A.1 Cost Minimization vs Profit Maximization.

If a firm is maximizing profit, it must be minimizing the cost. This implies that the profit maximization problem of the firm can be written using the minimum-cost function $C^*$ as

$$\max_z [pz - C^*(w, z)]$$  \hspace{1cm} (A1)

The first-order condition for a solution $z^* > 0$ to (22) is

$$p = \frac{\partial C^*}{\partial z}(w, z^*)$$  \hspace{1cm} (A2)

that is “output price = marginal cost”.

The solution to (A1) is the same as the profit maximizing supply $z^*(w, p)$ from (2) in lecture Notes. Further,

$$x_{pmax}^*(w, p) = x_{cmin}^*(w, z^*(w, p))$$  \hspace{1cm} (A3)
A.2 Optimal Portfolio with One Risky Asset

Consider a risky asset with return \( \tilde{r} \). The riskiness of the return is expressed by the fact that \( \tilde{r} \) can take any of \( S \) values \( r_1 \) through \( r_S \). That is, there are \( S \) states and the return in state \( s \) is \( r_s \).

Further, there is a risk-free asset with state-independent return \( \hat{r} \).

Agent’s portfolio choice problem is

\[
\max_a \sum_{s=1}^{S} \pi_s [v((w - a)\hat{r} + ar_s)],
\]

\( (A4) \)

where \( a \) is the amount of wealth invested in the risky asset and \( w \) is the initial wealth. Investment \( a \) is unconstrained in this problem.

A more convenient way to write maximization (1) is

\[
\max_a E[v((w - a)\hat{r} + a\tilde{r})].
\]

\( (A5) \)

The first-order condition for an interior solution \( a^* \) to (A5) is

\[
E[v'(w\hat{r} + a^*(\tilde{r} - \hat{r}))(\tilde{r} - \hat{r})] = 0.
\]

\( (A6) \)

**Theorem, A.2:** If an agent is strictly risk averse and has differentiable vNM utility function, then the optimal investment in the risky security is strictly positive, zero or strictly negative iff the risk premium on the risky security (i.e., \( E(\tilde{r}) - \hat{r} \)) is strictly positive, zero or strictly negative.