Protection and Exchange Rates in an Open Economy

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December 9, 2003

1 The Two-Factor Case

Suppose we are concerned with country 1. Its two factor rentals are determined as
the solution of the equations setting minimum-unit costs equal to the domestic prices
of the two tradables:

\begin{equation}
\begin{align*}
g_1^1(w_1^1, w_2^1) &= p_1^1 \\
g_2^1(w_1^1, w_2^2) &= p_2^1
\end{align*}
\end{equation}

where \( p_j^i \) denotes the price of commodity \( j \) on country \( 1 \)'s markets and \( w_k^i \) denotes
the rental of factor \( i \) in country \( 1 \). Denoting the solutions to (1) by \( \hat{w}_1^i(p_1^1, p_2^1) \), they
may be substituted into the minimum-unit-cost function for the nontradable good to obtain:

\begin{equation}
p_3^1 = p_3^1(p_1^1, p_2^1) = g_3^1(\hat{w}_1^1(p_1^1, p_2^1), \hat{w}_2^1(p_1^1, p_2^1)).
\end{equation}

The problem is simply to compute the partial derivatives of this function. We note for
future reference that since the minimum-unit-cost functions \( g_j^i(w_1^1, w_2^1) \) are homogeneous
of degree 1, so are the inverse cost functions \( \hat{w}_j^i(p_1^1, p_2^1) \), and therefore \( g_3^1(w_1^1, w_2^1) \)
is also homogeneous of degree 1.

Adopting the convention that commodity 2 requires a larger proportion of factor
2 to factor 1 than commodity 1, we have

\begin{equation}
|B^1| = \left| \begin{array}{cc} b_{11}^1 & b_{12}^1 \\ b_{21}^1 & b_{22}^1 \end{array} \right| = b_{11}^1 b_{22}^1 - b_{21}^1 b_{12}^1 = b_{11}^1 b_{12}^1 \left[ \frac{b_{22}^1}{b_{12}^1} - \frac{b_{12}^1}{b_{11}^1} \right] > 0,
\end{equation}

where by Shephard’s theorem the ratio of the input of factor \( i \) to the output of
commodity \( j \) is \( b_{ij}^k = \partial g_j^k(w_1^k, w_2^k)/\partial w_i^k \). Differentiating (2) we then obtain

\[
\left( \frac{\partial p_3^1}{\partial p_1^1}, \frac{\partial p_3^1}{\partial p_2^1} \right) = \left( b_{13}^1, b_{23}^1 \right) \left[ \begin{array}{cc} b_{11}^1 & b_{12}^1 \\ b_{21}^1 & b_{22}^1 \end{array} \right]^{-1}
\]
\[ B^1 = B^1 \begin{bmatrix} b_{13}^1 & b_{23}^1 \\ b_{12}^1 & b_{11}^1 \end{bmatrix} \begin{bmatrix} \frac{b_{22}^1}{b_{12}^1} - \frac{b_{21}^1}{b_{12}^1} \\ -b_{12}^1 \end{bmatrix} \]

Thus we see that in order for both partial derivatives \( \hat{p}_1 / \hat{p}_2 \) to be positive we must have
\[ \frac{b_{22}^1}{b_{12}^1} > \frac{b_{23}^1}{b_{13}^1} > \frac{b_{21}^1}{b_{11}^1}, \]
i.e., the factor-intensity ratio of the nontradables sector must lie strictly in between those of the two tradables sectors.

\[ (a) \] In the case of a tariff at the rate \( \tau_2 > 0 \) imposed by country 1 on its import of commodity 2 (to which the subscript 2 refers), expressed as a proportion of the world price, we have \( p_2^1 = (1 + \tau_2)p_2 \), or \( p_2^1 = T_2p_2 \), where \( T_2 = 1 + \tau_2 \) > 1 and \( p_2 \) is the world price of commodity 2. Thus, a rise in the tariff rate \( \tau_2 \) brings about a rise in \( p_2^1 \), given that \( p_2 \) is assumed to be given on the world market. A necessary
and sufficient condition for this to cause a rise in the price of the nontradable is then, from (4), that $b_{23}/b_{13} > b_{21}/b_{11}$, i.e., that the nontradables sector should use a larger proportion of factor 2 to factor 1 than the export sector.

The result is illustrated in Figure 1, where (5) are assumed to hold. $I_1$ is the isoquant for the export good (commodity 1), defined by $f_1(l_1, l_2) = 1/p_1$, showing the the combinations of inputs of the two factors $l_1, l_2$ that will yield an output of an amount of commodity 1 whose value (at the initial domestic price $p_1$) is equal to one unit of country 1’s currency. $I_2$ is the isoquant for the import good (commodity 2) defined by $f_2(l_1, l_2) = 1/p_2$. The common tangent to these two isoquants goes through point $a$ and $b$. Since it is assumed that commodity 3 is also produced, its isoquant $I_3$ (defined in the same way as the other two) must be tangential to the line through $a$ and $b$, at the point $c$. Now suppose the domestic price of the import good rises (say as a result of imposition of a tariff); then the isoquant $I_2$ must shift inward, say to $I_2'$. The diversification cone swings clockwise from $AOB$ to $A'O'B'$, and the new common tangent to $I_2'$ and $I_1$ now goes through points $a'$ and $b'$. Since it is assumed that commodity 3 continues to be produced, the isoquant $I_3$ must also shift inward, to $I_3'$, to the new tangency point $c'$. This means that the price of the nontradable good must rise.

Figure 2 illustrates an opposite case, in which instead of (5) holding, we have

(6) \[ \frac{b_{22}}{b_{12}} > \frac{b_{21}}{b_{11}} > \frac{b_{23}}{b_{13}}. \]

In this case, the nontradables sector uses an even lower ratio of factor 2 to factor 1 than the export sector. Starting from the initial situation in which the isoquants $I_2$, $I_1$, and $I_3$ line up with common tangents at $b$, $a$, and $c$, an increase in the domestic price of the import good (commodity 2) will (as before) result in an inward shift of $I_2$ to $I_2'$, and the new common tangent to $I_2'$ and $I_1$ goes through $b'$ and $a'$. This time, however, the isoquant for the nontradable must shift outward from $I_3$ at $c$ to $I_3'$ at $c'$; this means that the price of the nontradable must fall.

(b) In the case of a subsidy at the rate $\sigma_1 > 0$ imposed by country 1 on its export of commodity 1 (to which the subscript 1 refers), expressed as a proportion of the country-1 price (to which the superscript 1 refers), we have $p_1 = (1 - \sigma_1)p_1^1$ or $p_1 = T_1^1p_1^1$, where $T_1^1 = 1 - \sigma_1^1 < 1$. Equivalently, defining the subsidy factor

\[ T_1 = 1 + \sigma_1 = \frac{1}{1 - \sigma_1} = \frac{1}{T_1^1}, \]

where $\sigma_1 > 0$, we have $p_1^1 = T_1p_1 = p_1/T_1^1$ where $T_1 > 1$. Thus, a rise in the rate of subsidy $\sigma_1$ brings about a rise in $p_1$, given that $p_1$ is assumed to be given on the world market. From (4), a necessary and sufficient condition for this to cause a rise in the price of the nontradable is that $b_{22}/b_{12} > b_{21}/b_{11}$, i.e., that the import-competing sector should use a larger ratio of factor 2 to country 1 than the nontradables sector.
(c) Let $\chi$ denote the value of country 1’s currency in amounts of country 2’s. Then the nominal prices of the tradable goods on country 1’s markets will be related to those on the world market by

\begin{align}
    p_1^1 &= \frac{T_1}{\chi} p_1 = \frac{1}{\chi(1 - \sigma_1)} p_1 \quad \text{and} \\
    p_2^1 &= \frac{T_2}{\chi} p_2 = \frac{1 + \tau_2}{\chi} p_2.
\end{align}

We may note from (7) that if $T_1 = T_2 = T$, then a rise in this uniform rate $T$ of subsidy and tariff has exactly the same effect as a fall in $\chi$, i.e., as a devaluation of country 1’s currency. This was first observed by Keynes (1936). Under the same conditions, the domestic price ratio is equal to the foreign price ratio, hence there is no distortion in relative prices—an observation first made by Hicks (1951). These
two observations may be described as the Keynes-Hicks equivalence theorem.

d) Now let us now investigate the effect of a change in a subsidy or tariff factor on the price of the nontradable, when the exchange rate is fixed. We define

\[ \hat{p}_3^1(p_1, p_2, T_1, T_2, \chi) = \hat{p}_3^1(T_1 p_1 / \chi, T_2 p_2 / \chi). \]

The elasticity of the price of the nontradable with respect to the \( j \)th tariff factor is then, using (7) and the homogeneity of degree 1 of \( \hat{p}_3^1 \),

\[ \frac{T_j \hat{p}_3^1}{\hat{p}_3^1 \partial T_j} = \frac{p_j^1 \hat{p}_3^1}{\hat{p}_3^1 \partial p_j^1}. \]

Thus, the conditions for a subsidy or tariff to raise the price of the nontradable are exactly the same as the conditions for a rise in the domestic price of the export or import-competing good to raise the price of the nontradable.

e) Let us now assume that the monetary authority acts so as to stabilize a price index

\[ p^1 = \hat{c}_1^1 p_1^1 + \hat{c}_2^1 p_2^1 + \hat{c}_3^1 \hat{p}_3^1, \]

where the \( \hat{c}_i \) are nonnegative weights and \( \hat{c}_3^1 > 0 \). For definiteness we may think of these weights as defined by the partial derivatives of the demand functions with respect to income

\[ \hat{c}_i^1 = \frac{\partial h_i^1(p_1^1, p_2^1, p_3^1, Y^1)}{\partial Y^1}, \]

where the prices \( p_j^1 \) and income \( Y^1 \) are evaluated at their initial levels. Now substituting (7) into (10) and fixing the price level at \( \hat{p}_1^1 \), we obtain

\[ \hat{p}_1^1 = \hat{c}_1^1 T_1 p_1 / \chi + \hat{c}_2^1 T_2 p_2 / \chi + \hat{c}_3^1 \hat{p}_3^1(T_1 p_1 / \chi, T_2 p_2 / \chi) \]

\[ = \frac{\hat{c}_1^1 T_1 p_1 + \hat{c}_2^1 T_2 p_2 + \hat{c}_3^1 \hat{p}_3^1(T_1 p_1, T_2 p_2)}{\chi}, \]

where use has been made of the homogeneity of degree 1 of the function \( \hat{p}_3^1(p_1^1, p_2^1) \). The exchange rate is then determined from

\[ \chi = \frac{\hat{c}_1^1 T_1 p_1 + \hat{c}_2^1 T_2 p_2 + \hat{c}_3^1 \hat{p}_3^1(T_1 p_1, T_2 p_2)}{\hat{p}_1^1}, \]

the last equation of which defines the function \( \hat{\chi} \). We see then that

\[ \frac{T_1}{\chi} \frac{\partial \hat{\chi}}{\partial T_1} = \frac{p_1^1}{\hat{p}_1^1} \left\{ \hat{c}_1^1 + \hat{c}_3^1 \frac{b_{12}^1 b_{13}^1}{|B_1^1|} \left[ b_{22}^1 - \frac{1}{b_{13}^1} \right] \right\} \]

\[ \frac{T_2}{\chi} \frac{\partial \hat{\chi}}{\partial T_2} = \frac{p_2^1}{\hat{p}_1^1} \left\{ \hat{c}_2^1 + \hat{c}_3^1 \frac{b_{11}^1 b_{13}^1}{|B_1^1|} \left[ b_{23}^1 - \frac{1}{b_{13}^1} \right] \right\}. \]
Thus we see that as long as \( \bar{e}_j^1 \geq 0 \) for \( j = 1, 2 \) and \( \bar{e}_3^1 > 0 \), the inequalities (5) are sufficient for an increase in the rate of subsidy on exports or in the rate of tariff on imports to cause an appreciation of the exchange rate.

Now let us suppose that the subsidy and tariff factors are required to be uniform, i.e., \( T_1 = T_2 = T \). Then using once again the homogeneity of degree 1 of \( \bar{p}_3^1 \) we see that
\[
\frac{\partial \bar{X}}{\partial T} = 1.
\]
This is Keynes’s original proposition (1936).

2 The Three-Factor Case

Letting \( h_j^1(p_1^1, p_2^1, p_3^1, Y^1) \) denote country 1’s Marshallian demand function for commodity \( j \), where \( p_j^1 \) is the price of commodity \( j \) on country 1’s markets and \( Y^1 \) is country 1’s disposable national income, and letting \( \hat{y}_j^1(p_1^1, p_2^1, p_3^1, l^1) \) denote country 1’s supply or Rybczynski function, where \( l^1 \) is the vector of its factor endowments (of dimension at least 3), the equation
\[
h_3^1(p_1^1, p_2^1, p_3^1, \Pi^1(p_1^1, p_2^1, p_3^1, l^1) + D^1) = \hat{y}_3^1(p_1^1, p_2^1, p_3^1, l^1),
\]
—where \( \Pi^1 \) is country 1’s domestic-product function and \( D^1 \) is the deficit in its trade balance—implicitly defines the function
\[
p_3^1 = \tilde{p}_3^1(p_1^1, p_2^1, D^1, l^1).
\]
We verify the following

**Lemma.** The function \( \tilde{p}_3^1(p_1^1, p_2^1, D^1; l^1) \) is homogeneous of degree 1 in \( (p_1^1, p_2^1, D^1) \).

**Proof:** By Euler’s theorem, the assertion is equivalent to the identity
\[
\frac{\partial p_3^1}{\partial p_1^1} + \frac{\partial p_3^1}{\partial p_2^1} + \frac{\partial p_3^1}{\partial D^1} D^1 = \tilde{p}_3^1.
\]
Substituting (17) in (16) and then differentiating the resulting identity with respect to \( p_j^1 \) \((j = 1, 2)\) and \( D^1 \) we obtain
\[
\frac{\partial \tilde{p}_3^1}{\partial p_j^1} = -\frac{s_{3j}^1 - t_{3j}^1 - c_{3j}^1 z_j^1}{s_{33}^1 - t_{33}^1} \quad (j = 1, 2)
\]
and
\[
\frac{\partial \tilde{p}_3^1}{\partial D^1} = -\frac{c_3^1}{s_{33}^1 - t_{33}^1},
\]
where
\[
s_{ij}^1 = \frac{\partial h_i^1}{\partial p_j^1} + \frac{\partial h_i^1}{\partial Y^1} h_j^1, \quad t_{ij}^1 = \frac{\partial \hat{y}_i^1}{\partial p_j^1}, \quad c_i^1 = \frac{\partial h_i^1}{\partial Y^1}, \quad \text{and} \quad z_j^1 = x_i^1 - y_i^1.
\]
Now denoting the Slutsky and transformation matrices

\[ S^1 = [s^1_{i,j}]_{i,j=1,2,3} \quad \text{and} \quad T^1 = [t^1_{i,j}]_{i,j=1,2,3} \]

since the Slutsky matrix \( S^1 \) is negative semi-definite and (from the homogeneity of degree zero of the demand functions in \((p^1, p^1_1, p^1_2, Y^1)\)) satisfies \( S^1 p^1 = 0 \) (where \( p^1 = (p^1_1, p^1_2, p^1_3)^t \)), and likewise the transformation matrix \( T^1 \) is positive semi-definite (from the convexity of the domestic-product function in the prices) and satisfies \( T^1 p^1 = 0 \) (from the homogeneity of degree 1 of the domestic-product function in the prices), we have

\[ (s^1_{31} - t^1_{31})p^1_1 + (s^1_{32} - t^1_{32})p^1_2 + (s^1_{33} - t^1_{33})p^1_3 = 0. \]  

Finally we use country 1’s budget equation

\[ p^1_1 z^1_1 + p^1_2 z^1_2 = D^1. \]  

Combining (19), (20), (22), and (23) we see that

\[
\frac{\partial p^1_3}{\partial p^1_1} p^1_1 + \frac{\partial p^1_3}{\partial p^1_2} p^1_2 + \frac{\partial p^1_3}{\partial D^1} D^1 = - \frac{(s^1_{31} - t^1_{31} - c^1_3 z^1_1) p^1_1 + (s^1_{32} - t^1_{32} - c^1_3 z^1_2) p^1_2 + c^1_3 D^1}{s^1_{33} - t^1_{33}}
\]

\[ = - \frac{(s^1_{33} - t^1_{33}) p^1_3 - c^1_3 (p^1_1 z^1_1 + p^1_2 z^1_2 - D^1)}{s^1_{33} - t^1_{33}} = p^1_3, \]  

as desired. \( \square \)

Thus we see that the 2-factor case is the special case in which the function \( \tilde{p}^1_3 \) is independent of \( D^1 \) and \( l^1 \).

(a) Denoting the export-subsidy rate by \( \sigma_1 > 0 \) and the import-tariff rate by \( \tau_2 > 0 \), both of which are expressed as proportions of the corresponding world price denominated in country-1 currency, and letting \( \chi \) denote the price of country 1’s currency in amounts of the foreign (“world”) currency, the prices of the tradable goods in country 1 are related to the corresponding world prices as in (7) by

\[
p^1_1 = (1 + \sigma_1) p_1 / \chi = T_1 p_1 / \chi \quad \text{and} \quad p^1_2 = (1 + \tau_2) p_2 / \chi = T_2 p_2 / \chi,
\]

where \( T_1 = 1 + \sigma_1, \ T_2 = 1 + \tau_2 \) denote the corresponding tax factors. Country 1’s net revenue from these two instruments, expressed in its own currency, is given by

\[
D^1 = \sigma_1 (p_1 / \chi) z^1_1 + \tau_2 (p_2 / \chi) z^1_2 = (T_1 - 1) (p_1 / \chi) z^1_1 + (T_2 - 1) (p_2 / \chi) z^1_2 = T_1 (p_1 / \chi) z^1_1 + T_2 (p_2 / \chi) z^1_2
\]
where use has been made in the third expression of the fact that country 1’s trade must be balanced when expressed in country-2 prices, i.e., \( p_1 z_1^1 + p_2 z_2^1 = 0 \). Noting that the price and deficit terms (25) and (26) all contain the factor \( 1/\chi \), and recalling that country 1’s trade-demand functions

\[
(27)\quad \hat{h}_j^1(p_1^1, p_2^1, D^1, l^1) = \tilde{h}_j^1(p_1^1, p_2^1, \tilde{p}_3, \Pi^1(p_1^1, p_2^1, \tilde{p}_3, l^1) + D^1) - \hat{y}_j^1(p_1^1, p_2^1, \tilde{p}_3, l^1) \quad (j = 1, 2)
\]

—where the function \( \tilde{p}_3 \) of (17) is substituted for \( p_3 \) in the country’s consumer-demand and Rybczinski functions—are homogeneous of degree 0 in \( (p_1^1, p_2^1, D^1) \), country 1’s trades \( z_1^1, z_2^1 \) may be obtained by solving simultaneously the two equations

\[
(28)\quad \begin{align*}
z_1^1(\cdot) &= \hat{h}_1^1(T_1 p_1, T_2 p_2, T_1 p_1 z_1^1(\cdot) + T_2 p_2 z_2^1(\cdot), l^1) \\
z_2^1(\cdot) &= \hat{h}_2^1(T_1 p_1 T_2 p_2, T_1 p_1 z_1^1(\cdot) + T_2 p_2 z_2^1(\cdot), l^1)
\end{align*}
\]

which implicitly define the excess-demand functions

\[
(29)\quad \begin{align*}
z_1^1(\cdot) &= \tilde{z}_1^1(p_1, p_2, T_1, T_2, l^1), \\
z_2^1(\cdot) &= \tilde{z}_2^1(p_1, p_2, T_1, T_2, l^1).
\end{align*}
\]

Note that these functions are independent of \( \chi \), and homogeneous of degree 0 in the pair of variables \( T_1, T_2 \), as well as the pair \( p_1, p_2 \). Moreover, because of the trade-balance constraint (in foreign, or world, prices)

\[
(30)\quad p_1 z_1^1(\cdot) + p_2 z_2^1(\cdot) = 0
\]

—whence in particular

\[
(31)\quad p_1 \frac{\partial z_1^1}{\partial T_j} + p_2 \frac{\partial z_2^1}{\partial T_j} = 0
\]

—the functions (29) are of course not independent.

(b) We now consider the effect of an export subsidy or import tariff of the price of the nontradable, when the exchange rate is fixed. It will further be assumed that the subsidy and tariff factors \( T_1 \) and \( T_2 \) are initially equal to one another (which of course includes the special case in which they are both = 1, i.e., in which there is initially free trade). Substituting the functions (29) in the expression (26) for country 1’s net revenues from the tariff and subsidy, and then substituting (25) and (26) in turn in (17), we see that the dependence of the nominal price of country 1’s nontradable on the tax factors and the exchange rate is given by

\[
(32)\quad \begin{align*}
p_3^1 &= \tilde{p}_3(T_1 p_1 / \chi, T_2 p_2 / \chi, \Pi^1(p_1^1, p_2^1, \tilde{p}_3, l^1)) + T_2 p_2 \hat{z}_2^1(\cdot, l^1) \\
&= \chi^{-1} \tilde{p}_3(T_1 p_1, T_2 p_2, T_1 p_1 z_1^1(\cdot) + T_2 p_2 z_2^1(\cdot), l^1) \\
&= \hat{p}_3(p_1, p_2, T_1, T_2, \chi, l^1),
\end{align*}
\]

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where account is taken of the homogeneity of degree 1 of the function \( \bar{p}_3 \) of (17) in the variables \( p_1, p_2, D^1 \). The last equation of (32) defines the function \( \bar{p}_3 \); it gives the effect of an exogenous change in either tax factor or in the exchange rate on the domestic price of the nontradable good.

Differentiating \( \bar{p}_3 \) with respect to \( T_j \) we have

\[
\frac{\partial \bar{p}_3}{\partial T_j} = \frac{1}{\chi} \left[ \frac{\partial \bar{p}_3}{\partial p_j} p_j + \frac{\partial \bar{p}_3}{\partial D^1} \left( p_j z_j^1 + T_1 p_1 \frac{\partial z_1^1}{\partial T_j} + T_2 p_2 \frac{\partial z_2^1}{\partial T_j} \right) \right].
\]

Now using the assumption that \( T_1 = T_2 \) initially, combined with (31), and substituting (19) and (20) in (33), (33) becomes

\[
\frac{\partial \bar{p}_3}{\partial T_j} = \frac{p_2}{\chi} \left( \frac{\partial \bar{p}_3}{\partial p_j} + \frac{\partial \bar{p}_3}{\partial D^1} z_j^1 \right) = -\frac{p_j}{\chi} \frac{s_{3j}^1 - t_{3j}^1}{s_{33}^1 - t_{33}^1}.
\]

Expressed as an elasticity this becomes, using (25),

\[
\frac{T_j}{p_3} \frac{\partial \bar{p}_3}{\partial T_j} = \frac{p_2}{p_3} \left( \frac{\partial \bar{p}_3}{\partial p_j} + \frac{\partial \bar{p}_3}{\partial D^1} z_j^1 \right) = -\frac{1}{p_3} \frac{1}{s_{33}^1 - t_{33}^1} \left( s_{3j}^1 - t_{3j}^1 \right) (j = 1, 2),
\]

showing that the effect of a tariff factor, unlike the two-factor case, includes an “income effect” resulting from its impact on government revenues.

We shall say that tradable commodity \( j = 1, 2 \) is a net substitute of the nontradable commodity 3 if and only if \( s_{3j}^1 - t_{3j}^1 > 0 \), that is, if and only if the Slutsky substitution term less the production transformation term between commodities \( j \) and 3 is positive. We see then that: If the tariff rate on imports is initially equal to the subsidy rate on exports (in particular, if they are both initially equal to zero), and if the exchange rate is fixed, then an import tax will lead to a rise in the price of the nontradable if and only if the export good is a net substitute of the nontradable good, and an import tariff will lead to a rise in the price of the nontradable if and only if the import good is a net substitute of the nontradable good.

(c) Now let us assume that the exchange rate is flexible and the monetary authority stabilizes a price index (10) of all three commodities. Such a price index for country 1 will then be given by

\[
\bar{p}^1 = \bar{c}_1 p_1 + \bar{c}_2 p_2 + \bar{c}_3 p_3
\]

\[
= \bar{c}_1 T_1 p_1 / \chi + \bar{c}_2 T_2 p_2 / \chi + \bar{c}_3 p_3 \left( T_1 p_1, T_2 p_2, T_1 p_1 z_1^1 (\cdot) + T_2 p_2 z_2^1 (\cdot), l^1 \right) / \chi.
\]

Assuming the monetary authority to stabilize \( \bar{p}^1 \), the exchange rate is determined by

\[
\chi = (\bar{p}^1)^{-1} \left[ \bar{c}_1 T_1 p_1 + \bar{c}_2 T_2 p_2 + \bar{c}_3 p_3 \left( T_1 p_1, T_2 p_2, T_1 p_1 z_1^1 (\cdot) + T_2 p_2 z_2^1 (\cdot), l^1 \right) \right]
\]

\[
\equiv \chi(T_1, T_2, p_1, p_2, \bar{p}^1, l^1; \bar{c}_1, \bar{c}_2, \bar{c}_3),
\]

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which defines the function \( \tilde{\chi} \). We find then without difficulty that
\[
\frac{T_1}{\chi} \frac{\partial \tilde{\chi}}{\partial T_1} = \frac{p_1}{\bar{p}_1} \left[ \tilde{e}_1 - \frac{e_3^3}{e_3^3 - t_{33}} \frac{e_3^1}{e_3^3 - t_{33}} \right]
\]
\[
\frac{T_2}{\chi} \frac{\partial \tilde{\chi}}{\partial T_2} = \frac{p_2}{\bar{p}_1} \left[ \tilde{e}_2 - \frac{e_3^3}{e_3^3 - t_{33}} \frac{e_3^1}{e_3^3 - t_{33}} \right].
\]

References


