Econ 8402: International Trade & Payments
Problem Set #2

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Marshalian offer functions with constant elasticity of substitution

Let the inhabitants of two countries have their demand functions generated by the same constant-elasticity-of-substitution utility function for two commodities,

\[ U(x_1, x_2) = (x_1^{-\beta} + x_2^{-\beta})^{1/\beta} \]

where \( \beta = 1/\sigma - 1 > -1 \), and \( \sigma > 0 \) is the constant elasticity of substitution. Let \( x_i^k \) be the rate of consumption of commodity \( i \) in country \( k \). Assume that in each country, \( k \), there is a fixed rate of production \( y_i^k = \omega_i^k \) of commodity \( i \). Country \( k \)'s trade-demand function \( z_i^k = \hat{h}_i^k(p_1, p_2, D^k; \omega^k) \) is the excess over \( \omega_i^k \) of its ordinary demand function \( h_i^k(p_1, p_2, Y^k) \) (generated by the utility function (1)), where \( Y^k = p_1\omega_1^k + p_2\omega_2^k + D^k \) is its disposable national income, \( D^k \) is the deficit in its balance of payments on current account (which will be assumed to be zero), and \( p_i \) is the world price of commodity \( i \). Material balance \( z_1^1 + z_2^1 = 0 \) will be assumed, where \( z_i^k = x_i^k - y_i^k \).

Country 2's inverse trade-demand function \( \hat{r}_2(z_1^1) \) is defined implicitly as the solution of the equation

\[ z_1^2 = \hat{h}_1^2(\hat{r}_1(z_1^2), 1, 0; \omega^k), \]

which holds for \( \partial \hat{h}_1^2 / \partial p_1 < 0 \). With balanced trade \( D^k = p_1z_1^k + p_2z_2^k = 0 \) assumed, country 2’s Marshallian reciprocal demand (or “offer”) function is defined as

\[ -z_2^2 = F^2(z_1^2) \equiv \hat{r}_1(z_1^2)z_1^2 \]

(similarly for country 1). Country 2’s Marshallian elasticity of demand for imports (assuming \( z_1^2 > 0 \)) is defined as

\[ \eta^2 = -\frac{p_1}{\hat{h}_1^2} \frac{\partial \hat{h}_1^2}{\partial p_1}. \]

In parts (b) and (c) below, the following two cases will be taken up:

(i) \( (\omega_1^1, \omega_1^2) = (2, 1) \) and \( (\omega_2^1, \omega_2^2) = (1, 2) \),

(ii) \( (\omega_1^1, \omega_1^2) = (1, 0) \) and \( (\omega_2^1, \omega_2^2) = (0, 1) \)
(a) Derive the formula for the demand function \( x_i = h_i(p_1, p_2, Y) \) generated by maximizing (1) subject to the budget constraint \( p_1x_1 + p_2x_2 \leq Y \). (ii) Further, derive the formula for country \( k \)'s trade-demand function

\[
z_i^k = \hat{h}_i(p_1, p_2, D^k; \omega^k) = h_i(p_1, p_2, p_1\omega_i^k + p_2\omega_2^k + D^k) - \omega_i^k.
\]

(b) For the case \( \sigma = 1 \), derive explicit formulas for the two offer functions \( -z_2^2 = F^2(z_2^1) \) and \( -z_1^1 = F^1(z_2^1) \). In each of cases (i) and (ii), sketch their graphs in the ranges \( 0 \leq z_1^1 \leq 1, 0 \leq z_2^1 \leq 1 \), and determine the world equilibrium. Discuss the stability of this equilibrium in terms of the simplified Marshall-Samuelson dynamic-adjustment process

\[
\begin{align*}
 z_1^2 &= P_1(z_1^2, z_1^1) \equiv F^1(z_1^1) - z_1^2 \\
 z_2^2 &= P_2(z_1^2, z_2^1) \equiv F^2(z_1^1) - z_2^1.
\end{align*}
\]

(c) For the case \( \sigma = 0.1 \), obtain grids of the functions \( F^1(z_2^1) \) and \( F^2(z_2^1) \) in the ranges \( 0 \leq z_1^1 \leq 1, 0 \leq z_2^1 \leq 1 \) and intervals of 0.1, for each of cases (i) and (ii). HINT: This will require use of root-solving techniques. Determine the world equilibrium and discuss its stability properties.