Question 1

Consider the following version of the Hopenhayn model. In a stationary equilibrium of this model, the price is constant over time at a level $p$. Let $\pi(\bar{A}; p)$ be the profit from producing in the industry in a period given a productivity parameter $\bar{A}$ and a price $p$. $\pi(\bar{A}; p)$ is strictly increasing in both arguments. Assume that $\pi(0; p) < 0$ for all $p$.

New entrants draw their productivity type $\bar{A}$ from a distribution $G(\bar{A})$ with density $g(\bar{A})$ and support $[0; \bar{A}]$. The productivity parameter for those already in the industry evolves as follows: With probability $(1 - \pm)$ it remains the same as it was last period, $\bar{A}_{t+1} = \bar{A}_t$. With probability $\pm$ it depreciates to $\bar{A} = 0$ and remains at zero thereafter.

At the beginning of period $t$, after observing one’s productivity $\bar{A}_t$, an individual can choose to exit before realizing the current profit $\pi(\bar{A}_t; p)$. For example, if an individual finds that his or her $\bar{A}$ has depreciated to 0, the individual can exit before suffering the loss $\pi(0; p)$. New entrants, after observing their $\bar{A}$, can also exit before producing. However, there is an entry cost $c_e$ that cannot be recovered.

Demand in the industry at price $p$ is $D(p)$.

Let $M^*$ denote the equilibrium measure of entry in this industry and $p^*$ the stationary equilibrium price. Show how $M^*$ and $p^*$ are determined. What happens to $M^*$ and $p^*$ as $c_e$ is increased?

Question 2

Suppose that demand in an industry is given by $Q_t = D(p_t)$, where $p_t$ is the industry price at time $t$ and $Q_t$ is the industry quantity.

There are two kinds of firms, entrants and incumbents. An entrant pays a cost of $c$ at the beginning of the period and obtains a unit of capital. With this unit of capital, the entrant produces a unit of output in the period (there are no other costs of production).

An incumbent firm begins period $t$ with a particular capital stock $k_t$. The incumbent firm makes an investment decision to expand its capital at rate $i_t$ so that the end-of-period capital stock
of the ..rm is \( q_t = k_t(1 + i_t) \). The output of the ..rm in the period equals the end-of-period capital stock \( q_t \).

The cost of investment in a period depends on the expansion rate \( i_t \), the beginning capital level \( k_t \), and a cost parameter \( \mu \) in the following way,

\[
c(i_t; k_t; \mu) = \mu k_t h(i_t),
\]

where the function \( h(i_t) \) is strictly increasing in \( i_t \) and strictly concave and \( h(0) = 0 \). The cost parameter \( \mu \) take two values, \( \mu_L < \mu_H \). The probability of having the low cost \( \mu_L \) is \( \tilde{\pi} \) and this is i.i.d. across time for each ..rm and i.i.d. across ..rms.

The timing of the model works as follows. At the beginning of the period there is a set of incumbent ..rms that vary in their initial capital level \( k_t \). Then each incumbent ..rm ..nds out whether it is low cost \( \mu_L \) or high cost \( \mu_H \) in the period. Then each incumbent makes its investment decision while new entrants simultaneously make their entry decisions.

There is depreciation in the model that depends on the age of the ..rm. If a ..rm has been in production for less than two periods, then capital depreciates at rate \( \pm \) between periods. If a ..rm has been in production for three periods, then its capital depreciates at a 100 percent rate. To illustrate this, consider a new entrant in period \( t \). Recall it ends period \( t \) with a unit of capital. Such an entrant will begin period \( t + 1 \) with \( (1 \pm \pm) \) units of capital. If it invests at a rate \( i_{t+1} \) in the period, it will end the period with \( (1 \pm \pm)(1 + i_{t+1}) \) units of capital. It will begin period \( t + 2 \) with \( (1 \pm \pm)^2(1 + i_{t+1}) \) units of capital and end the period with \( (1 \pm \pm)^2(1 + i_{t+1})(1 + i_{t+2}) \) units. At this point is has been in production for three periods so it will begin period \( t + 3 \) with zero units of capital.

The discount factor is \( \tilde{\pi} \).

(a) Take as given that the market price in the industry is constant over time at level \( p \). Analyze the problem of incumbent ..rms.

(b) Show how the stationary equilibrium price and the stationary equilibrium distribution of ..rms is determined.

(c) Suppose that \( \tilde{\pi} = 0 \). Show that there exists a unique level of \( \delta \), denoted \( \delta_0 \), such that in the stationary equilibrium the total output of age-1 ..rms equals the total output of age-0 ..rms (entrants). Assuming \( \delta = \delta_0 \) does this model exhibit a skewed size distribution of ..rms?
Question 3

Recall the transportation model discussed in class. At that time, we discussed a social planner’s problem. Here we consider a market allocation in the same economy.

Individuals are uniformly distributed on the real line with a unit density at each point. The labor endowment is uniformly distributed and it has measure one on a unit interval. There is a fixed cost of \( A \) labor units to set up a plant. The marginal cost to produce one unit of output is one unit of labor.

Assume that it is costless for individuals to commute to work. However they must return to their home to consume. Assume that there is a transportation cost to ship the final good to individuals. The cost to ship a unit of final good a distance \( d \) is \( td \) units of labor.

Consider the following two-stage game. In stage 1, a set of firms locate at points on the real line. In stage 2, the firms set prices. Suppose that firms can price discriminate based on the location of consumers. Let \( p(x_f; x_c) \) be the delivered price that a firm located at \( x_f \) offers to consumers located at \( x_c \). (The price of output is denominated in units of labor, which, again, is perfectly mobile). Firms in stage 2 set their prices simultaneously.

A symmetric-spacing allocation is an allocation where all plants are equally spaced. Define a symmetric-spacing equilibrium. Characterize the spacing levels \( s \) that can be supported as symmetric-spacing equilibria. Define a zero-profit symmetric-spacing equilibrium. Show that it is unique. Let \( s^0(t) \) be the distance between plants in the zero-profit symmetric spacing equilibrium as a function of the transportation cost \( t \). Show that \( s^0(t) \) strictly decreases in \( t \). Show that plant output and employment strictly decrease in \( t \).

Take as given that the following is true:

\[ 5s^u(t) < s^0(t) < s^m(t); \]

where \( s^u(t) \) is the distance that solves the social planners problem discussed in class (I have verified that the above holds with numerical methods). Can the distance \( s^u(t) \) be supported as a symmetric-spacing equilibrium?