Pareto: Manuel of Political Economy*

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Introduction

Vilfredo Pareto was born in Paris on 15 July 1848 to an exiled Italian father (follower of Mazzini) and a French mother, but the family returned to Italy ten years later after a political amnesty had been declared there. Pareto's education was steeped in the classics and mathematics, followed by training in Turin as an engineer. He became director-general of the Italian railways, during which time he also wrote impassioned articles supporting free trade and vigorously opposing the protectionist and militarist policies of the Italian government, and he ran unsuccessfully for parliament. As Bousquet recounts (1928, p. 18), his scientific interest in economics was inspired by Pantaleoni whom he chanced to meet in a train. Pantaleoni's *Principii* persuaded him to overcome his objection to Walras's "sterile approach" and to study Walras's general-equilibrium theory, whereupon he made contact with Walras and started publishing numerous scientific articles in the *Giornale degli Economisti.* Nominated by Walras to suceed him in his chair at Lausanne, he took up his new career in 1893 at the age of 45. All this preceded the great works by which he is known, starting with the *Cours* (1896–7), continuing with the *Systèmes socialistes* (1902–3), and proceeding to the *Manuel* (1906, 1909). This work is a translation into French by Alfred Bonnet of Pareto's *Manuale di economia politica* (so-called presumably because of its small size (13.4 × 8.8 cm.)), which Pareto revised, omitting the Preface, adding some new passages and omitting others, and substituting a totally new mathematical appendix. Because of space limitations, in this survey it will not be possible to cover the fascinating "Introduction to Social Science" in Chapter II, nor the important discussion of Pareto's law of income distribution in Chapter VII, but we will confine ourselves to Pareto's main contributions to economic theory, namely consumption, production, and welfare. For more comprehensive treatments the reader is referred to Chipman (1976) and Malinvaud (1992).

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1 Consumption and exchange

It is commonly thought that economists regard economic agents as rational, "maximizing", agents. However, Pareto perceived things in a very different light. He distinguished between "logical" (today we say "rational") and "nonlogical" (nonrational) actions of one and the same economic agent. The feature he emphasized as characteristic of rational actions was that of frequent repetition; learning from one's mistakes leads after trial and error to a fairly uniform behavior. In his words (III.1): "we take into account only actions that are repeated; this allows us to assume that the link between these actions is a logical one", making it possible to infer that "the subjective fact is perfectly adjusted to the objective one." Thus, the purchases of consumers on competitive markets may be assumed to be rational; but Pareto would not have agreed with some modern approaches extending this hypothesis to non-repeated (or at least seldom repeated!) decisions such as marriage and divorce.

Pareto's contributions to the theory of the consumer have had a profound influence on economic thought as evidenced by Hicks (1939a) and Samuelson (1947). Pareto's long-term concern in the theory of consumer behavior was to derive conditions under which the "law of demand" holds, by which is meant the proposition that the demand, x_i, for a commodity varies inversely with its price, p_i. The treatment in the *Manuel* (Appendix, §§52–55, pp. 579–585) is based on his theoretical work developed in a series of articles published in 1892, in which the consumer is assumed to maximize a utility function $\Phi(x_1, x_2, \ldots, x_n)$ (called by Pareto the "total ophelimity") subject to a budget constraint (cf. Chipman 1976, §2.1, pp. 67–75). In his 1892 work Pareto conceived of demand for a commodity as a function of the prices of the n commodities available and the consumer's income; the treatment in the *Manuel* (also that of Pareto (1911)) replaces this by an excess-demand function in a situation of pure exchange, expressed as a function of the n prices and the consumer's n initial endowments; but the method of analysis is substantially the same.

Pareto came very close to obtaining the decomposition into an income term and a substitution term subsequently derived by Slutsky (1915), who based his analysis on Pareto's (for details see Chipman 1976, §2.1, pp. 67–75). He did succeed in showing (p. 583) that in the special case in which the marginal utilities ("elementary ophelimities") $\partial\Phi/\partial x_i$ depend on the quantity of x_i only, the law of demand holds. This led Pareto to interest himself in the measurability of utility.

The term "ophelimity" (*Manuel*, III.30) had already been introduced by Pareto in the *Cours* (1896, §5) to denote the quantity of satisfaction that the individual maximizes, on the ground that "utility" is not the appropriate word in the case of the satisfaction an addict derives from morphine. Regarding the measurability of "ophelimity" that he had assumed in the *Cours*, in the *Manuel* (III.35) Pareto acknowledged the "weak point ... which has been brought to light mainly by ... Fisher" (cf. Fisher 1892, Ch. IV, §5, p. 88): "We have assumed that this thing called *pleasure, value in use, economic utility*, or *ophelimity* is a quantity; but this has not yet been proved. And supposing it were, how should we go about measuring this quantity? It is a mistake to believe that we can in general infer the value of ophe-

limity from the law of supply and demand." It is important to note an important distinction made by Pareto here: (1) whether utility is measurable, and (2) if so, whether its measurement can be derived from market observations. The second obviously implies the first, but the converse is clearly not true. To the second question Pareto gave the following answer: "This can be done only in one particular case, in which the unit of measurement of ophelimity remains arbitrary; this is the case with commodities each of whose ophelimity depends solely upon the quantity of that commodity, and is independent of the quantities of other commodities consumed" (III.35). (Here, by ophelimity Pareto means his "elementary ophelimity" (*marginal* utility).) Presumably fortified by this result he left open the possibility that there might be other ways to measure utility.

This was indicated in his treatment of related goods. Pareto showed (IV.32, pp. 264–5) that if one could define a quaternary ordering of commodity combinations, such that one could say whether the preference for bundle A over bundle B was greater than, equal to, or less than the preference for bundle B over bundle C, then one could define a utility ("ophelimity") index that is unique up to linear transformations. Of course, such an index could be obtained only from introspection, and even then, one would be unable to obtain much precision, hence: "This is the difficulty in considering ophelimity as a quantity, except as a mere hypothesis." But under this hypothesis, given a total ophelimity index $\Phi(x, y)$, he defined commodities X and Y to be independent if $\Phi_{xy} \equiv \partial^2\Phi/\partial x\partial y = 0$; as having a "dependence of the first kind" if $\Phi_{xy} > 0$ (IV.8–9, pp. 251–2), and a "dependence of the second kind" if $\Phi_{xy} < 0$ (IV.14, p. 256). The first kind is what has come to be known as that of complementarity, although Pareto confined this term to the case of *perfect* complements (commodities that have to be consumed in fixed proportions); the second kind is that of substitutability; very unfortunately, as was pointed out by Schultz (1938, p. 23n) (see also Wicksell (1913, pp. 137–8) and Georgescu-Roegen (1975, p. 236, note 54)), in the Appendix (§47, p. 576) Pareto inadvertently interchanged the two formulas (63) and (64) providing the relevant inequalities. These definitions had already been proposed by Auspitz and Lieben (1889, p. 482; 1914, I, pp. 318–319), then adopted by Fisher (1892, p. 65), and once again (in a footnote citing both Auspitz and Lieben and Fisher, as well as Marshall (1891, Appendix, Note VI) proposed by Edgeworth (1897, pp. 20–21n))—a paper that was referred to by Pareto (1902, pp. 1116–7; 1906c, p. 446).

In the text of the *Manuel* (IV.49) Pareto made the strong assertions (pp. 272–3): "If the consumption of [the] commodities is independent, or if there is a dependence of the first kind between them, the demand for a commodity always falls as the price of this commodity rises", and (p. 273): "If consumption of [the] commodities is independent, or if there is a dependence of the first kind between them, the demand for each of these commodities always increases when income increases". As we know from Slutsky (1915), the first proposition follows from the second; and we saw above that Pareto proved the first when consumption is independent. He does not seem to have provided proofs for the case of complementarity, but the result is nevertheless correct (cf. Chipman 1976, p. 78, note 14). He also correctly pointed

out that goods could be inferior if they were substitutes according to his definition.

Pareto's proposition that utility could be measured from market observations only in the case of "independent consumption" (representation of preferences by an additively separable utility function) was challenged by Wicksell (1913, p. 136), who argued that the utility function $\Phi(x, y) = 2A\sqrt{xy}$ permitted measurable utility even though it was not additively separable, apparently in the belief that measurability followed from the mere possibility of specifying a precise analytic form for a utility function and its partial derivatives. But here Wicksell overlooked that the monotone increasing transformation $\log(\Phi(x, y)^2)$ of his utility function is in fact additively separable. Wicksell also mistakenly understood Pareto to claim (IV.33) that independence of commodities implies that marginal utility is inversely proportional to the quantity consumed (1913, p. 135), whereas Pareto showed this to hold only under Marshall's assumption (1891, p. 182n) of "constant" (independent-of-prices) marginal utility of income (Appendix, §§56–59, pp. 585–8).

In the Appendix (§§12–21) Pareto provided an argument, derived from Pareto (1906b, 1971), to the effect that information on a consumer's desired temporal order of consumption could be used to obtain a measure of utility. However, this argument, based on a surprising confusion between order of integration and order of consumption, and between integrability and exactness of a differential equation, has been universally criticized and may be considered as an aberration (cf. Wicksell (1913, pp. 137, 150–151), Samuelson (1950), Georgescu-Roegen (1975, p. 258), Malinvaud (1992, pp. 167ff.)).

2 Production and distribution

Pareto's main concern appears to have been to develop a theory of production that could simultaneously cover the cases of both fixed and variable technical coefficients, as well as be free from the hypothesis of constant returns to scale.

In three footnotes in the *Cours* (1897: §714[1], pp. 83–84; 717[2], pp. 85–86; 719[2], pp. 88–90), Pareto had criticized two aspects of the theory of marginal productivity: (1) failure to take account of the fact that certain inputs had to be combined in fixed proportions, and (2) the assumption of constant returns to scale. (These criticisms were in turn criticized by Stigler (1941, pp. 364–8).) A famous footnote in Pareto (1901, p. 10, note 1: 1955, p. 9, note 1; 1966, p. 131, note 6) criticizes Walras (1900, pp. 374–375) for committing the first of these "errors":

> We have provided in the *Cours*, §719 note, the equations that determine the fabrication coefficients.
>
> The theory that claims to determine them by consideration of marginal productivities is erroneous. This treats as independent variables, quantities that are not independent, and the equations that are written down to determine the minimum [cost] are inadmissible. Such are equations (3) of Mr. Walras's *Éléments d'écon. pol. pure*, 4th edition, p. 375. coefficients de fabrication.

The quantities treated as "independent variables" are the factor inputs, as arguments of Walras's production function (his formula (2)). Walras's equation (3) criticized by Pareto expresses the marginal productivity of a factor as the ratio of its price to the price of the product.

Shortly thereafter, Pareto (1902, p. 1117, note 55; 1906c, p. 447, note 1) added Wicksteed (1894) to this criticism:

> The theory of marginal productivity, as expounded in this work [Wicksteed, 1894], contains an error which was pointed out by Pareto, *Cours*, §714[1]. This error appears again in Walras, *Économie politique* (1900 ed.), pp. 374–375. The author treats as independent variables, quantities that are not independent.

Wicksteed (1906, p. 554n; 1910, p. 373n) withdrew §6 of his work (1894, pp. 32–43) which assumed constant returns to scale, in the light of these as well as Edgeworth's criticisms.

In a book review of Wicksteed (1894) requested but turned down by Edgeworth for the *Economic Journal* (cf. Jaffé, 1964), Barone had pointed out that the assumption of constant returns to scale was not needed to prove that distribution of the product to factors according to marginal productivity exhausts the product, and this claim of his was subsequently put forward by Walras (1896, Appendix III, pp. 485–492); it was only necessary that competitive firms operate at minimum average cost, i.e., that constant returns to scale hold in the small. This argument had also occurred to Wicksell (cf. Stigler, 1941, Chs. X, XII); see also Samuelson (1947, p. 86). However, Barone and Walras apparently overlooked the fact that under conditions of decreasing followed by increasing average costs, industry supply functions would be discontinuous, leading to limit cycles of exit and entry of marginal firms, unless one adopted an idealization of a continuum of firms. While Pareto could have justified his approach by such an idealization, he did not do so, relying instead on his and Walras's practice of resting the existence of equilibrium on the equating of numbers of equations and unknowns. But modern economists (e.g., Debreu, 1959) have still had to fall back on Wicksteed's hypothesis.

Pareto's treatment in the *Manuel* is largely confined to the Appendix (§§77–79, pp. 101–108), and may be summarized as follows (cf. Schultz (1929) and Neisser (1940)).

He starts out (§78) with m relations which we may denote $l_i = F_i(y_1, y_2, \dots, y_n)$, where l_i is the amount of the ith out of m factors of production, and y_j is the output of the jth commodity. These m relations are described by Pareto (p. 607) as "technical conditions of production" but they are better interpreted as combinations of these with resource-allocation constraints, since in the special case assumed by Pareto (p. 608) in which each F_i is additively separable, we have

$$F_i(y) = c_i + \sum_{j=1}^{n} \int_0^{y_j} b_{ij}(\eta_j) d\eta_j \quad (i = 1, 2, \dots, m),$$

where c_i is the overhead and the $b_{ij}(y_j) = \partial F_i/\partial y_j$ are the "production coefficients" (p. 607) or "fabrication coefficients" (p. 608). If each F_i is homogeneous of degree 1 then this reduces to the resource-allocation constraints

$$l_i = F_i(y) = \sum_{j=1}^{n} b_{ij}(y_j) \cdot y_j \quad (i = 1, 2, \ldots, m).$$

But even this leaves the production coefficients dependent solely on output, and not subject to variation as functions of prices. Pareto's procedure, then, is to treat the *functions* $F(\cdot)$ as variable, and therefore to subject the subset of *variable* production-coefficient functions $b_{ij}(\cdot)$ to a production constraint, and then proceed to a cost-minimization process, employing the calculus of variations (Appendix, §§101–5). See Schultz (1929) for a summary, and Neisser (1940) for a critical evaluation.

An important criticism was made by Hicks (1975, pp. 25–28) of Pareto's extension to production of what he called Edgeworth's "proposition", namely the Pareto-optimality of competitive exchange equilibrium, in the presence of overhead costs. His criticism was directed towards Pareto's assumption (Appendix, §90, p. 619) that optimality requires not only the equality of price and marginal cost, but also that of total revenue and total cost. Hicks showed that this error explains "the contortions of Chapter VI" involving the "line of complete transformations".

3 General equilibrium and welfare

Pareto is largely known today (cf. Arrow (1951), Debreu (1959)) for his criterion now known as "Pareto optimality", and the proposition that this is characteristic of competitive equilibrium—the phrase "Pareto 'optimum'" having been first introduced by Little (1950, p. 89). It was pointed out by Hicks (1975) that the criterion, and its relevance to the theory of pure exchange, was already implicit in Edgeworth (1881). However, in the article in which Pareto first applied the concept (Pareto, 1894, p. 58), he acknowledged the influence of persuasive conversations with Pantaleoni and Barone, who in turn were influenced by the discussion in Marshall, who had provided the following characterization: "a position of equilibrium of demand and supply is a position of maximum satisfaction in this limited sense, that the aggregate satisfaction of the two parties increases until that position is reached" (1891, Book V, Ch. XII, p. 506). Of course, Marshall in turn was no doubt influenced by Edgeworth, and perhaps also by Walras (1889). And Barone's well-known 1908 contribution was in turn strongly influenced by Pareto (1894).

Pareto proved in a series of articles, culminating in the *Manuel*, what is now described as the "first welfare theorem", namely that a competitive equilibrium is Pareto-optimal, i.e., has the property that it is not possible for all individuals to be better off than they are at this equilibrium. In applying this idea to the problem faced by a socialist state, he came very close to enunciating the "second welfare theorem" to the effect that (under certain conditions, of course), any Pareto optimum can be sustained by a competitive equilibrium. The famous "compensation principle", long thought to have been originally formulated by Kaldor (1939)and

Hicks (1939b), was fundamental to Pareto's method of proof; probably it was the 1935 English translation of Barone (1908) which provided the main channel of communication to English-speaking economists.

Pareto built on Walras (12$^{\text{me}}$ Leçon, 1889, §116, pp. 142–3; 1896, 1900, §117, pp. 122–3) who, however, never defined what he meant by "maximum satisfaction":

> What, exactly, is this condition [of maximum satisfaction]? It always consists in the attainment of equality between the ratio of the *raretés* [*marginal utilities*] of any two commodities and the price of one in terms of the other . . .

Walras's definition is more one of competitive equilibrium than of "Pareto optimality". However, Pareto was convinced that Walras had a deeper meaning in mind, and tried to give effect to it. Unfortunately, as Wicksell (1913, p. 141) pointed out (see also Georgescu-Roegen (1975, p. 228), Chipman (1976, p. 97–98)), Pareto marred his contribution by stating his definition very carelessly (1909, VI.33, p. 33):

> We shall say that the members of a community enjoy, in a certain situation, *maximum ophelimity* when it is impossible to move slightly away from this position ⌈in such a way that the ophelimity enjoyed by each member of the community increases or decreases.⌉ That is to say, every small displacement from this position must necessarily have the effect of increasing the ophelimity enjoyed by some individuals and decreasing that enjoyed by others . . . .

The first sentence contains an ambiguous passage (which we enclose by the signs ⌈⌉) which is probably the result of a bad translation from the Italian; the latter reads: "so as to benefit, or harm, all the members of the community." But even this contains the strange "or harm", which suggests that Pareto was translating from the first-order conditions of the calculus rather than expressing the economics of the situation. Similar wording is employed in the Appendix (§89, pp. 617–618). The second sentence leaves out of account the possibility that all may be harmed. However, only a few pages later Pareto states, for the case of two individuals and two commodities—in which he introduces for the first time the back-to-back diagram (now generally known as the "Edgeworth box"!)—the following statement for *finite* movements (VI.37, p. 356): ". . . if we move along a straight line a finite distance away from the equilibrium position, the ophelimities enjoyed by the two individuals may vary in such a way that the one increases while the other decreases, or that they both decrease; but they cannot both increase."

This highly satisfactory statement is followed, however, by a puzzling qualification: "This, however, is true only for commodities whose ophelimities are independent, or in the case in which these commodities are linked by a dependence of the first kind." How can we explain this strange qualification?

The answer is to be found in the fact that Pareto did not have at his disposal the mathematical concept of *quasi-concavity*, which he needed for his general proof. Instead he had to rely on *concavity* of the utility (ophelimity) function, expressed

(Appendix, §48, p. 577) by the oscillating principal minors of the Hessian of his ophelimity function (as opposed to the minors of the *bordered* Hessian—introduced in his derivation of the law of demand (§52, p. 580) but not applied to the welfare problem). He was able to show that the utility function is concave in the cases of independence and complementarity, but only in those cases. The above statement in the text suggests that he thought that these conditions were also *necessary* for quasi-concavity, whereas they were only *sufficient* for concavity (and therefore for quasi-concavity). His mathematical treatment was contained in the Appendix (§§121–125, pp. 650–654), where he was able to show that the second differential of the utility function was negative under these assumptions. For details, see Chipman (1976, pp. 99–107).

Coming back to the compensation principle, Pareto (Appendix, §89–91, pp. 617–619) considered the expression

$$\sum_{i=1}^{N} \frac{1}{\varphi_{i1}} d\Phi_i, \tag{3.1}$$

where $d\Phi_i$ is an increment in individual i's utility ("total ophelimity") and $\varphi_{i1} = \partial\Phi_i/\partial x_{i1}$ is the marginal utility ("elementary ophelimity") of commodity 1 to individual i, commodity 1 being the numéraire. This measures, in units of commodity 1, a net gain or loss to the community; for, if it is positive in a given situation, this means that it would be possible, by appropriate redistribution of commodity 1 among the N individuals, to make them all better off. Hence the situation is not Pareto optimal. Similarly, if the expression is negative, by an appropriate redistribution everybody could be made worse off. Thus, Pareto optimality is attained when and only when the expression (3.1) is equal to zero, in which case it is not possible to make some individuals better off without making some others worse off.

In his final contribution to welfare economics (1913), Pareto introduced a social-welfare function that may be interpreted as expressing the revealed preferences of the community rather than any a priori principles of justice as in Bergson (1938) and Samuelson (1947). In contrast (p. 338) he equated the above expression to dU for an increment in total utility, and characterized points with $dU = 0$ by the new terminology "points of type P", having the property that "it is not possible by departing from them to benefit or harm all the members of the community; we can depart from them only by benefiting some of the individuals and harming others."

Conclusion

Pareto's influence on modern economic theory has been profound. He was the first to make ordinal utility a systematic part of the subject, in his observation (quite consistently with his use of utility measurement for other purposes, e.g., classification of commodities into substitutes and complements) that all the propositions concerning equilibrium of consumers follow simply from the ordinal ranking of indifference surfaces, without the need to assume measurability of utility (*Manuel* III.54, p. 169n). This became the the leading theme of Hicks (1939a). Equally profound

has been his impact on welfare economics; he turned what in Walras was a tautology into an important theorem which remains today the main justification for the implementation of the system of competitive markets.

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