Welfare Effects of Trade-Diverting Customs Unions: A Quantitative Approach

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Welfare Effects of Trade-Diverting Customs Unions: A Quantitative Approach

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Summary. The seminal analysis of trade-diverting customs union introduced by Gehrels and Lipsey in the 1950s is reconsidered from a quantitative point of view. It is found that, if utility functions are assumed to have a constant elasticity of substitution, then when this elasticity of substitution is low (say less than one-quarter), the precustoms-union tariff rates would have to be extremely high (above 1800%) in order for the trade-diverting customs union to be welfare-improving. This seems to accord well with Viner’s view.

1. Introduction

The seminal contributions of Jacob Viner (1931, 1950) to the analysis of customs unions led to his conclusion (1950, p. 44) that if the union was “trade-creating”, i.e., one that led to trade in commodities that had not previously been traded, then it was beneficial to the partner countries, and possibly but not likely beneficial to third countries; whereas if the union was “trade-diverting”, i.e., one that led to a shift in the locus of production of traded goods from low-cost third countries to higher-cost partner countries, then it would be definitely detrimental both to the partner countries themselves and to the rest of the world.

This conclusion was challenged by Meade (1955, pp. 39–41) on the ground that it overlooked the possible benefits to consumers from substitution in consumption resulting from a trade-diverting customs union. Shortly thereafter the same point was made independently by Gehrels (1956) and Lipsey (1957), who employed a geometric analysis illustrated by Figure 1.1 below.

Suppose country 1 is engaged in trade of two commodities with countries 2 and 3, both of which produce under constant costs and are sufficiently large relative to country 1 that under free trade country 3’s cost ratio forms the world price ratio, whereas under a customs union with country 2 it is country 2’s cost ratio that determines the price ratio for the union. Country 1 specializes in the production of commodity 1, producing an amount indicated by the point A. Under free trade its budget line is AW whose slope (equal to 1 in the illustration) is country 3’s cost ratio; the free-trade equilibrium is given by the point of tangency Q⁰ of the indifference curve U⁰ with this budget line. Under a customs union with country 2, which produces commodity 2 at a higher cost than country 3—indicated by the slope of the line AP—equilibrium takes place at the point of tangency of the indifference curve U* with the budget line AP. Let this indifference curve intersect AW at a point
Q*, and draw a tangent to U* at Q*. The slope of this tangent (referred to the vertical axis) is the (nondiscriminatory) tariff factor that country 1 must impose on its import of commodity 2 from countries 2 and 3 in order to reach the same welfare level as obtained under the customs union (it is assumed that the proceeds of the tariff are distributed to country 1’s consumers). It is clear that the same construction can be created for an indifference curve slightly below U* (say U1, not shown) which intersects AW at Q1 (also not shown), and therefore that there exists a sufficiently high pre-customs-union tariff so that country 1 would benefit from moving from this situation to the customs union. This is the welfare-improving trade diversion.

The problem can be posed in two ways: (1) For a given initial nondiscriminatory tariff (say, that given by the tangent at Q*), if one can find a country whose cost ratio lies between AW and AP, then a customs union with that country will be welfare-improving; (2) For a given cost ratio AP of a prospective customs-union partner, one can find a pre-customs-union nondiscriminatory tariff high enough so that movement from that tariff to the customs union will be beneficial. The problem is usually posed in the first way (cf. Lipsey (1957, 1960), Bhagwati (1971, 1973), Kirman (1973), Johnson (1974)). However, Lipsey (1960, pp. 499-500) claimed, joined by Meade (1955) and Corden (1965), that Viner in order to obtain his result must have assumed that commodities are consumed in fixed proportions; this suggests the second formulation, which looks for a sufficient condition for trade-diverting customs union to be welfare-improving. Actually, one can imagine that the family of indifference curves of Figure 1.1 is bounded from below and asymptotic to (but not reaching) an L-shaped curve—any consumption bundles below it being considered intolerable; in this case, if the
budget line AP passed below and to the left of this bounding curve, there would not exist a nondiscriminatory tariff that would make the country worse off than under the customs union. Thus, Lipsey’s assumption of L-shaped indifference curves is not needed to get Viner’s result—whether or not Viner intended this assumption.

Viner himself (1976) specifically and categorically rejected Lipsey’s assertion concerning his implicit assumption. However, Michaely (1976) responded that Viner’s argument that there can be no welfare gains from moving from a nondiscriminatory tariff to a trade-diverting customs union would then be incorrect. While it is not my aim in this paper to reconstruct “what Viner really meant”, I present an argument that he probably would have found congenial.

An example corresponding to Figure 1.1 is given in subsection 3.4 below, in which the utility function has an elasticity of substitution of one-quarter and the tangent at \( Q^* \) has a slope of slope of 0.051297 (referred to the horizontal axis) whose reciprocal (the tariff factor) is 19.4943. This corresponds to a tariff rate of 1849.43%. One would have to go back to the Peronist regime in Argentina to find tariffs of comparable magnitude, and even then they were not that high. But as Table 3.1 shows (for the parameters assumed in our illustration), the critical tariff factor increases as the elasticity of substitution decreases, so that at an elasticity of substitution of 1/33, the pre-customs-union tariff rate would have to exceed 700 million percent in order for a trade-diverting customs union to be welfare-improving. A perhaps more accurate description of Lipsey’s theorem is that for any customs-union equilibrium, one can find a nondiscriminatory tariff which would make the country even worse off than under the customs union, while (in his model) free trade would always make it better off.

2. The Formal Model

According to Lipsey’s model (Lipsey (1957) and (1960)), there are three countries, two commodities, and constant costs (hence one factor) in each country. Country 3 is “large,” so that its cost ratio determines the world prices. Countries 1 and 2 initially have nondiscriminatory (most-favored-nation) tariffs on their imports of commodity 2 from country 1, and they both export commodity 1 to country 3 and do not trade with each other. They then form a customs union, which makes the existing tariff on imports of commodity 2 from country 3 prohibitive; country 1 then exports commodity 1 to and imports commodity 2 from country 2, at country 2’s cost ratio.

Let \( k \) index the three countries, 1, 2, and 3, and \( j \) the two commodities 1 and 2 (wheat and clothing in Lipsey’s example). Let \( t \) index three situations: \( t = 0 \) corresponds to free trade; \( t = 1 \) corresponds to the initial situation in which countries 1 and 2, prior to their customs union, impose nondiscriminatory tariffs on country 3’s export good; and \( t = 2 \) corresponds to the situation
after the formation of a customs union between countries 1 and 2, in which a common prohibitive tariff is imposed against country 3’s export good. The following notation will be used:

\[ p^k_j (t) = \text{price of commodity } j \text{ in country } k \text{ in situation } t. \]
\[ y^k_j (t) = \text{output of commodity } j \text{ in country } k \text{ in situation } t. \]
\[ x^k_j (t) = \text{consumption of commodity } j \text{ in country } k \text{ in situation } t. \]
\[ Y^k(t) = \text{nominal national income of country } k \text{ in situation } t. \]
\[ y^k_j = \text{maximum output of commodity } j \text{ in country } k. \]
\[ \tau^k_j (t) = \text{tariff rate on country’s import of commodity } j \text{ in situation } t. \]
\[ T^k_j (t) = \text{tariff factor } = 1 + \tau^k_j (t). \]

Production possibilities in country \( k \) are given by

\[ \frac{y^k_1}{y^k_1} + \frac{y^k_2}{y^k_2} \leq 1 \quad (k = 1, 2, 3), \quad (2.1) \]

where \( y^k_j \) is country \( k \)'s maximum output of commodity \( j \), and it is assumed that

\[ \frac{y^1_j}{y^1_1} < \frac{y^2_j}{y^2_2} < \frac{y^3_j}{y^3_1}, \quad (2.2) \]

i.e., country 3 is the low-cost producer of commodity 2, country 2 the next-lowest-cost producer, and country 1 the high-cost producer. I shall assume that preferences are homothetic and identical within and as between countries. Figure 2.1 illustrates a free-trade world equilibrium corresponding to the maximum outputs of the three countries given in Table 2.1, in which a world indifference curve in tangential to the world production-possibility set at a point on the frontier corresponding to country 3’s cost ratio; the three countries’ production-possibility sets are indicated by the three marked areas. Figure 2.2 illustrates the corresponding customs-union equilibrium in which countries 1 and 2 trade only with each other, and a customs-union indifference curve is tangential to the customs union’s production-possibility set at a point on its frontier corresponding to country 2’s cost ratio.\(^1\)

<table>
<thead>
<tr>
<th>Commodity 1</th>
<th>Country 2</th>
<th>Country 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Commodity 1</td>
<td>( \hat{y}^1_1 = 2.0 )</td>
<td>( \hat{y}^2_2 = 4.0000 )</td>
</tr>
<tr>
<td>Commodity 2</td>
<td>( \hat{y}^2_2 = 0.5 )</td>
<td>( \hat{y}^3_2 = 2.5600 )</td>
</tr>
<tr>
<td>Cost ratios</td>
<td>( \hat{\rho}^3_2 = 4.0 )</td>
<td>( \hat{\rho}^3_2 = 1.5625 )</td>
</tr>
</tbody>
</table>

\(^1\) Diagrams such as Figure 2.1 and 2.2 are found in Lipsey (1970, pp. 82ff.), but without the world or customs-union indifference curves. The 45°-line indicates (as in Lipsey’s diagrams) the position of the kinks in the symmetric indifference curves when the elasticity of substitution becomes zero.
Fig. 2.1. World production-possibility set defined by the parameters of Table 2.1, and a symmetric maximizing world indifference curve tangential to country 3’s production-possibility frontier.

Fig. 2.2. Customs-union production-possibility set, showing a maximizing symmetric customs-union indifference curve tangential to country 2’s production-possibility frontier.
In the initial pre-customs-union situation,
\[ p_1^k(1) = p_1^0(0); \quad p_2^k(1) = [1 + \tau_2^k(1)]p_2^0(0) = T_2^k(1)p_2^0(0) \quad (k = 1, 2), \quad (2.3) \]
where \( \tau_2^k(1) \) is the (nondiscriminatory) tariff rate, and \( T_2^k(1) = 1 + \tau_2^k(1) \) the corresponding tariff factor, imposed by country \( k = 1, 2 \) on its imports of commodity 2 (which it obtains only from the low-cost producer, country 3). Since country 3’s cost ratio determines the world prices, country 3 produces both commodities and hence, since it is assumed to have no tariffs,
\[ p_1^3(0)\bar{y}_1^3 = p_2^3(0)\bar{y}_2^3. \quad (2.4) \]

Denoting the cost ratios by
\[ \rho_2^k = \frac{\bar{y}_2^k}{\bar{y}_2^3}, \quad (k = 1, 2, 3), \quad (2.5) \]
we have from (2.2) and (2.5)
\[ \rho_2^1 < \rho_2^2 < \rho_2^3. \quad (2.6) \]

We have initially
\[ \bar{y}_1^k(1) = \bar{y}_1^3; \quad \bar{y}_2^k(1) = 0 \quad (k = 1, 2). \quad (2.7) \]

Now let us consider country 1’s situation before and after the customs union. Before the customs union, its disposable national income is
\[ Y^1(1) = p_1^3(0)\bar{y}_1^1 + \tau_2^1(1)p_2^3(0)x_2^1(1), \quad (2.8) \]
the second term of which consists of the tariff revenues. In order to allow for simple computations, we will assume that preferences are identical and homothetic within countries. The demand for commodity \( j \) in country \( k \) is given by
\[ x_j^k = h_j^k(p_1^k, p_2^k, Y^k) = Y^k c_j^k(p_1^k, p_2^k), \quad (2.9) \]
where \( c_j^k(p_1^k, p_2^k) = h_j^k(p_1^k, p_2^k, 1) \). Since the demand function can be generated by a homogeneous-of-degree-1 utility function \( U^k(x_1^k, x_2^k) \), defining the indirect utility function of country \( k \) by
\[ V^k(p_1^k, p_2^k, Y^k) = U^k[h_1^k(p_1^k, p_2^k, Y^k), h_2^k(p_1^k, p_2^k, Y^k)] \quad (2.10) \]
and using homogeneity and (2.9), we have
\[ V^k(p_1^k, p_2^k, Y^k) = \frac{Y^k}{C^k(p_1^k, p_2^k)}, \quad (2.11) \]
where
\[ C^k(p_1^k, p_2^k) = \frac{1}{U^k[c_1^k(p_1^k, p_2^k), c_2^k(p_1^k, p_2^k)]} \quad (2.12) \]
may be interpreted as a cost-of-living index. Clearly, \( C^k \) is homogeneous of degree 1. From the Antonelli-Allen-Roy (AAR) partial differential equation
\[ h^k_j(p^k_1, p^k_2, Y^k) = \frac{\partial V^k(p^k_1, p^k_2, Y^k)}{\partial Y^k}/\partial p^k_j \]  

(2.13)

we have from (2.9)

\[ c^k_j(p^k_1, p^k_2) = \frac{1}{C^k(p^k_1, p^k_2)} \frac{\partial C^k(p^k_1, p^k_2)}{\partial p^k_j}. \]

(2.14)

Thus, \( c^k_j \) is homogeneous of degree \(-1\).

Substituting (2.9) in (2.8) and using (2.3) we obtain for country 1 in the pre-customs-union situation,

\[
Y^1(1) = \frac{p^1_1(0) \bar{y}^1_1}{1 - \tau^1_2(1) p^3_2(0) c^1_2 \left( p^3_1(0), T^1_2(1) p^2_2(0) \right)}
= \frac{p^1_1(0) \bar{y}^1_1}{1 - \tau^1_2 p^3_2 c^1_2 \left( 1, T^1_2(1) p^2_2 \right)},
\]

(2.15)

the last expression following from (2.4), (2.5), and the homogeneity of degree \(-1\) of \( c^1_2 \). Further, country 1’s cost-of-living function in the pre-customs-union situation is

\[
C^1(p^1_1(0), T^1_2(1) p^3_2(0)) = p^3_3(0) C^1 \left( 1, T^1_2(1) p^3_2 \right),
\]

(2.16)

from (2.4), (2.5), and the homogeneity of degree 1 of \( C^1 \). From (2.11), country 1’s potential welfare in the initial (pre-customs-union) situation is then given by

\[
W^1(1) = V^1 \left( p^1_1(0), T^1_2(1) p^3_2(0), Y^1(1) \right)
= \frac{\bar{y}^1_1}{1 - \tau^1_2 p^3_2 c^1_2 \left( 1, T^1_2(1) p^3_2 \right)} C^1 \left( 1, T^1_2(1) p^3_2 \right).
\]

(2.17)

After the customs union we have

\[
p^1_1(2) = p^3_1(0); \quad p^1_2(2) = \frac{p^2_1(2)}{p^1_1(2)} p^3_1(0) = p^3_2 p^3_1(0),
\]

(2.18)

the last equality following from the assumption that in the customs-union situation the exchange between countries 1 and 2 is carried out at country 2’s cost ratio, hence country 2 produces both commodities and we have

\[
\frac{p^2_2(2)}{p^1_2(2)} = \frac{\bar{y}^2_1}{\bar{y}^2_2} = p^2_2.
\]

(2.19)

Country 1’s disposable national income is now given simply by

\[
Y^1(2) = p^3_1(0) \bar{y}^1_1,
\]

(2.20)

and its potential welfare is, from (2.11) and (2.18),
\[ W^1(2) = V^1\left(p_1^2(0), \rho_2^2 p_2^2(0), Y^1(2)\right) \]
\[ = \frac{\hat{p}_1^2(0) \tilde{y}_1}{C^1\left(p_2^2(0), \rho_2^2 p_2^2(0)\right)} = \frac{\hat{g}_1^1}{C^1(1, \rho_2^2)}. \quad (2.21) \]

The ratio of (2.21) and (2.17) is
\[ \frac{W^1(2)}{W^1(1)} = \frac{1 - \tau_2^1(1) \rho_2^2 c_2^1\left(1, T_2^1(1) \rho_2^2\right)}{C^1\left(1, \rho_2^2\right)} \]
\[ \times \frac{C^1\left(1, T_2^1(1) \rho_2^2\right)}{C^1(1, \rho_2^2)}. \quad (2.22) \]

We need to find sufficient conditions for welfare improvement, i.e., for the expression in (2.22) to be greater than unity.

From (2.22) and (2.14), the inequality \( W^1(2)/W^1(1) > 1 \) is equivalent to
\[ \frac{C^1\left(1, T_2^1(1) \rho_2^2\right) - C^1(1, \rho_2^2)}{T_2^1(1) \rho_2^2 - \rho_2^2} > \frac{\partial C^1\left(1, T_2^1(1) \rho_2^2\right)}{\partial \rho_2^2}. \quad (2.23) \]

Now, strict concavity of the function \( C^1 \) implies
\[ \frac{C^1\left(1, T_2^1(1) \rho_2^2\right) - C^1(1, \rho_2^2)}{T_2^1(1) \rho_2^2 - \rho_2^2} > \frac{\partial C^1\left(1, T_2^1(1) \rho_2^2\right)}{\partial \rho_2^2}. \quad (2.24) \]

If it were the case that \( \rho_2^2 \geq \rho_2^3 \), then (2.24) would imply (2.23) for any \( T_2^1(1) > 1 \); but on the contrary, \( \rho_2^3 < \rho_2^2 \) from (2.6). What needs to be shown is that for given \( \rho_2^3 < \rho_2^2 \), there exists a \( T_2^1(1) \) such that (2.23) holds. The problem can be posed as follows.

\[ \text{Fig. 2.3. A country's cost-of-living function } C(1, p) \text{ and its supporting tangent line from the point } P \text{ whose coordinates are } 1 \text{ (country 3's cost ratio) and } C(1, R) \text{ where } R \text{ is the partner country's cost ratio.} \]
In Figure 2.3 (where we set $\rho_3^1 = 1$, $\rho_2^1 = R$, $p_1^1 = 1$, $p_2^1 = p$, $T_2^1(1) = T$, $T_2^2(1) = T^*$, and $C^1 = C$), from the point P with coordinates $\left(\rho_2^1, C^1(1, \rho_2^1)\right) = (1, C(1, R))$, draw a tangent (if possible) to the curve $\left(p_1^1, C^1(1, p_1^1)\right) = (p, C(1, p))$. Let the point of tangency be denoted Q, with coordinates $\left(p_1^1, C^1(1, p_1^1)\right) = (T^*, C(1, T^*))$, where we define $T_2^1(1) = p_1^1$. Then $T_2^1(1)$ satisfies the critical equality

$$
\frac{C^1(1, T_2^1(1)p_2^1) - C^1(1, \rho_2^1)}{T_2^1(1)p_2^1 - \rho_2^1} = \frac{\partial C^1(1, T_2^1(1)p_2^1)}{\partial p_2^1}.
$$

(2.25)

In Figure 2.3, the slope $[C(1, T^*) - C(1, R)]/(T^* - 1)$ is given by the tangent of the angle QPA at P, i.e., the ratio QA/AP, which is the same as the slope of $C(1, p)$ at $p = T^*$. Now choose any $T_2^1(1) > T_2^1(1)$, and let S be the point with components $\left(T_2^1(1)p_2^1, C^1(1, T_2^1(1)p_2^1)\right) = (T, C(1, T))$. Draw a straight line from P to S. Since $C^1$ is increasing and strictly concave, this line must pass under the point Q and pass through S; therefore its slope, which is given by the expression on the left side of the inequality (2.23), must be greater than the slope of $C^1$ at S, which is the given by the expression on the right side of the inequality (2.23). In Figure 2.3, the slope $[C(1, T) - C(1, R)]/(T - 1)$ is given by the tangent of the angle SPB at P, i.e., the ratio SB/BP, which is steeper than the slope of $C(1, p)$ at $p = T$. In the next section we show how to compute $T_2^2(1)$ explicitly in some simple cases.

The above development, from (2.8) up to this point, has all been carried out in terms of the point of view of country 1. However, it is clear that an exactly similar analysis holds for country 2, since before the customs union they were both importing commodity 2 from country 3 (the low-cost country) subject to a nondiscriminatory tariff, and after the customs union they were both trading exclusively with each other at country 2’s cost ratio. Therefore all the above development holds equally well for country 2, with country-2 superscripts taking the place of country-1 superscripts. It follows that if country 1 gains from the customs union, so must country 2. And since country 3 is so large that its cost ratio determines the world price ratio, it neither gains nor loses from trade nor from variations in the patterns of

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2 That this need not always be possible for an arbitrary increasing concave function $C(1, p)$ can be seen from the example $C(1, p) = 1 + p - e^{-p}$. This curve is asymptotic to the line $1 + p$. The basic equality (2.25) below, written $[C(1, T^*) - C(1, R)]/(T^* - 1) = \partial C(1, T^*)/\partial p$, reduces to $1 - T^*e^{-T^*} = R - e^{-R}$. Now $f(T^*) = 1 - T^*e^{-T^*}$ and $g(R) = R - e^{-R}$ are both increasing concave functions, $f$ ranging from $1 - e^{-1}$ to 1 and $g$ ranging from $1 - e^{-1}$ to $\infty$; $g(R) < 1$ for $R < R^* = 1.2784645276$, for $1 < R < R^*$ there exists a $T^*$ satisfying (2.25). For example, if $R = 1.1$ then $g(R) = 0.767128916$ and (2.25) is satisfied for $T^* = 2.28257403496$. However, for $R > R^*$ (as in our example where $R = 1.5625$), no solution of (2.25) exists. This corresponds to the situation mentioned in the Introduction.
trade. Thus it can be said that “the world” gains from the customs union—provided the pre-customs-union tariff was sufficiently high. But it can also be said that “the world” also gains, and gains considerably more, from free trade—without the need for any such qualification.

3. The Case of CES Utility Functions

We shall consider four special cases of the CES (constant-elasticity-of-substitution) utility function

$$U(x_1, x_2) = [\alpha_1 x_1^{-\beta} + \alpha_2 x_2^{-\beta}]^{-1/\beta} \quad (\alpha_i > 0; \quad \beta > -1) \tag{3.1}$$

where $\sigma = 1/(1 + \beta)$ is the constant elasticity of substitution (country superscripts have been suppressed in (3.1) for convenience). When $\beta \rightarrow 0$, (3.1) reduces (as is well known—cf., e.g., Chipman (1966, pp. 57–9)) to the Cobb-Douglas function

$$U(x_1, x_2) = x_1^{\alpha'_1} x_2^{\alpha'_2} \quad \text{where } \alpha'_i = \alpha_i / (\alpha_1 + \alpha_2) \tag{3.2}$$

with $\sigma = 1$. At the other extreme, when $\beta \rightarrow \infty$ (3.1) reduces to the Kantorovich form ascribed to Viner by Lipsey,

$$U(x_1, x_2) = \min \left( \frac{x_1}{\alpha_1}, \frac{x_2}{\alpha_2} \right) \tag{3.3}$$

with $\sigma = 0$. We shall also consider the intermediate cases $\beta = 1$, which leads to the simple form

$$U(x_1, x_2) = \frac{1}{\alpha_1 + \alpha_2} \tag{3.4}$$

with $\sigma = 1/2$, and $\beta = -1/2$, which leads to the form

$$U(x_1, x_2) = (\alpha_1 x_1^{1/2} + \alpha_2 x_2^{1/2})^2 \tag{3.5}$$

with $\sigma = 2$.

Setting ratios of marginal utilities equal to price ratios and using the budget constraint, the unit-income demand functions of (2.9) are seen to be

$$c_i(p_1, p_2) = \frac{\alpha_i^{\frac{\beta}{\beta + 3}} p_i^{\frac{-\beta}{\beta + 3}}}{\alpha_1^{\frac{\beta}{\beta + 3}} p_1^{\frac{-\beta}{\beta + 3}} + \alpha_2^{\frac{\beta}{\beta + 3}} p_2^{\frac{-\beta}{\beta + 3}}} \quad \text{for } i = 1, 2, \tag{3.6}$$

hence from (2.12),

$$C(p_1, p_2) = \left( \frac{\alpha_1^{\frac{\beta}{\beta + 3}} p_1^{\frac{\beta}{\beta + 3}} + \alpha_2^{\frac{\beta}{\beta + 3}} p_2^{\frac{\beta}{\beta + 3}}} {\alpha_1^{\frac{\beta}{\beta + 3}} p_1^{\frac{\beta}{\beta + 3}} + \alpha_2^{\frac{\beta}{\beta + 3}} p_2^{\frac{\beta}{\beta + 3}}} \right)^{\frac{1}{\beta}}. \tag{3.7}$$
This will be recognized as the dual of the utility function (3.1), having the same relationship to it as the minimum-unit-cost function of production theory has to the constant-returns-to-scale production function. (This is so because the cost-of-living function minimizes expenditure at the given prices and utility, subject to the utility function.) Since the utility function (3.1) has elasticity of substitution \( \sigma = 1/(1 + \beta) \), its dual cost-of-living function (3.7) has elasticity of substitution \( \sigma^* = 1/\sigma = 1 + \beta \) hence the dual coefficient \( \beta^* \) has the form \( \beta^* = 1/\sigma^* - 1 = -\beta/(1 + \beta) \).

In the examples to follow we shall assume \( \alpha_1 = \alpha_2 = 1 \) and use the parameter values given by Table 2.1, and since these give a unit cost ratio to country 3, the latter’s prices will be taken as numéraire, i.e., we shall take \( p_3^3(0) = p_3^2(0) = 1 \). Since, then, \( p_3^k(t) = 1 \) for \( k = 1, 2, 3 \) and \( t = 0, 1, 2, \) and \( p_3^1 = 1 \), we shall to simplify notation denote \( p_3^k(1) = 1 \) and \( p_3^k(1) = p \) as well as \( T_3^k(1) = T \) for \( k = 1, 2, \) and \( R_3^2 = R \). Since \( R_3^2 > p_3^1, R > 1 \). The cost-of-living function and its partial derivative with respect to the price of the second good then take the form

\[
C(1, p) = \left(1 + p^{\frac{\beta}{1+\beta}}\right)^{\frac{1+\beta}{\beta}} \quad \text{and} \quad \frac{\partial C(1, p)}{\partial p} = \left(1 + p^{\frac{-\beta}{1+\beta}}\right)^{1/\beta}.
\]

The critical equality (2.25) may then be written

\[
\frac{\left(1 + T^{*\frac{\beta}{1+\beta}}\right)^{\frac{1+\beta}{\beta}} - \left(1 + R^{\frac{\beta}{1+\beta}}\right)^{\frac{1+\beta}{\beta}}}{T^* - 1} = \left(1 + T^{*\frac{\beta}{1+\beta}}\right)^{1/\beta}.
\]

This simplifies to

\[
\left(1 + T^{*\frac{-\beta}{1+\beta}}\right)^{1/\beta} \left(1 + T^{*\frac{\beta}{1+\beta}}\right) = \left(1 + R^{\frac{\beta}{1+\beta}}\right)^{\frac{1+\beta}{\beta}}.
\]

Denoting the left side of (3.10) as \( F(T^*) \), we note that it can also be expressed as

\[
F(T^*) = \left(1 + T^{*\frac{\beta}{1+\beta}}\right)^{1/\beta} \left(1 + T^{*\frac{-\beta}{1+\beta}}\right)
= \left(1 + T^{*\frac{-\beta}{1+\beta}}\right)^{1/\beta} + \left(1 + T^{*\frac{\beta}{1+\beta}}\right)^{1/\beta},
\]

showing, incidentally, that if \( T^* \) is a solution of (3.10), so is its reciprocal, \( 1/T^* \).

3 This is easy to explain. In Figure 1.1 there will be another point such as \( Q^* \) but northwest of \( Q^* \) where the indifference curve \( U^* \) intersects \( AW \), and because of the assumed symmetry of the utility function and the assumed unit slope of country 3’s production-possibility frontier, the tangent to \( U^* \) at this point will have a slope \( T^{*-1} < 1 \) (referred to the vertical axis, i.e., \( p_2^3/p_3^3 = T^{*-1}(p_3^3/p_3^2) \)) which is the reciprocal of the slope \( T^* > 1 \) (again, referred to the vertical axis, i.e., \( p_3^2/p_3^1 = T^*(p_3^3/p_3^2) \)) of the tangent at \( Q^* \). But this corresponds to a subsidy. Writing \( T^{*-1} = 1/(1 + s) \), where \( s > 0 \) is a subsidy, we may interpret it as an export subsidy granted by country 3 on its exports of commodity 1 to countries.
Now, since $R > 1$, if $\beta > 0$, then $R^{\frac{\beta}{1+\beta}} > 1$ and thus $(1 + R^{\frac{\beta}{1+\beta}})^{\frac{1+\beta}{\beta}} > 2^{\frac{1+\beta}{\beta}}$. Likewise, if $-1 < \beta < 0$, then $R^{\frac{\beta}{1+\beta}} < 1$ and thus again $(1 + R^{\frac{\beta}{1+\beta}})^{\frac{1+\beta}{\beta}} > 2^{\frac{1+\beta}{\beta}}$. So the right side of (3.10) is always greater than $2^{\frac{1+\beta}{\beta}}$. On the other hand, $F(1) = 2^{\frac{1+\beta}{\beta}}$, while $\lim_{T^* \to 0} F(T^*) = \infty$. Consequently there exists a $T^* \in (0, 1)$ which satisfies (3.10), and therefore its reciprocal $1/T^* \in (1, \infty)$ must also be a root of (3.10). This shows that in the case of CES utility functions (the special case $\beta = 0$ is treated in subsection 3.2 below), as long as the potential partner’s cost ratio $R$ exceeds that of the “rest of the world” (assumed equal to unity), there exists a critical tariff factor $T^* > 1$ such that countries 1 and 2 will be indifferent between the status quo and forming a customs union.

In the computations to follow we shall assume that countries 1 and 2 impose the critical tariff factors $T^*$ for each elasticity of substitution, so that their welfare levels are exactly the same before and after the customs union. Under the critical pre-customs-union tariff their welfare levels are given by, using (2.17), (2.14), and (3.8) and performing a number of simple manipulations,

$$W^k(1) = \tilde{y}_1^k \cdot \left( C(1, T^*) - (T^* - 1) \frac{\partial C(1,T^*)}{\partial p} \right)$$
$$= \tilde{y}_1^k \cdot \left( 1 + T^*^{\frac{\beta}{1+\beta}} \right)^{-1/\beta} \left( 1 + T^*^{\frac{1}{1+\beta}} \right)^{-1} (k = 1, 2). \tag{3.12}$$

By assumption, this must be equal to their welfare levels under the customs union:

$$W^k(2) = \tilde{y}_1^k \cdot C(1, R)^{-1} = \tilde{y}_1^k \cdot \left( 1 + R^{\frac{\beta}{1+\beta}} \right)^{-1/\beta} \left( 1 + R^{\frac{1}{1+\beta}} \right)^{-1} (k = 1, 2). \tag{3.13}$$

By our assumption, (3.12) and (3.13) must be equal, as indeed follows from (3.10).

Under free trade, on the other hand, the countries’ welfare levels are given by

$$W^k(0) = \tilde{y}_1^k C(1, 1)^{-1} = \tilde{y}_1^k 2^{-\frac{1+\beta}{\beta}} (k = 1, 2, 3). \tag{3.14}$$

This also gives country 3’s welfare level in all situations, since it neither gains nor loses from trade. The expression for the partners’ welfare levels under the critical non-discriminatory tariff as well as under the customs union, relative to what their welfare levels would be under free trade, is therefore

$$\left( 1 + T^*^{\frac{\beta}{1+\beta}} \right)^{-1/\beta} \left( 1 + T^*^{\frac{1}{1+\beta}} \right)^{-1} 2^{\frac{1+\beta}{\beta}} = \left( 1 + R^{\frac{\beta}{1+\beta}} \right)^{-\frac{1+\beta}{\beta}} \left( 1 + R^{\frac{1}{1+\beta}} \right)^{-\frac{1+\beta}{\beta}}. \tag{3.15}$$

---

1 and 2, i.e., $p_1^k = (1 + s)p_1^2$ and $p_2^k = p_2^0$ for $k = 1, 2$. This is possible, but we rule it out by assumption. On this question see the interchange between Bhagwati (1973, p. 896) and Johnson (1974, p. 620).
3.1 Elasticity of substitution equal to 2

In this case $\beta = -1/2$ ($\sigma = 2$), and using the value $R = 25/16$ from Table 2.1, (3.10) reduces to

$$8T^{*2} - 25T^{*} + 8 = 0$$

with reciprocal roots $T_1^* = 0.361914$ and $T_2^* = 2.763086$. We reject the first root since it implies a negative tariff rate, and conclude that the critical tariff factor is $T^* = 2.763086$, hence the critical tariff rate is 1.763086, or 176.31%.

We see from formula (3.15) that at the critical pre-customs-union tariff factors and under the customs union, countries 1 and 2’s welfare relative to what they could achieve under free trade are given by

$$W^k(1) = \frac{(1 + T^*)^2}{2(1 + T^{-2})} = \frac{1 + R^{-1}}{2} = 0.82,$$

that is, their welfare under the customs union are 82% of the welfare they would enjoy under free trade.

3.2 Unitary elasticity of substitution

In this Cobb-Douglas case ($\beta \to 0, \sigma \to 1$) we have, for countries $k = 1, 2$,

$$C(1, p) = 2p^{1/2}, \quad \text{and} \quad \frac{\partial C(1, p)}{\partial p} = p^{-1/2}. \quad (3.16)$$

The critical equality (2.25) becomes, for $R = \rho^2_2 = 1$,

$$T^{*\frac{1}{2}} + T^{*-\frac{1}{2}} = 2R^{\frac{1}{2}}, \quad (3.17)$$

or

$$T^* - 2R^\frac{1}{2}T^{*\frac{1}{2}} + 1 = 0.$$

Multiplying through by $T^{*\frac{1}{2}}$ and defining $\lambda = T^{*\frac{1}{2}}$ gives, for $R = 25/16$, the quadratic equation

$$f(\lambda) = \lambda^2 - \frac{5}{2}\lambda + 1 = 0$$

with roots

$$\lambda_1 = 1/2 \quad \text{and} \quad \lambda_2 = 2$$

yielding the two critical values 1/4 and 4 for $T^*$. Since the first of these corresponds to a negative tariff, we accept only the second. Thus, the pre-customs-union tariff rate that countries 1 and 2 would have had to have set on their imports of commodity 2 from country 3 in order to subsequently be on the verge of forming a customs union must have been 3, or 300%.

At this critical tariff factor, from (3.12), (3.13), and (3.16), the welfare levels of countries 1 and 2 are
\[ W^k(1) = \hat{g}^k_1 \left( C(1, T^*) - (T^* - 1) \frac{\partial C(1, T^*)}{\partial p} \right)^{-1} = \hat{g}^k_1 (T^* + T^* - \frac{1}{2})^{-1} = W^k(2) = \hat{g}^k_1 2R^{-\frac{1}{2}}, \] (3.18)

the two expressions being equal by (3.10). For \( R = 25/16 \) this gives a welfare level of \( 0.4\hat{g}^k_1 \). Under free trade, on the other hand, the countries’ welfare levels are \( \hat{g}^k_1/C(1, 1) = 0.5\hat{g}^k_1 \), so the welfare levels of the partners to the customs union are 80% of what they could achieve under free trade.

### 3.3 Elasticity of substitution of one-half

In this case \( \beta = 1 \) (\( \sigma = 1/2 \)), and the basic equation (3.10) reduces to, for \( R = 25/16 \),

\[ T^* = \frac{49}{16} T^{*1/2} + 1 = 0. \]

Defining \( \lambda = T^{*1/2} \), this reduces to

\[ f(\lambda) = \lambda^2 - 3.0625\lambda + 1 = 0. \]

As before, \( f(\lambda) \) has two reciprocal roots \( \lambda_1 = 0.371627 \) and \( \lambda_2 = 2.690873 \), leading to \( T^*_1 = 0.138106 \) and \( T^*_2 = 7.240800 \). The first is rejected (implying a negative tariff rate) and the second implies a minimum tariff rate of 6.2408, or 624.08%.

Formula (3.15) gives in this case

\[ 4(1 + T^*(-1/2))^{-1}(1 + T^{*1/2})^{-1} = 4 \left( 1 + \frac{5}{4} \right)^{-2} = 4 \cdot \frac{16}{81} = 0.79012345 \]

for the welfares of the partners to the customs union (as well as under the critical pre-customs-union nondiscriminatory tariff level) relative to the welfares they could achieve under free trade.

### 3.4 Elasticity of substitution of one-quarter

The computations involved in this case are typical of the general case in which, unlike the cases \( \sigma = 2 \) and \( \sigma = 1/2 \), the problem does not reduce to solving a simple quadratic equation. In the case \( \beta = 3 \) (\( \sigma = 1/4 \)) (3.10) gives, for \( R = 1.5625 \),

\[ (1 + T^* - 3/4)^{1/3}(1 + T^{*1/4}) = 3.208891866. \]

Raising both sides to the third power we obtain

\[ T^{*3/4} + 3T^{*1/2} + 3T^{*1/4} - 31.04191785 + 3\frac{1}{4}T^{* - 1/4} + 3T^{* - 1/2} + T^{* - 3/4} = 0 \]

Multiplying through by \( T^{*3/4} \) and defining \( \lambda = T^{*1/4} \) this reduces to the 6th-degree polynomial
\[ f(\lambda) = \lambda^6 + 3\lambda^5 + 3\lambda^4 - 31.04191785\lambda^3 + 3\lambda^2 + 3\lambda + 1 = 0. \]

This has exactly two real roots, \( \lambda_1 = 0.47590807407 \) and \( \lambda_2 = 2.10124613236. \)

Raising these to the fourth power we get the reciprocal roots \( T_1^* = 0.051297 \) and \( T_2^* = 19.494303, \) of which the first leads to a negative tariff.

### 3.5 Zero elasticity of substitution

Finally let us take up the case \( \beta \to \infty \) \( (\sigma \to 0) \). We have in this case

\[ C(1, p) = 1 + p \quad \text{and} \quad \frac{\partial C(1, p)}{\partial p} = 1. \]

Thus in this limiting case the cost-of-living function \( C(1, p) \) is linear and no longer strictly concave, so no tangent can be drawn to it from any point above it. The basic inequality (2.23) becomes \( T - 4 > T - 1, \) or \(-3 > 0\), which is manifestly impossible, showing that there does not exist a tariff factor which will make countries 1 and 2 as least as well off under the customs union.

### 3.6 Summary

Table 3.1 summarizes the basic results for this model for a wide range of elasticities of substitution.\(^5\) It will be noted that for very high elasticities of substitution, e.g. \( \sigma = 25 \), the critical tariff factor is reasonable (in this case 1.72454, implying a tariff rate of 72.5%), and the partners to the customs union are able to achieve a welfare level that is 97% of what they could have achieved under free trade. However, the situation is very different as the elasticity of substitution becomes small. It will be noted from the table that as this elasticity becomes smaller, the critical tariff factor increases while the welfare of the partners relative to their prospective welfare under free trade fall to about 78%. Even with an elasticity of substitution of 1/8, the critical tariff factor is 117.483, implying a tariff rate of 11.648%. At the elasticity of substitution 1/33, the critical tariff factor is 7,456,540, implying a tariff rate of over 700 million percent—all this to achieve a welfare level that is only 78% of the free-trade level.

Among the interesting generalizations we can make from the above illustrations are the following:

---

\(^4\) The way in which the right side of (3.10) enters into the construction of the polynomial assures that there will be exactly two changes of sign in the polynomial’s sequence of coefficients, and therefore exactly two real roots, by Descartes’ rule of signs.

\(^5\) The computations of the critical tariff factors in the general case were carried out by the bisection method using a Fortran routine found in Press et al. (1986, p. 247). The bisection was applied to the interval \([10^{-6}, 1]\) to obtain a root whose reciprocal is the critical tariff factor \( T^*. \)
\begin{table}
\centering
\begin{tabular}{cccc}
\hline
$\beta$ & $\sigma$ & $T^*$ & $W^k(2)/W^k(0)$ \\
\hline
-0.96 & 25 & 1.72454 & 0.971533 \\
-0.95 & 20 & 1.75613 & 0.964187 \\
-0.9375 & 16 & 1.78900 & 0.954925 \\
-0.923 & 13 & 1.82892 & 0.944188 \\
-0.9 & 10 & 1.87831 & 0.927713 \\
-0.875 & 8 & 1.93361 & 0.911310 \\
-0.8 & 5 & 2.09517 & 0.874142 \\
-0.75 & 4 & 2.20340 & 0.857748 \\
-0.66 & 3 & 2.38649 & 0.839524 \\
-0.5 & 2 & 2.76309 & 0.820000 \\
0 & 1 & 4.00000 & 0.800000 \\
1 & 1/2 & 7.24080 & 0.790123 \\
2 & 1/3 & 12.0930 & 0.780879 \\
3 & 1/4 & 19.4943 & 0.785269 \\
4 & 1/5 & 30.8649 & 0.784306 \\
5 & 1/6 & 48.4001 & 0.783666 \\
6 & 1/7 & 75.5065 & 0.783210 \\
7 & 1/8 & 117.483 & 0.782868 \\
8 & 1/9 & 182.573 & 0.782603 \\
9 & 1/10 & 283.612 & 0.782390 \\
10 & 1/11 & 440.590 & 0.782217 \\
11 & 1/12 & 684.652 & 0.782072 \\
12 & 1/13 & 1064.34 & 0.781950 \\
13 & 1/14 & 1655.31 & 0.781845 \\
14 & 1/15 & 2575.56 & 0.781755 \\
15 & 1/16 & 4009.01 & 0.781675 \\
16 & 1/17 & 6242.69 & 0.781605 \\
17 & 1/18 & 9723.81 & 0.781543 \\
18 & 1/19 & 15150.4 & 0.781488 \\
19 & 1/20 & 23613.3 & 0.781437 \\
20 & 1/21 & 36812.3 & 0.781392 \\
21 & 1/22 & 2417.533 & 0.781247 \\
22 & 1/23 & 339.792 & 0.781218 \\
23 & 1/24 & 203.250 & 0.781121 \\
24 & 1/25 & 312.340 & 0.781100 \\
25 & 1/26 & 479.490 & 0.781081 \\
26 & 1/27 & 745.540 & 0.781063 \\
\hline
\end{tabular}
\caption{Elasticities of substitution ($\sigma$), critical tariff factors ($T^*$), and welfares of partners to the customs union relative to the levels they could achieve under free trade.}
\end{table}
1. In order for the customs union to be beneficial to the partner countries (and therefore to the world as a whole), it is necessary for the pre-customs-union tariff levels to be extremely high, the height increasing as the elasticity of substitution decreases. Only in the limiting case of zero elasticity of substitution is it impossible to find pre-customs-union tariffs so high that the partners gain from the customs union.

2. A gain to the partners from forming a customs union is possible only if the pre-customs-union tariff was sufficiently high, and even in this case there is a greater gain from moving to free trade; while there will always be a gain from moving to free trade.

3. One must ask what would have motivated countries to introduce such high tariffs in the first place. The same rationale that “explains” why two countries would form a customs union predicts much more strongly that they would remove all tariff barriers and engage in free trade. Thus, it is not enough for a theory of customs unions to show that the arrangement would be beneficial to the partner countries.

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