FDI is massive. As noted in Ramondo and Rodriguez-Clare, worldwide sales of multinationals is on the order of twice that of total world exports. To get started on the topic, let’s begin with Helpman, Melitz, and Yeaple (AER, 2004). It is a relatively straightforward paper that highlights key issues.

1 Helpman, Melitz, and Yeaple (2004)

The paper extends the Melitz model to add a new margin. A firm can pay a fixed cost to setup serve a foreign market through FDI and thereby avoid the fixed cost.

1.1 The model

- $N$ countries use labor in $H$ differentated goods sectors plus one homogeneous goods sector. Let $\beta_h$ be spending share on sector $h$, $1 - \sum \beta_h$ be the residual share for the homogeneous goods sector.

- Country $i$ endowed with $L^i$ units of labor, wage rate is $w^i$ (talk about endogenizing wage later)

- Take a particular sector and leave sector index $h$ implicit.

  - There is a fixed entry cost $f_E$ in units of labor
  
  - On entry, firm draws $a$, labor per unit of output, distribution $G(a)$
  
  - After seeing $a$, can choose to exit. If stay and produce, must pay $f_D$
  
  - If firm want to export, it must pay $f_E$ in units of labor for each foreign market.

  In addition, it faces an iceberg transportation cost of $\tau_{ij}$ to ship from $i$ to $j$. 

A firm with a homebase at $i$ can choose to set up a plant at $j$ and avoid the transportation cost $\tau^{ij}$. This entails a fixed cost $f_I > f_E$. (Note, they do not allow exports out of an FDI plant. That is, a U.S. firm setting up a plant in France can’t use the French plant to export to Belgium.)

Demand for each product derived the usual CES way. The elasticity of substitution is $\varepsilon = 1/(1-\alpha)$.

* Thus firm level demand takes form $q = A^i p^{-\varepsilon}$ from country $i$
* Profit maximizing price $p = \frac{w^i a}{\alpha}$.
* So foreign consumers pay $\frac{w^i a}{\alpha} \tau^{ij}$, $i$ buying from $j$

Assume for all $i$ and $j$

$$\left(\frac{w^i}{w^j}\right)^{\varepsilon-1} f_I > \left(\tau^{ij}\right)^{\varepsilon-1} f_X > f_D$$

### 1.2 Analysis of the Model

- Start with case where $w^i = 1$ for all $i$ (This will be the case if all countries are interior with the homogeneous good, and labor units in each country is suitable scaled.)

- Operating profit of serving domestic market has form

$$\pi^i_D = (p-a)q - f_D = (p-a)A^i p^{-\varepsilon} - f_D$$

$$= \left(\frac{a}{\alpha} - a\right) A^i \left(\frac{a}{\alpha}\right)^{-\varepsilon} - f_D$$

$$= a^{1-\varepsilon} \left(\frac{1}{\alpha} - 1\right) A^i \left(\frac{1}{\alpha}\right)^{-\varepsilon} - f_D$$

$$= a^{1-\varepsilon} B^i - f_D$$
• Additional profits from exporting $i$ to $j$ (i.e., conditioned upon $f_D$ sunk) are

\[
\pi_{X}^{ij} = (p - a\tau^{ij}) A^{i} p^{-\varepsilon} - f_X = (\tau^{ij} a)^{1-\varepsilon} B^{j} - f_X
\]

Note, can think of the $\tau^{ij}$ as an adjustment to the firm’s labor cost of serving market $j$.

• Additional profits from going with FDI (i.e. conditioned upon $f_D$ being sunk) are

\[
\pi_{I}^{ij} = a^{1-\varepsilon} B^{j} - f_I.
\]

• Let $\theta \equiv a^{1-\varepsilon}$ be an index of productivity. Then have

\[
\pi_{X}^{ij}(\theta) = \theta (\tau^{ij})^{1-\varepsilon} B^{j} - f_X
\]
\[
\pi_{I}^{ij}(\theta) = \theta B^{j} - f_I
\]

FDI preferred to exports iff

\[
\theta B^{j} - f_I > \theta (\tau^{ij})^{1-\varepsilon} B^{j} - f_X
\]
\[
\theta \left( B^{j} - (\tau^{ij})^{1-\varepsilon} B^{j} \right) > f_I - f_X
\]

so have a cutoff rule $\hat{\theta}_I$ where indifferent.

• Cutoff where indifferent between exports and no exports

\[
\theta (\tau^{ij})^{1-\varepsilon} B^{j} - f_X = 0
\]
\( \hat{\theta} \) solves this. So

\[
\hat{\theta}_I = \frac{f_I - f_X}{B^j - (\tau^{ij})^{1-\varepsilon} B^j} \\
\hat{\theta}_X = \frac{f_X}{(\tau^{ij})^{1-\varepsilon} B^j}
\]

Finally, standard Melitz cutoff \( \hat{\theta}_D \) cutoff for opening domestic,

\[
\pi^i_D(\hat{\theta}_I) = \theta B^i - f_D = 0
\]

- (i) intercepts: \( 0 > \pi^i_D(0) > \pi^i_X(0) > \pi^i_I(0) \)
- (ii) slopes. If \( B^i = B^j \), \( \pi^i_D(\theta) > \pi^i_X(\theta) \), so this implies \( \hat{\theta}_X > \hat{\theta}_D \).
- (iii) slopes: \( \pi^i_X(\theta) < \pi^i_I(\theta) \). So there exists a \( \hat{\theta}_I \) where indifferent.
- (iv) Evaluate \( \pi_I(\theta) \) at \( \hat{\theta}_X \),

\[
\text{at } \hat{\theta}_E, 0 = \theta (\tau^{ij})^{1-\varepsilon} B^j - f_X \\
> \theta (\tau^{ij})^{1-\varepsilon} B^j - (\tau^{ij})^{1-\varepsilon} f_I \\
= (\tau^{ij})^{1-\varepsilon} [\theta B^j - f_I] \\
= (\tau^{ij})^{1-\varepsilon} \pi^j_I(\theta)
\]

where the inequality follows from \( f_I > (\tau^{ij})^{\varepsilon-1} f_X \), assumed earlier. This implies \( \theta_X < \theta_I \).

- Picture:

- Bring back the cutoff formulas for comparative statics

\[
\hat{\theta}_I = \frac{f_I - f_X}{B^j - (\tau^{ij})^{1-\varepsilon} B^j} \\
\hat{\theta}_E = \frac{f_X}{(\tau^{ij})^{1-\varepsilon} B^j}
\]
Figure 1: If increase $\tau^{ij}$ note that $\hat{\theta}_I$ increases (less likely to want to export), while $\hat{\theta}_E$ decreases (more likely to want to set up a foreign affiliate).

1.3 Exports versus FDI Sales

$$\frac{s_X^{ij}}{s_I^{ij}} = \frac{\tau^{1-\varepsilon} (V(a_X) - V(a_I))}{V(a_I)} = \tau^{1-\varepsilon} \left[ \frac{V(a_X)}{V(a_I)} - 1 \right]$$ (1)

for

$$V(a) = \int_0^a y^{1-\varepsilon} dG(y)$$

Assume Pareto distribution for $\frac{1}{a}$.

$$F(x) = 1 - \left( \frac{b}{x} \right)^k$$

where $b$ is the scale parameter $x \in [b, \infty)$ and $k$ is the shape parameter. $\log x$ has standard deviation $1/k$, For finite variance require $k > 2$. The paper assumes $k > \varepsilon + 1$. 

5
Given a draw $x = \frac{1}{\alpha}$, the sales of domestic firm is proportional to $s = a^{1-\varepsilon} = x^{\varepsilon-1}$. Or

$$(s)^{\frac{1}{\varepsilon-1}} = x$$

So what is distribution of sales?

$$\text{Pr}(\hat{s} \leq s) = \text{Pr}(x^{\varepsilon-1} \leq s)$$

$$= \text{Pr}(x \leq (s)^{\frac{1}{\varepsilon-1}})$$

$$= 1 - \left( \frac{b}{(s)^{\frac{1}{\varepsilon-1}}} \right)^k$$

$$= 1 - b^k \left( \frac{1}{s} \right)^{\frac{k}{\varepsilon-1}}$$

So it looks like distribution of sales is Pareto with shape parameter $k/(\varepsilon - 1)$. However, the paper claims the distribution of firm sales is Pareto with shape parameter $k - (\varepsilon - 1)$. Can’t see where this comes from. Maybe we can figure this out in class.

Let’s try another claim, this one does work and we get a $k - \varepsilon + 1$ term.

$$\frac{V(a_1)}{V(a_2)} = \frac{\int_0^{a_1} y^{1-\varepsilon} dG(y)}{\int_0^{a_2} y^{1-\varepsilon} dG(y)} = \frac{\int_0^{a_1} y^{1-\varepsilon} k b^k y^{k-1} dy}{\int_0^{a_2} y^{1-\varepsilon} k b^k y^{k-1} dy} = \frac{\int_0^{a_1} y^{k-\varepsilon} dy}{\int_0^{a_2} y^{k-\varepsilon} dy}$$

$$= \left[ \frac{y^{k-\varepsilon+1}}{y^{k-\varepsilon+1}} \right]_0^{a_1} = \left[ \frac{y^{k-\varepsilon+1}}{y^{k-\varepsilon+1}} \right]_0^{a_2} = \left( \frac{a_1}{a_2} \right)^{k-(\varepsilon-1)}$$

where note:

$$G(y) = \text{Pr}(a \leq y)$$

$$= \text{Pr} \left( \frac{1}{\alpha} \geq \frac{1}{y} \right)$$

$$= (by)^k$$

$$G'(y) = k b^k y^{k-1}$$
Note that $\frac{1}{a} \geq b$, so $a \leq \frac{1}{b}$.

Let’s plug the above into (1),

$$\frac{s^i_X}{s^j_I} = \tau^{1-\varepsilon} \left[ \frac{V(a_X)}{V(a_I)} - 1 \right]$$

$$= \tau^{1-\varepsilon} \left[ \left( \frac{a_X}{a_I} \right)^{k-(\varepsilon-1)} - 1 \right]$$

Recall the cutoffs from above

$$\hat{\theta}_I = a^{1-\varepsilon}_I = \frac{f_I - f_X}{B^I - (\tau^{ij})^{1-\varepsilon} B^J}$$

$$\hat{\theta}_X = a^{1-\varepsilon}_X = \frac{f_X}{(\tau^{ij})^{1-\varepsilon} B^J}$$

So for symmetric case, derive the key equation

$$\frac{s^i_X}{s^j_I} = \tau^{1-\varepsilon} \left[ \left( \frac{f_I - f_X}{B^I - (\tau^{ij})^{1-\varepsilon} B^J} \right)^{k-(\varepsilon-1)} - 1 \right]$$

$$= \tau^{1-\varepsilon} \left[ \left( \frac{f_I - f_X}{f_X} \frac{1}{(\tau^{ij})^{\varepsilon-1}} \right)^{k-(\varepsilon-1)} - 1 \right]$$

where use $B^i = B^j$. Note that by an earlier assumption $a_I < a_X$, so the first term in the brackets must be greater than 1.

We see in this key equation:

- **proximity-concentration tradeoff.** Increase $f_X$ then falls. I
- Increase $\tau$ then fall? That part looks messy...
- If decrease $k$, then ratio falls. (Smaller $k$ means more dispersion.) That, is more heterogeneity, then more FDI versus exports. This is the key result of the paper, what is new beyond the proximity-concentration tradeoff which is straightforward.
The main job of the rest of the paper is to bring this key equation to data, to test how well it works.

1.4 To Data

Let’s look at the key equation again. Let $FP = f_I - f_X$ be the "plant level" fixed cost. Now take U.S. as the originating country, let $\tau_{h,j}$ be the cost of shipping to $j$ from U.S. in industry $h$. Assume cost of setting up an export channel is the same everywhere.

\[
\frac{s_X^{hj}}{s_I^{hj}} = \tau_{h,j}^{1-\varepsilon} \left[ \left( \frac{FP_h}{f_X} \frac{1}{\tau_{h,1}^{\varepsilon-1}} \right)^{\frac{k_{h} - (\varepsilon k_{h} - 1)}{k_{h} - 1} - 1} \right]
\]

This equation motivates comparative statics relationship. Run a regression. Feed it:

- BEA data on U.S. multinational sales abroad for $s_I^{hj}$
- Standard export data
- Allow $\tau_{h,j}$ to vary across destinations and industries. Use trade data to figure out an estimate of transportation costs and tariffs.
  - To get info on transportation costs, use import data into the United States (assuming here transportation cost of importing and exporting the same. The are two numbers reported, c.i.f. (value including "cost insurance and freight") as well as f.o.b. (free on board). So can take difference for an estimate of physical transportation costs.
  - Can get tariff measures
- Assume $f_X$ is the same for all industries, so don’t have to measure it
- Calculate $FP$ from taking expenditures on nonproduction workers on average by industry in the U.S. Outlandish...(but they like they result)
• Estimate $k - (\varepsilon - 1)$ from estimating Zipf’s plots for individual industries, regress log rank on log sales. Note that with Pareto, the right tail of $x = \frac{1}{a}$ is

$$1 - F(x) = \left(\frac{b}{x}\right)^k$$

$$\ln(1 - F(x)) = k \ln b - k \ln x$$

so if you observed productivity $x$, then $k$ is recovered from such a regression. Analogously, if sales are Pareto, then get $k - (\varepsilon - 1)$ from a Zipf plot of industry sales.

1.5 Results: See Table

1.6 Comments

• Relation to Holmes (2005), sale office location paper?

• How fit in with Holmes and Stevens (2010) line that big firms within narrowly defined industries do different things?

2 Arkolakis, Costinot, Rodríguez-Clare: New Trade Models, Same Old Gains?

Claim that if looking at aggregates, micro data doesn’t change answers for the welfare impacts of trade. Take a demand curve $D(p)$ and calculate consumer surplus at a high price $p^H$. (If $P(Q)$ is inverse demand this is

$$CS(p) = \int_0^{Q(p)} P(Q)dQ.$$ 

If $p$ is lowered to $p^L$, the change in consumer surplus is $CS(p^L) - CS(p^H)$, apart from any micro model that is the foundation for $D(p)$.
The paper works out the Armington example. Than maps Eaton Kortum, Dixit Stiglitz, Chaney, into this same general framework as Armington. That mapping is actually the main interest. However, in the interest of time, we are just going to go over the Armington example

2.1 Armington

Take $n$ countries, $L_i$ is labor in county $i$. CES utility in each county. Associated price index is

$$P_j = \left[ \sum_{i=1}^{n} (w_i \tau_{ij})^{1-\sigma} \right]^{\frac{1}{1-\sigma}}$$

By the way, this is the first time I have ever seen a discussion of the Armington model that I could understand. Country $i$, as part of its endowment, is the only one able to produce in sector $i$. There is some kind of transportation cost shipping this stuff around. Having $\tau_{ij} > 1$ for $i \neq j$ is what gives the preference for the locally-produced good. Note above, we are assuming competitive markets and price of good $i$ at $j$ equals the cost $w_i \tau_{ij}$ to deliver it.

Also, value $X_{ij}$ of country $j$’s total imports from country $i$ is equal to

$$X_{ij} = \left( \frac{w_i \tau_{ij}}{P_j} \right)^{1-\sigma} Y_j$$  \hspace{1cm} (2)

Think of a foreign shock that can impact welfare of country $j$ Let there be a shock to $L_i$ or $\tau_{ij}$ except leave $\tau_{jj}$ alone. Let labor in country $j$ be numeriare, so keeps $w_j$ fixed.

- What is the change in real income $W_j = Y_j/P_j$
- trade balance $Y_j = w_j L_j$, so $d \ln Y_j = 0$ (since $L_j$ fixed $w_j$ fixed at numeriare).
Now

\[
d\ln W_j = d\ln Y_j - d\ln P_j
\]

\[
= - \frac{1}{1 - \sigma} \frac{d}{\sum_{i=1}^{n} (w_i \tau_{ij})^{1-\sigma}} \sum_{i=1}^{n} (w_i \tau_{ij})^{1-\sigma}
\]

\[
= - \left[ \sum_{i=1}^{n} (w_i \tau_{ij})^{-\sigma} [d\ln w_i + d\ln \tau_{ij}] \right] \sum_{i=1}^{n} (w_i \tau_{ij})^{1-\sigma}
\]

\[
= - \sum_{i=1}^{n} \lambda_{ij} [d\ln w_i + d\ln \tau_{ij}]
\]

for

\[
\lambda_{ij} = \frac{(w_i \tau_{ij})^{-\sigma}}{\sum_{i=1}^{n} (w_i \tau_{ij})^{1-\sigma}}
\]

\[
= \frac{X_{ij}}{Y_j} = \frac{\left( \frac{w_i \tau_{ij}}{P_j} \right)^{1-\sigma}}{Y_j} = \left( \frac{w_i \tau_{ij}}{P_j} \right)^{1-\sigma}
\]

\[
= \frac{(w_i \tau_{ij})^{1-\sigma}}{\sum_{i=1}^{n} (w_i \tau_{ij})^{1-\sigma}}
\]

OK, what am I missing, let’s fix this. But let’s take as given this is true.

• By equation (2),

\[
d\ln \lambda_{ij} - d\ln \lambda_{jj} = d\ln \left( \frac{w_i \tau_{ij}}{P_j} \right)^{1-\sigma} - d\ln \left( \frac{w_j \tau_{jj}}{P_j} \right)^{1-\sigma}
\]

\[
= (1 - \sigma) d\ln (w_i \tau_{ij})
\]

\[
= (1 - \sigma) (d\ln w_i + d\ln \tau_{ij})
\]
Combine this with above

\[
d \ln W_j = - \sum_{i=1}^{n} \lambda_{ij} \left[ d \ln w_i + d \ln \tau_{ij} \right]
\]

\[
= - \sum_{i=1}^{n} \lambda_{ij} \left[ \frac{d \ln \lambda_{ij}}{1 - \sigma} - d \ln \lambda_{ij} \right]
\]

\[
= \sum_{i=1}^{n} \lambda_{ij} \frac{d \ln \lambda_{ij}}{1 - \sigma} = \frac{d \ln \lambda_{ij}}{1 - \sigma} + \frac{\sum_{i=1}^{n} \lambda_{ij} d \ln \lambda_{ij}}{1 - \sigma}
\]

\[
= \frac{d \ln \lambda_{ij}}{1 - \sigma} + 0(?)
\]

Now integrate before and after the shock?

\[
\hat{W} = \hat{\lambda}^{\frac{1}{1-\sigma}}
\]

where \( \hat{v} = v'/v \) is the change in an variable between the initial and new equilibrium.

• Welfare changes in country \( j \), whatever the origin of the foreign shock—can be inferred from the changes in the share of the domestic expenditure \( \lambda_{jj} \), using the trade elasticity, here \( \varepsilon = 1 - \sigma \).

  - welfare depends upon terms of trade impacts (3)

  - But terms of trade can be inferred from changes in relative demand for goods from different countries (4)

• Example: for US \( \lambda_{jj} = .93 \).

• Estimates of \( 1 - \sigma \) is range of \( -5 \) to \( -10 \).

• compare autarky.

\[
1 - \lambda^{-\frac{1}{\varepsilon}} = 1 - .93^{-\frac{1}{.07}} = 1 - 1.007 = -.007
\]

\[
= 1 - .93^{-\frac{1}{.07}} = 1 - 1.014 = -.014
\]
• Now map into more general models

3 Ramondo and Rodríguez-Clare:

Sorry, nothing yet...
References


Holmes and Stevens (2010), "An Alternative Theory of the Size Distribution with an Application to Trade.