Speculative Trade under Ambiguity *

Jan Werner

March 2014.

Abstract: Ambiguous beliefs may lead to speculative trade and speculative bubbles. We demonstrate this by showing that the classical Harrison and Kreps (1978) example of speculative trade among agents with heterogeneous beliefs can be replicated with agents having common ambiguous beliefs. More precisely, we show that the same asset prices and pattern of trade can be obtained in equilibrium with agents’ having recursive multiple-prior expected utilities with common set of probabilities.

While learning about the true distribution of asset dividends makes speculative bubbles vanish in the long run under heterogeneous beliefs, it may not do so under common ambiguous beliefs. Ambiguity need not disappear with learning over time, and speculative bubbles may persist forever.
Extended Abstract

Ambiguous beliefs may lead to speculative trade and speculative bubbles. We demonstrate this by showing that the classical Harrison and Kreps (1978) example of speculative trade among agents with heterogeneous beliefs can be replicated with agents having common ambiguous beliefs. More precisely, we show that the same asset prices and speculative pattern of trade can be obtained in equilibrium with agents’ having multiple-prior expected utilities with common set of probabilities.

The key question of Harrison and Kreps (1978) was whether equilibrium prices in asset markets can persistently exceed all agents’ valuations of the asset where valuation is defined by what an agent would be willing to pay if obliged to hold the asset forever. If price exceeds all valuations, then agents who buy the asset must intend to sell it in the future. They engage in speculative trade.

Harrison and Kreps considered a model of infinite-time asset markets where risk-neutral agents have heterogeneous expectations about the asset payoffs and cannot short sell the asset. In this model, equilibrium asset price $p_t$ at date $t$ must satisfy the relationship

$$p_t = \max_i E_t^i[p_{t+1} + d_{t+1}],$$

(1)

where $d_{t+1}$ denotes dividend at date $t + 1$ and $E_t^i$ stand for date-$t$ conditional expectation under $i$’th agent one-period-ahead belief. If beliefs are such that there is no single agent who is more optimistic at all dates and states than other agents about the asset’s high dividend next period, then there is no single agent whose expectation is the maximizing one in (1) at all dates and states. This property of one-period-ahead beliefs is called perpetual switching, see Morris (1996). Under perpetual switching, asset prices (strictly) exceed every agent’s valuation. There is also persistent speculative trade because the agent whose belief is the maximizing one in (1) holds the asset at date $t$ while agents whose beliefs give strictly lower expectations sell the asset if they had some holdings from previous date.

We consider the same model of asset markets except for specification of agents’ beliefs. Instead of heterogeneous but exact beliefs in Harrison and Kreps (1978), agents have common ambiguous beliefs described by sets of one-period-ahead probabilities. Their decision criterion is the recursive multiple-prior expected utility - an extension of Gilboa and Schmeidler (1989) maxmin criterion to dynamic set-
ting due to Epstein and Schneider (2003). Our key observation is that equilibrium pricing relationship (1) continues to hold with expectation $E^i_t$ of agent $i$ being now the one-period-ahead belief that minimizes expected value of that agent’s date-$(t + 1)$ consumption (more precisely, date-$(t + 1)$ continuation utility) over the set of multiple probabilities. We call those probabilities effective one-period-ahead beliefs, and they feature in the valuation of agents’ willingness to pay for the asset if obliged to hold it forever. If there is sufficient heterogeneity of agents’ equilibrium consumption plans, then effective beliefs have switching property and - as in Harrison and Kreps (1978) - equilibrium prices exceed all agents’ valuations. Further, there is speculative trade. Heterogeneity of equilibrium consumption is generated by heterogeneous initial endowments. Initial endowments play a critical role under common ambiguous beliefs, in contrast to the case of heterogeneous beliefs where they do not matter.

Two objections have been frequently raised against Harrison and Kreps’ model of speculative trade. The first is that it departs from the common-prior assumption. This objection does not apply to our model. Agents in our model have common priors, or more precisely, common set of priors. The second is that agents have dogmatic beliefs and do not learn from observations of realized dividends over time. In response to this objection, Morris (1996) introduced learning in the model of speculative trade. He considered an i.i.d. dividend process parametrized by a single parameter of its distribution (probability of high dividend) that is unknown to the agents. Agents have heterogeneous prior beliefs about that parameter. Morris (1996) showed that, as the agents update their beliefs over time, their posterior beliefs will exhibit switching property that leads to speculative trade. It is a standard result in Bayesian learning (see Blackwell and Dubins (1962)) that agents posterior beliefs converge to the true parameter of the dividend distribution. This implies that asset prices converge to agents’ valuations as time goes to infinity.

A slightly different model of speculative trade under heterogeneous beliefs with learning has been studied in Slawski (2008). In his model the dividend process is a Markov chain. Agents don’t know the true transition matrix and have heterogeneous priors on a set of possible transition matrices. Slawski (2008) provides conditions under which there is speculative trade in equilibrium when agents update their beliefs. Under fairly general conditions, posterior beliefs converge to true
parameter as time goes to infinity, and asset prices converge to agents’ valuations.

We show that Morris (1996) model of speculative trade under heterogeneous beliefs with learning can be replicated with common ambiguous beliefs. Agents in our model have common set of prior beliefs about the parameter of the distribution of dividends. They update their beliefs prior-by-prior in Bayesian way. If their consumption plans exhibit sufficient heterogeneity, effective posterior beliefs have switching property and there result speculative trade. The critical condition is again heterogeneity of agents' initial endowments. As agents update their ambiguous beliefs over time, ambiguity fades away and - as in Morris (1996) - prices converge to valuations of the asset.

Learning and updating of beliefs can be significantly different under ambiguity than with no ambiguity. It is well known, see Epstein and Schneider (2007) and the references therein, that learning with multiple priors may leave some ambiguity remaining in the long run. A different specification of the set of prior beliefs in our version of Morris' model leads to some ambiguity remaining in the long run despite belief updating. Asset prices do not converge to agents’ valuations. In fact, one can view the dividend process in our version of Harrison and Kreps’ model as sequences of independently and indistinguishably (IID) distributed random variables, see Epstein and Schneider (2003b). Such processes have persistent ambiguity that is unaffected by learning.

In their comprehensive study of the dot.com bubble of 2000-2001 Ofek and Richardson (2003) concluded that short-sales restrictions and heterogeneity of investors' beliefs were the main reasons for the dramatic rise and fall of prices of internet stocks during that period. Short sales restrictions on internet stocks were particularly stringent because of the so-called lockups. Their main argument in support of belief heterogeneity was relatively low level of institutional holdings of internet stocks. Individual investors tend to have more diverse beliefs. Our findings offer a different interpretation of Ofek and Richardson’s analysis.

The results of this paper are in stark contrast to the existing literature on implications of ambiguous beliefs (or ambiguity aversion) on equilibrium in asset markets. Inspired by the portfolio-inertia result of Dow and Werlang (1992), the literature has strived to demonstrate that non-participation in trade by some agents with ambiguous beliefs may arise in equilibrium. In Cao, Wang and Zhang
agents have heterogeneous ambiguity and those with the highest degree of ambiguity opt out of trading risky assets in equilibrium. Mukerji and Tallon (2001) show that ambiguous beliefs concerning idiosyncratic risk may lead to break down of trade of some assets.

The paper is organized as follows. In Section 2 we review the model of Harrison and Kreps (1978) of speculative trade under heterogeneous beliefs. In Section 3 we show how the same asset prices and asset holdings can be obtained in equilibrium with agents having recursive multiple-prior expected utilities with common set of probabilities. In Section 4 we extend the model and identify the property of switching beliefs that is shown to give rise to speculative trade. A model of speculative trade with learning under ambiguity is presented in Section 5.

References