Part I

1 Optimal tariff and retaliation

Suppose two countries are trading two goods, with constant production by country $k$ per period of $\omega_k^T$ units of good $i$, and consumption $x_k^i$ in each country $k$ given by a demand function generated by a utility function $U(x_k^i, x_k^j) = x_k^i x_k^j$. Starting from free trade, country 1 imposes an optimal tariff on its import of good 2 from country 2, and thereafter country 2 in retaliation imposes an optimal tariff on its import of good 1 from country 1. The constant rates of output are given by the matrix

$$\begin{bmatrix}
\omega_1^1 & \omega_1^2 \\
\omega_2^1 & \omega_2^2
\end{bmatrix} = \begin{bmatrix}
2 & 1 \\
1 & 2
\end{bmatrix}.$$ 

(a) Derive an exact expression for country 1’s tariff-modified offer function $-z_1^T = F^1(z_2^T; T_2)$ for any tariff factor $T_2 = 1 + \tau_2$ that it imposes on its import of good 2 from country 2.

(b) Assuming that country 2 is not initially imposing any tariff, determine the magnitude of country 1’s optimal tariff.

(c) Given the formula you obtained in (a), where $T_2$ is country 1’s optimal tariff factor on good 2 against country 2, find country 2’s retaliatory optimal tariff factor $T_1$ on good 1 against country 1.

(d) Determine whether both countries lose from this two-step process or whether one of them gains.

(e) (optional) Derive the expression for country 2’s elasticity of demand for imports, $\eta^2 = -(p_1/z_1^2) \left( \partial z_2^T / \partial p_1 \right)$, where $z_2^T(p_1, p_2, 0, \omega^2)$ is country 2’s tariff-free excess-demand function, and show that

(i) this elasticity is not constant;

(ii) at the point $(z_1^T, z_2^T)$ of country 1’s optimal tariff, the optimal tariff rate you obtained is given by Johnson’s formula $\tau_2 = 1/(\eta^2 - 1)$. 


Part II

2 Autarky and trade under external economies of scale

Suppose a small open economy with one factor of production (labor) produces two commodities with production functions

\[ y_1 = v_1^{1/2}, \quad y_2 = v_2^2, \]

where \( y_i \) is the output of commodity \( i \) and \( v_i \) is the input of labor into industry \( i \), these satisfying the resource-allocation constraint

\[ v_1 + v_2 = l, \]

\( l \) being the labor endowment. Assume that individual firms operate under the assumption that their production functions are of the form \( y_i = k_i v_i \), where (unknown to them) \( k_1 = v_1^{-1/2} \) and \( k_2 = v_2 \) respectively, hence they equate the average cost of produced commodities (their perceived marginal cost) to the price. Finally, let consumers in the country have identical utility functions of the form

\[ U(x_1, x_2) = x_1 x_2, \]

where \( x_i \) is the consumption of commodity \( i \).

Let the labor endowment \( l \) and the world price ratio \( p = p_1/p_2 \) be assumed to have the values

\[ l = 3/4 \quad \text{and} \quad p = 1/4. \]

(a) Find the equilibrium of this country under autarky.

(b) Under free trade, show that there will be

(i) an equilibrium in which the country specializes in commodity 1, and in which its potential welfare (as measured by the utility function) is lower than it was under autarky.

(ii) an equilibrium in which the country produces positive amounts of both commodities, exports commodity 2 and imports commodity 1, and is better off than it was under autarky.

3 Effect of a capital inflow on a small open economy

Let a small open economy produce two tradables and one nontradable with at least three factors of production, and let world prices of the tradables, denominated in foreign currency, be given. Let preferences in this economy be aggregable. Denote the domestic-currency prices of the export, import, and nontradable good by \( p_1, p_2, \) and \( p_3 \), respectively, and let the foreign-currency prices of the country’s export and import good be denoted \( p_1^* \) and \( p_2^* \); further, define the open economy’s exchange rate, \( \chi \), as the value of its currency in terms of foreign currency, so that \( p_1 = p_1^*/\chi \) and \( p_2 = p_2^*/\chi \). Starting from balanced trade, let this country receive an exogenous capital inflow of \( D^* \) expressed in foreign currency, so that its balance-of-payments deficit is
$D = D^*/\chi$ expressed in its own currency. Assume that the country’s central bank pursues a policy to stabilize the general price level, defined by

$$\bar{c}_1 p_1 + \bar{c}_2 p_2 + \bar{c}_3 p_3 = \bar{p},$$

where $\bar{c}_i > 0$ for $i = 1, 2, 3$ and $\bar{c}_1 + \bar{c}_2 + \bar{c}_3 = 1$, and the $\bar{c}_i$ may be interpreted as the amounts consumed of these commodities in the initial equilibrium.

Find the effect of the capital inflow on (a) the country’s exchange rate, and (b) the domestic price of its nontradable.