KOOPMANS, Tjalling Charles (1910–1985)[1]

Koopmans was born in the village of 's Graveland, Netherlands (southeast of Amsterdam) on 28 August 1910, of Sjoerd Koopmans (principal of a bible school) and Wijtske van der Zee (also a teacher). He studied mathematics and statistics at the University of Utrecht under the theoretical physicist Hans Kramers, where he obtained a master's degree in 1933 and published a paper (that is still cited) on quantum mechanics (reproduced in Koopmans 1970). His interests shifted to economics, and he studied under Jan Tinbergen at the University of Amsterdam in 1934, where he met Truus Wanningen whom he was to marry in 1936. Tinbergen was then a professor at the Netherlands School of Economics in Rotterdam and lectured once a week in Amsterdam. While at Amsterdam, Koopmans spent five months in Oslo with Ragnar Frisch, where at Frisch's request he gave some lectures on new developments in statistics by R. A. Fisher and J. Neyman, but failed to persuade Frisch of the merits of their probability models. Koopmans's Ph. D. dissertation on linear regression was supervised by Kramers, in consultation with Tinbergen on the economic aspects; since Kramers had by then moved to the University of Leiden, the Ph. D. was granted by that institution in November 1936. It was published the following year.

When Tinbergen was called to the League of Nations in Geneva in 1936–8 to do research on business cycles, Koopmans took over his class at Rotterdam, and worked on his monograph on tankships which was published in 1939. After Tinbergen's return to Rotterdam in 1938, Koopmans took his place in Geneva. While there he became acquainted with James Meade, who interested him in welfare economics, and he attended a conference on Tinbergen's work held at Oxford, where he met Jacob Marschak. World War II broke out, and since Koopmans had received an invitation from the renowned statistician Samuel Wilks at Princeton, he decided to make the move with his family in 1939. He was engaged as a research assistant at Princeton and taught a course in statistics at New York University. Meanwhile, Marschak had left Oxford to become a professor at the New School of Social Research, and Koopmans attended his seminars. In 1941 he was hired as an economist at the Penn Mutual Life Insurance Company in Philadelphia, and in 1942 he worked as a statistician at the British Merchant Shipping Mission in Washington, during which time he did research on optimal routing of ships. In 1943 Marschak became director of research at the Cowles Commission for Research in Economics at the University of Chicago, and in 1944 Koopmans accepted his invitation to join Cowles as a research associate. Koopmans was appointed an associate professor of economics in 1946, and professor from 1948 to 1955. He became director of research at Cowles in 1948, continuing to 1954.

The years at Chicago saw a meteoric rise in Koopmans's reputation. In 1950 he

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was elected president of the Econometric Society; his two major Cowles Commission monographs were published in 1950–51, and a third in 1953. In 1955, owing to disagreements on methodology with the economics department at Chicago, he moved to Yale along with the Cowles Commission which was renamed the Cowles Foundation. He became its director from 1961 to 1967. In 1969 he was elected to the National Academy of Sciences; he became a member of the National Research Council's Committee on Nuclear and Alternative Energy Systems in 1975–8; and in 1975 he shared the Nobel Memorial Prize in Economic Sciences with Leonid Kantorovich. In 1978 he agreed to assume the presidency of the American Economic Association after the death of its president-elect Jacob Marschak; he had previously turned it down owing to the pressure of his research. After a series of strokes, he died in New Haven on 28 April 1985.

There were three major research programmes that occupied Koopmans's lifetime work. These will be discussed in turn.

Econometrics

Koopmans's 1937 thesis, whose contribution is well described by Malinvaud (1972) and Christ and Hurwicz (1987), introduced concepts from probability theory and modern statistical inference into econometrics, and thus preceded Haavelmo's important 1944 monograph and 1943 and 1947 contributions, which ushered in the famous 'simultaneous-equations' approach introduced by Koopmans (1945, 1949a, 1950c). Koopmans's contribution was to turn Haavelmo's approach and his own previous work into a rigorous statistical methodology, which will now be briefly described.

Economists had long been troubled by the fact that in estimating a demand function one had to allow for the fact that all the econometrician could observe was points of intersection of demand and supply curves. Koopmans perceived that in order to estimate a demand function, demand would have to be independent of some variables that enter into the supply function (such as rainfall in the case of an agricultural product), and likewise to estimate a supply function, the supply would have to be independent of some variables affecting demand (income, for example). This led to his formulation of the *identification problem* (Koopmans 1949a, Koopmans, Rubin, and Leipnik 1950c).

The identification problem may be illustrated by the following system of two equations, which subsumes the above demand-and-supply model as a special case:

$$\text{(S)} \qquad \begin{aligned} y_{t1} &= y_{t2}\gamma_{21} + x_{t1}\beta_{11} + x_{t2}\beta_{21} + x_{t3}\beta_{31} + \varepsilon_{t1}, \\ y_{t1} &= y_{t2}\gamma_{22} + x_{t1}\beta_{12} + x_{t2}\beta_{22} + x_{t3}\beta_{32} + \varepsilon_{t2}. \end{aligned}$$

Here, y_{t1} and y_{t2} stand for the quantity and price (of wheat, say) at time t, $x_{t1} \equiv 1$ is the constant term, x_{t2} is rainfall and x_{t3} another variable, at time t, and the ε_{tj}

are random variables with zero means. The first equation is the demand function, the second equation the supply function. The y_{tj} are the endogenous, and the x_{tj} the exogenous variables in the system (S), which Koopmans called the 'structure'. Provided $\gamma_{21} \neq \gamma_{22}$ (the demand and supply curves are not parallel), the structure (S) may be solved to obtain what he called the 'reduced form':

$$\text{(R)} \qquad \begin{aligned} y_{t1} &= x_{t1}\pi_{11} + x_{t2}\pi_{21} + x_{t3}\pi_{31} + \eta_{t1}, \\ y_{t2} &= x_{t1}\pi_{12} + x_{t2}\pi_{22} + x_{t3}\pi_{32} + \eta_{t2}, \end{aligned}$$

expressing the two endogenous variables (quantity and price) as functions of the three exogenous ones. Now from (R) we have

$$\begin{aligned} y_{t1} - y_{t2}\gamma_{21} &= x_{t1}(\pi_{11} - \pi_{12}\gamma_{21}) + x_{t2}(\pi_{21} - \pi_{22}\gamma_{21}) + x_{t3}(\pi_{31} - \pi_{32}\gamma_{21}) \\ &\qquad + \eta_{t1} - \eta_{t2}\gamma_{21}, \\ y_{t1} - y_{t2}\gamma_{22} &= x_{t1}(\pi_{11} - \pi_{12}\gamma_{22}) + x_{t2}(\pi_{21} - \pi_{22}\gamma_{22}) + x_{t3}(\pi_{31} - \pi_{32}\gamma_{22}) \\ &\qquad + \eta_{t1} - \eta_{t2}\gamma_{22}. \end{aligned}$$

For these two equations to agree with the respective two equations of the structure (S), we must have

$$\begin{aligned} &\beta_{11} = \pi_{11} - \pi_{12}\gamma_{21}, \quad \beta_{21} = \pi_{21} - \pi_{22}\gamma_{21}, \quad \beta_{31} = \pi_{31} - \pi_{32}\gamma_{21}; \\ &\beta_{12} = \pi_{11} - \pi_{12}\gamma_{22}, \quad \beta_{22} = \pi_{21} - \pi_{22}\gamma_{22}, \quad \beta_{32} = \pi_{31} - \pi_{32}\gamma_{22}. \end{aligned}$$

Since the π_{ij} can always be estimated consistently by ordinary least squares, they may be taken as given, hence the first of the above set is a system of three equations in the four unknowns $\beta_{11}, \beta_{21}, \beta_{31}$, and γ_{21} (and similarly for the second). Thus these parameters cannot be 'identified' unless an a priori restriction is made. So if we assume that rainfall x_{t2} does not enter into the demand equation, we have $\beta_{21} = 0$, which leads to $\gamma_{21} = \pi_{21}/\pi_{22}$, which can be substituted into the other two equations to obtain β_{11} and β_{31}—*as long as* $\pi_{22} \neq 0$. But $\pi_{22} \neq 0$ if and only if $\beta_{22} \neq 0$, which can be seen as follows. Given that $\gamma_{21} \neq \gamma_{22}$ and $\beta_{21} = 0$ by assumption, if $\pi_{22} \neq 0$ then from the above equations $\beta_{22} \neq \beta_{21} = 0$; conversely, if $\pi_{22} = 0$ then $\beta_{22} = \beta_{21} = 0$. This is a special case of the necessary and sufficient 'rank condition' for identifiability (Koopmans et al. 1950c), namely that the rank of the 'rank-criterion matrix' be equal to one less than the number of equations (in this case, $\text{rank}([0, \beta_{22}]) = 1$). The necessary 'order condition' states that the number of restrictions imposed on the parameters of an equation be at least one less than the number of equations.

If in the above example one imposes an additional restriction on the first equation of (S), say $\beta_{31} = 0$, then we have $\gamma_{21} = \pi_{21}/\pi_{22} = \pi_{31}/\pi_{32}$ and the five parameters must be estimated *jointly* subject to $\pi_{i1} + \pi_{i2}\gamma_{21} = 0$ for $i = 2, 3$. Most of the work in simultaneous-equations estimation consists in estimating the parameters π_{ij} of the reduced form jointly with the γ_{ij} subject to restrictions of the above sort. Koopmans, Rubin, and Leipnik (1950c) derived such estimators when linear restrictions are imposed on the parameters of all the equations of the system, using the method of maximum likelihood, when the ε_{tj} are assumed to be jointly normally distributed. This is known as the method of 'full-information maximum likelihood' (FIML). Although not published until 1950, this paper had already been completed in 1945 (see Koopmans 1945, p. 455n). Later, Anderson and Rubin (1949) developed the simpler 'limited-information maximum likelihood' (LIML) method in which account is taken only of restrictions on one equation. A still simpler method for the latter was later developed by Theil (1958) called 'two-stage least squares' (2SLS), and an equivalent one by Basmann (1957). The FIML and LIML methods were both implemented by Klein (1950). Finally a simpler full-information method was developed by Zellner and Theil (1962), called 'three-stage least squares' (3SLS). It was proved by Koopmans, Rubin, and Leipnik (1950c), and more simply in Koopmans and Hood (1953c), that the FIML estimators are (asymptotically) consistent; the same is true of the other estimators.

In his 1945 paper and elsewhere, Koopmans argued strongly in favor of simultaneous-equations methods as opposed to single-equation least-squares methods of estimation, basing himself on the superior asymptotic properties of the former. Later work by Bergstrom (1962), Basmann (1974), and others showed, however, that the properties of the former are not necessarily better in small samples. For example, in the above illustration, if one estimates γ_{21} by $\hat{\gamma}_{21} = p_{21}/p_{22}$, where the p_{ij} are the least-squares estimators of the π_{ij}, then if the ε_{tj} are normally distributed so are the p_{ij}, hence $\hat{\gamma}_{21}$ has no mean, variance, or other moments. Consequently its density function will have 'thick tails', leading to frequent outliers. One might want to dismiss this objection on the ground that prices and quantities, being intrinsically positive, cannot actually be normally distributed. However, the logical problem remains. Today the emphasis has shifted towards examining finite-sample properties of alternative estimators (Ullah 2005, Ch. 7).

Activity analysis

Koopmans's novel approach to production theory stems in good part from his early 1939 work on tanker freight rates and his subsequent work on utilization of the transportation system (see Koopmans 1942, 1949b), culminating in his monograph (1951a).

Economists are used to dealing with systems of production functions of the general form

$$q_j = f_j(u_{1j}, u_{2j}, \ldots, u_{nj}, v_{1j}, v_{2j}, \ldots, v_{mj}), \quad j = 1, 2, \ldots, n$$

where u_{ij} is the amount of the ith intermediate input and v_{ij} the amount of the ith primary input into the process of producing q_j units of commodity j, where the primary inputs are subject to the resource-allocation constraints $\sum_{j=1}^{n} v_{ij} \leq l_i$ for $i = 1, 2, \ldots, m$, l_i being the endowment of the ith factor of production in the economy.

A special case of the above is that in which f_j has the form (for fixed a_{ij}, b_{ij})

$$\min\left(\frac{u_{1j}}{a_{1j}}, \frac{u_{2j}}{a_{2j}}, \ldots, \frac{u_{2n}}{a_{2n}}, \frac{v_{1j}}{b_{1j}}, \frac{v_{2j}}{b_{2j}}, \ldots, \frac{v_{mj}}{b_{mj}}\right).$$

This is the case of a Walrasian 'fixed-coefficients' technology, since $u_{ij} = a_{ij}q_j$ and $v_{ij} = b_{ij}q_j$ for all i, j. The a_{ij} are the celebrated 'input-output coefficients' of Leontief (1951), and the b_{ij} for $m = 2$ his labour and capital coefficients. Defining the $n \times n$ input-output matrix $A = [a_{ij}]$ and the the $m \times n$ factor-output matrix $B = [b_{ij}]$, given the column vector $q = (q_1, q_2, \ldots, q_n)'$ of gross outputs, the column vector of net outputs is defined by $y = (I - A)q$, where $I - A$ is the 'Leontief matrix'. The resource-allocation constraint is $Bq \leq l$, where l is the $m \times 1$ column vector of factor endowments. In terms of the net outputs y (Leontief's 'bill of goods'), the resource-allocation constraint is $B(I - A)^{-1}y \leq l$.

Koopmans's model (1951b) lies in between these two extremes, although it has the additional advantage of allowing for joint production. A *commodity* is either a final good, an intermediate good, or a primary factor. With n goods (final or intermediate) and m factors, there are $N = n + m$ commodities. Inputs, whether primary or intermediate, are treated as negative commodities. Defining $y_{\text{pri}} = -Bq \geq -l \equiv \eta$, and the vector $\tilde{y} = (y', y'_{\text{pri}})'$, and stacking the above matrices $I - A$ and $-B$ on top of one another to obtain an $N \times n$ matrix C, and finally assuming $y \geq 0$ (the economy is closed to international trade), we obtain

$$\tilde{y} = \begin{bmatrix} y \\ y_{\text{pri}} \end{bmatrix} = \begin{bmatrix} I - A \\ -B \end{bmatrix} q = Cq \quad \text{where} \quad q \geq 0 \quad \text{and} \quad \tilde{y} \geq \begin{bmatrix} 0 \\ \eta \end{bmatrix}.$$

The columns of the matrix C constitute 'activities' in Koopmans's sense, and the gross outputs q_j are the nonnegative 'activity levels'. In the case of the above Leontief model, there is one activity for each industry. Koopmans's model allows for arbitrarily many of them, and imposes the two assumptions of additivity (activities can be added to each other) and divisibility (activities can be uniformly expanded or contracted at any scale). Of course, these imply constant returns to scale.

An illustration can be given in which there are two industries and two activities in each industry, so that

$$\begin{bmatrix} 1 - a_{11}^1 & 1 - a_{11}^2 & -a_{12}^1 & -a_{12}^2 \\ -a_{21}^1 & -a_{21}^2 & 1 - a_{22}^1 & 1 - a_{22}^2 \\ -b_{11}^1 & -b_{11}^2 & -b_{12}^1 & -b_{12}^2 \end{bmatrix} \begin{bmatrix} q_1^1 \\ q_1^2 \\ q_2^1 \\ q_2^2 \end{bmatrix} = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}.$$

Normalizing so that each activity uses up one unit of the one primary factor, and separating out the two industries, we have

$$\begin{bmatrix} \dfrac{1-a_{11}^1}{b_{11}^1} & \dfrac{1-a_{11}^2}{b_{11}^2} \\ \dfrac{-a_{21}^1}{b_{11}^1} & \dfrac{-a_{21}^2}{b_{11}^2} \\ -1 & -1 \end{bmatrix} \begin{bmatrix} b_{11}^1 q_1^1 \\ \\ b_{11}^2 q_1^2 \end{bmatrix} + \begin{bmatrix} \dfrac{-a_{12}^1}{b_{12}^1} & \dfrac{-a_{12}^2}{b_{12}^2} \\ \dfrac{1-a_{22}^1}{b_{12}^1} & \dfrac{1-a_{22}^2}{b_{12}^2} \\ -1 & -1 \end{bmatrix} \begin{bmatrix} b_{12}^1 q_2^1 \\ \\ b_{12}^2 q_2^2 \end{bmatrix} = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}.$$

Here, the new activity levels $b_{1j}^k q_j^k$ can no longer be identified with gross outputs. In this illustration, each production function, depicting the net output of the industry as a function of the input from the other industry, together with an input of 1 unit of labour, is a broken curve with two linear segments. It is clear that in general the classical smooth production function is replaced by a broken curve with a number of linear segments (see Koopmans 1953b). In Koopmans's view, this represented more closely than the neoclassical production function the decisions to be made on the part of entrepreneurs, which he felt should not be excluded from economic analysis.

It was shown in Chs. VII–X of Koopmans 1951a (by P. A. Samuelson, T. C. Koopmans, K. J. Arrow, and N. Georgescu-Roegen) that in economies with a single primary factor of production, each industry (producing just one final commodity) would efficiently use just one process of production. This result (called the 'substitution theorem' but subsequently renamed the 'nonsubstitution theorem') justified Leontief's assumption of fixed technical coefficients. But H. Simon showed (Ch. XV) under the same assumptions that a technological change in just one industry would trigger changes in input-output coefficients in all the other industries (his 'trigger effect').

Koopmans's main concern was with productive efficiency, defined as a state in which it is impossible to increase one component of $\tilde{y}$ without reducing some other. The set of $\tilde{y} = Cq$ for $q \geq 0$ defines a convex polyhedral cone in N-dimensional space. He showed (p. 63) that for a point $\tilde{y}$ in this cone to be efficient it is necessary and sufficient that there exist a positive price vector p such that $p'C \leq 0$ (profits are nonpositive on all possible activities) and $p'\tilde{y} = 0$ (profits are zero on the efficient activity $\tilde{y}$). Given the data on the technology, such efficient points can be computed using the simplex method of linear programming (Dantzig 1951a).

Of particular interest is Koopmans's application of this theory to the transportation problem (Koopmans 1942, 1949b, Koopmans and Reiter 1951c), derived independently of an earlier formulation by Hitchcock (1941). In the simplest case, that of transportation of cargo between ports i and j, since ships must make a round trip, in general some must travel empty in one direction, so the marginal cost of shipping must be greater in the full than in the empty direction. In the two-port case, if t_{ij} is the cost of loading, moving, and discharging cargo from i to j, and s_{ij} is the cost of moving empty from i to j, then the marginal cost (expressed relatively to the rental of the relevant shipping capacity) of moving some additional cargo from i to j is, in the case in which there is initially empty (ballast) traffic from i to j, given by $p_{ij} = t_{ij} - s_{ij} < t_{ij}$, since the ship will be moving from i to j in any event. In the case in which there is initially fully loaded traffic from i to j, the formula for the marginal cost is $p_{ij} = t_{ij} + s_{ji} > t_{ij}$, since to the *direct cost* t_{ij} one must add the *indirect cost* s_{ji} of moving the ship back empty from j to i. As shown in Koopmans (1949b, p. 190) and Koopmans and Reiter (1951c, p. 237), these formulas can be simplified by introducing the idea of a *locational potential* p_i, which is the price of the intermediate commodity 'ship appearance at port i', which of course is an input into the 'production' of the commodity 'moving cargo from i to j'. If ballast traffic is currently moving from i to j, then $s_{ij} = p_j - p_i > 0$, since port j has a higher locational potential than port i. On the other hand, if fully loaded traffic is currently moving from i to j, then $s_{ji} = p_i - p_j$, but now $p_i > p_j$ since port i has higher locational potential than port j, hence the marginal cost of moving cargo from i to j is $p_{ij} = t_{ij} + s_{ji} = t_{ij} + p_i - p_j > t_{ij}$. In both cases, and in fact in the general case of multiple ports, the formula for the marginal cost is $p_{ij} = t_{ij} + p_i - p_j$ (Koopmans and Reiter 1951c, p. 254), where $p_i - p_j$ is the *indirect cost* of shipping an extra unit from i to j—positive when the current movement of fully loaded traffic is from i to j, and negative in the opposite case. A solution to the transportation problem was derived by Dantzig (1951b) using his simplex method, and related to Koopmans's treatment in terms of a potential function.

While Koopmans (1949b, pp. 191–2) held that the international shipping market in peacetime years had been highly competitive and that 'a perfectly competitive market automatically brings about pricing according to marginal cost', it was admitted by Koopmans and Reiter (1951c, p. 257) that 'the type of contract that would make the p_i observable as market prices has to our knowledge not been in use in ocean shipping or in any other transportation market'. Koopmans noted (1949b, p. 192) that domestic railroad transportation in the U.S., being subject to government regulation, was highly inefficient, as evidenced by equal transport costs in opposite directions. Both the 1949 and 1951 contributions contained computations of efficient routes for empty international shipping using 1925 and 1913 data respectively.

Koopmans's deep concern with efficiency and economic welfare led him to write a masterful exposition of this general topic in the first of his *Three Essays* (1957), which is probably his best-known work.

Intertemporal economics

Koopmans showed an early interest in the formulation of preferences over time, concentrating at first (1950a, 1953a) on the desirability of postponing unnecessary future decisions (see also Koopmans 1964). His important 1960 paper, followed by his 1964 paper with Diamond and Williamson, presented a set of postulates on consumer preferences over time that would justify representing these preferences by a utility function of the form $U(x_1, x_2, \ldots) = \sum_{t=1}^{\infty} \alpha^{t-1} u(x_t)$, where α (satisfying $0 < \alpha = 1/(1+\rho) < 1$) is a discount factor and $\rho > 0$ is a positive rate of time preference. Such a discount factor of course gives a greater weight to the present than to the future, and is an indication of 'impatience' in Irving Fisher's terminology. A special postulate ensures constancy of α which in turn implies 'cardinality' of the utility index. The 1964 paper with Diamond and Williamson introduced a still stronger cardinal property of 'time perspective'. His two 1972 articles provided rigorous expositions of these very difficult topics.

Koopmans's important 1965 paper dealt with the normative problem of optimal economic growth. He recognized (p. 226) the position of a number of economists that 'the balancing of the interests of different generations is an ethical or political problem', and that one should therefore take an eclectic view. This was fortified by his observation (p. 229) that 'ignoring realities in adopting "principles" may lead one to search for a nonexistent optimum, or to adopt an "optimum" that is open to unanticipated objections'. He chose a simple Solow-type model in which a single commodity does duty as capital and consumer good, with a neoclassical production function and labour growing at a constant rate, and applied both a utility function with a constant discount factor and the 'overtaking principle' (which goes back to Ramsey 1928) in which the discount factor α equals 1. He treated continuous time, so that the discount factor $\alpha = 1/(1 + \rho)$ is replaced by $\alpha = e^{-\rho}$. He showed (not surprisingly) that $\rho = 0$ is the smallest rate of time preference for which an optimal path exists. An optimum for $\rho = 0$ exists only if welfare comparisons are made on a per capita basis: 'There seems to be no way, in an indefinitely growing population, to give equal weight to all individuals living at all times in the future', hence 'the open-endedness of the future imposes mathematical limits on the autonomy of ethical thought' (p. 254). In the case $\rho > 0$ he found that the optimal path converges to the 'golden rule' path in which the marginal productivity of capital is equal to the rate of growth of the population. He found a similar result using the overtaking criterion. A witty and authoritative exposition was added by Koopmans (1967).

Most of Koopmans's last works (1973, 1974, 1980, 1987) were devoted to the consideration of natural resources, which had been left out of account in most of the growth models until then. This work was influenced by his activities with the National Research Council.

In his important 1973 article, Koopmans was influenced by Hotelling's 1939 treatment of the economic theory of the mine owner and Gale's fascinating 1967 treatment of the problem of the 'cake eater' given an infinite time to eat a (nonperishable) cake. He combined this with Ramsey's 1928 model of a consumer good produced by labour and capital with labour constant and the marginal productivity of capital equal to zero for sufficiently high input of capital ('capital saturation'), expressing instantaneous utility as the sum of specific utility functions of the consumer good and the extracted resource. In terms of this model he found (p. 247) that 'discounting future utilities favors an earlier generation over any surviving later generation, and shortens the period of survival'. This model was further analyzed in Koopmans (1974), and an interesting general discussion was given in Koopmans (1980) where he expressed approval of the approach of Dasgupta and Heal (1974) in which the extracted resource is instead treated as a third factor of production for the single consumer good. His concern in this paper with econometric estimates of demand functions with constant elasticities led to his last and posthumously-published paper with Uzawa (1990).

The last of the four works on exhaustible resources, a cooperative venture with an economist (Nordhaus) and two geologists (Gordon and Skinner), which was virtually completed before his death and published posthumously, shows his stamp in Chapters 1 and 4–5 with the construction and solution of a linear programming model of copper production—activity analysis applied to the intertemporal domain much as he had applied it to the interspatial one. Chapter 1 provides a beautiful exposition of the model and its dual, with emphasis on efficiency and the corrresponding shadow prices. The most interesting conclusion is that intertemporally efficient production of copper—which is currently extracted from a geochemically scarce ore, and which has aluminium as a close (but more expensive) substitute—requires that copper be extracted until it is depleted, then replaced (when possible) by the superabundant aluminium, and (when this is not possible) by the abundant but probably prohibitively expensive rock source. As in Hotelling (1939), the efficiency price of copper rises exponentially. The existence of futures as well as spot prices was assumed (pp. 10–11); since futures prices are rarely quoted beyond two years, the question of the efficiency of the market system was left open.

Concluding remarks

There have been several significant appreciations of Koopmans's work and accounts of his life: those of the editors' introduction to Koopmans (1970), Malinvaud (1972), Werin and Jungenfelt (1976), Christ and Hurwicz (1987), and Scarf (1995). To these one may add Koopmans's autobiography (1975). Most of his significant papers have been reproduced in Koopmans (1970, 1985), where one will also find bibliographies.

Koopmans clearly had a major impact on the economics profession. Prior to Koopmans, many economists were satisfied with vague or incompletely-thought-out hypotheses and conjectures in the place of rigorous proofs. It is a mark of his contributions, quite apart from their substantive nature, that he had a deep influence on the nature of economic research.

Koopmans was a man of strong convictions and gentle temperament. He was very musical. This writer is in possession of a copy of his 14-page score, ‘Piano Pieces for Children of all Ages’, performed and presented during a visit to our home on 6 December 1980.

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John S. Chipman