 - \pi^*_{VI} \\
= (1 - \gamma) \left( \frac{\gamma^\gamma p^*_1}{(w^o)^\gamma} \right)^{\frac{1}{1-\gamma}} - \frac{1}{2} (1 - \gamma) \left( \frac{\gamma^2}{w^o + r} \right)^{\frac{1}{1-\gamma}} \\
= (1 - \gamma) \gamma^{2\gamma \frac{1}{1-\gamma}} \left[ \left( \frac{1}{1+\gamma} \right)^\frac{1}{1-\gamma} (w^o)^{-\frac{1}{1-\gamma}} - \frac{1}{2} (w^o + r)^{-\frac{1}{1-\gamma}} \right] \\
\geq (1 - \gamma) \gamma^{2\gamma \frac{1}{1-\gamma}} (w^o)^{-\frac{1}{1-\gamma}} \left[ \left( \frac{1}{1+\gamma} \right)^\frac{1}{1-\gamma} - 2^{-\frac{1}{1-\gamma}} \right] \\
> 0
\]

The second equality uses \( \alpha = 1 \), and the third equality follows by substituting in for \( p^*_1 \). The first inequality uses \( w^o \geq r \), and the last inequality follows since \( \gamma \in (0, 1) \) and thus \( \frac{1}{1+\gamma} > \frac{1}{2} \). Firms with transaction costs lower than \( \tilde{\tau} \) will choose to specialize, and firms with transaction costs higher than \( \tilde{\tau} \) will be vertically integrated. It remains to show whether \( \tilde{\tau} > 0 \) for lower values of \( \delta \).

Since \( \tilde{\delta}_i(\theta) \) is the lowest value of \( \theta \) for which a firm of structure \( i \) faces the unconstrained wage, it follows that at \( \delta = \max \{ \tilde{\delta}_1(\theta), \tilde{\delta}_2(\theta) \} \), \( p^*_1 \) and \( p^*_2 \) will still be equilibrium prices. We can use this fact to infer that \( \delta_1(\theta) > \tilde{\delta}_2(\theta) \). Specifically, at prices \( p^*_1 \) and \( p^*_2 \), simple algebra gives that
\[
\pi^o_1(p^*_1) > \pi^o_2(p^*_2) > \pi^*_1(p^*_1) = \pi^*_2(p^*_2),
\]
and hence
\[ \frac{\pi_1^o(p_1^*)}{\pi_2^o(p_2^*)} > \frac{\pi_1^o(p_1^*)}{\pi_1^o(p_1^*) + \theta} \]
or equivalently
\[ \tilde{\delta}_1(\theta) \equiv \frac{\pi_1^o(p_1^*) - \pi_1^*(p_1^*)}{\pi_1^o(p_1^*) + \theta} > \frac{\pi_2^o(p_2^*) - \pi_2^*(p_2^*)}{\pi_2^o(p_2^*) + \theta} \equiv \tilde{\delta}_2(\theta). \]

This tells us that as we lower \( \delta \), task 1 firms will face the constrained wage first. Recall that
\[ \pi_1(w_1(\theta, \delta); p_1) = (1 - \delta) \pi_1^o(\theta) - \delta \theta \]
and
\[ \pi_2(w_2^*(\theta, \delta); 1 - p_1) \]
for \( \delta \geq \tilde{\delta}_1(\theta) \),
\[ \pi_1(w_1(\theta, \delta); p_1) = \pi_2^*(1 - p_1) \]
for \( \tilde{\delta}_2(\theta) \leq \delta < \tilde{\delta}_1(\theta) \), and
\[ \pi_1(w_1(\theta, \delta); p_1) = \pi_2(w_2(\theta, \delta); 1 - p_1) \]
for \( \delta < \tilde{\delta}_2(\theta) \). So \( \tilde{p}_1(\delta) = p_1^* \) and \( \tilde{p}_2(\delta) = p_2^* \) for \( \delta \geq \tilde{\delta}_1(\theta) \).

From this point forward in the proof, we compare the vertically integrated firm’s profits with a type 2 firm’s profits since in the \( \alpha = 1 \) case they only depend on \( \delta \) through \( p_2 \). So define
\[ \pi_2(\delta) = \gamma \frac{\gamma}{1 - \gamma} (1 - \gamma) \frac{\gamma}{\gamma} \frac{1}{p_2(\delta)} \frac{1}{r - \gamma} \]
for \( \delta \in [0, 1] \) using again the limit result 14 and
\[ \lim_{\alpha \rightarrow 0} \left[ 1 + \frac{1 - \gamma}{\gamma \alpha} \right]^{-\frac{(1 - \alpha)\gamma}{1 - \gamma}} = 0. \]

When \( \delta \) is high, we need to look at
\[ \pi_{VI}^*(\delta) + \theta = \pi_2(\delta) + \theta - \tau \]
\[ \tilde{\tau} (\delta) = \pi_2 (\delta) - \pi^*_{VI} (\delta). \]

When \( \delta \) is low, we need to look at
\[ \pi_{VI} (\delta) + \theta = \pi_2 (\delta) + \theta - \tau \]
\[ \tilde{\tau} (\delta) = \pi_2 (\delta) - \pi_{VI} (\delta), \]
where the bar over \( \pi \) indicates that these are profits at the union’s constrained wage

Considering first the high \( \delta \) case, we know that \( \tilde{\tau} (\delta) > 0 \) because
\[ \pi_2 (p_2 (\delta)) \geq \pi^*_1 (p_1^*) = \pi^*_2 (p_2^*) > \pi^*_{VI}. \]

Now considering the low \( \delta \) case, we know that for \( \delta < \hat{\delta} (\theta) \),
\[ \pi^*_1 < \pi_{VI} (\delta). \]

So we compare \( \pi^*_1 (p_1^*) \) with \( \pi_{VI} (\delta) \).
\[
(1 - \gamma) \left( \gamma^\gamma p_1^* \right) \frac{1}{1 - \gamma} - \frac{1}{2} (1 - \gamma) (1 - \delta) \left( \frac{\gamma}{w^o + r} \right) \frac{1}{1 - \gamma} - \delta \theta \geq 0
\]
\[ \Leftrightarrow \delta \left( \frac{1}{2} (1 - \gamma) \left( \frac{\gamma}{w^o + r} \right) \frac{1}{1 - \gamma} - \theta \right) \geq \frac{1}{2} (1 - \gamma) \left( \frac{\gamma}{w^o + r} \right) \frac{1}{1 - \gamma} - (1 - \gamma) \left( \frac{\gamma^\gamma p_1^*}{(1 - \gamma) (w^o)} \right) \frac{1}{1 - \gamma}\]
This looks intractable.

Instead directly compare \( \pi_2 (p_2 (\delta)) \) with \( \pi_{VI} (\delta) \). Use
\[
\pi_{VI} (\delta) = \frac{1}{2} (1 - \gamma) (1 - \delta) \left( \frac{\gamma}{w^o + r} \right) \frac{1}{1 - \gamma} - \delta \theta
\]
to get
\[
(1 - \gamma) \left( \gamma^\gamma \hat{p}_2 (\delta) \right) \frac{1}{1 - \gamma} - \frac{1}{2} (1 - \gamma) (1 - \delta) \left( \frac{\gamma}{w^o + r} \right) \frac{1}{1 - \gamma} - \delta \theta \geq 0
\]
\[\Leftrightarrow \delta \left( \frac{1}{2} (1 - \gamma) \left( \frac{\gamma}{w^o + r} \right) \frac{1}{1 - \gamma} - \theta \right) \geq \frac{1}{2} (1 - \gamma) \left( \frac{\gamma}{w^o + r} \right) \frac{1}{1 - \gamma} - (1 - \gamma) \left( \frac{\gamma^\gamma \hat{p}_2 (\delta)}{r^\gamma} \right) \frac{1}{1 - \gamma}\]

Look at \( \delta = 0 \).
\[
(1 - \gamma) \left( \frac{\gamma^\gamma \hat{p}_2 (\delta)}{r^\gamma} \right) \frac{1}{1 - \gamma} \geq \frac{1}{2} (1 - \gamma) \left( \frac{\gamma}{w^o + r} \right) \frac{1}{1 - \gamma}\]
Firms should be indifferent between integration and specialization at \( \delta = 0 \). This follows from the above inequality if \( r = w^o \). When \( r \neq w^o \), it is not so clear.

The case with \( \alpha \in (\frac{1}{2}, 1) \) should be checked on the computer.

4. Changes over Time

This does comparative statics with the model parameters.

(1) First issue what happens when \( \tau \) decreases. Say goes from \( \tau = \infty \) to something else. From previous section we know we get vertical integration. here we have new proposition as to how this affects union rents. Again have proposition for \( \alpha_1 = 0, \alpha_2 = 1 \) case, can appeal to the computer for the general case.

(2) Next look at what happens with changes in union power.

(i) Start with what happens when \( w_0 \) decreases?

(ii) What happens when \( \delta \) decreaes. Distinuish between a cut in \( \delta \) for all firms and a cut in \( \delta \) for half the firms. In the first, expect that have more integration. In the second, expect less integration. For example, suppose half of the firms get a decrease. What is this? Changes in labor law makes it harder to organize new firms. But unions are still in the old
firms. Now if $\delta$ goes down for half, then these new unorganized firms come in. Eventually, as $\delta$ decreases for all firms (goes to zero) then all firms go back to being integrated.

Another way to model this. Suppose that firms can pay up front a payment $\phi$ so that their $\delta = 0$. Others have $\delta = \delta_0 > 0$. Suppose the before $\phi = \infty$. Now unions are weak so that $\phi$ is small. Interpretation, $\phi$ is cost to shut down operations and reopen. Now no need to shut down if one is specializing in the capital intensive task. So leave these guys open.

In this discussion have a paragraph talking about how unions are an absorbing state. Argue that if Toyota had build a car plant in Kentucky in 1920 rather than 1985 if would be union now. But new environment makes it harder to unionize new plants.

See if this result can be extended to general $\alpha$.

5. Conclusion

Maybe mention example of Wal-Mart outsourcing the distribution centers of distribution centers. Cite Brenner, Eidlin, and Candaele that for California distribution centers (where threat of unionization is the greatest) Wal-Mart uses agency temp employees (p. 27)

6. References


Alonso, Ricardo, Wouter Dessein, and Niko Matouschek “When Does Coordination Require Centralization?” July 2006,


