Firm Growth and Unemployment

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Work in Progress/Big Picture

□ Firm heterogeneity plays a central role in modern models of aggregate productivity, growth, and trade.

□ Models of growth and firm heterogeneity often feature continuous labor inputs and frictionless labor markets.

□ Does not help in interpreting rich body of evidence on job creation and destruction, unemployment, vacancies, labor market flows.

□ Supply of jobs in search models is often infinitely elastic...

□ Let's try to find a tractable way to include search frictions in the labor market in a model of firm heterogeneity that fits the data.

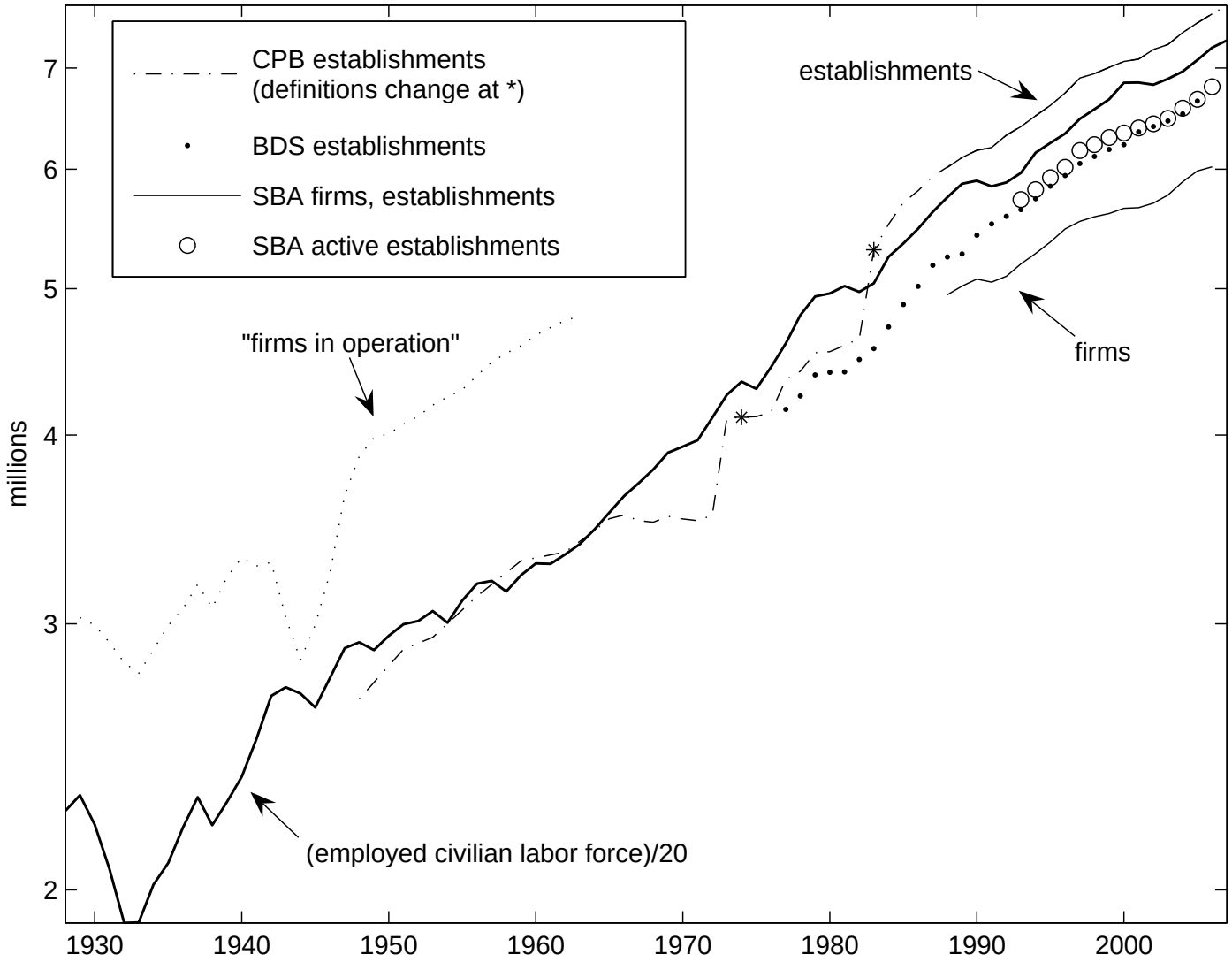
Some Related Work

- Fujita and Ramey [2007]
- Moscarini and Postel-Vinay [2008]
- Veracierto [2009]
- Acemoglu and Hawkins [2010]
- Helpman, Itskhoki, Redding [2010]
- Schaal [2010]
- Elsby and Michaels [2011]
- Kaas and Kircher [2011]
- Lentz and Mortensen [2010, *Annual Review of Economics*]
- Growth papers surveyed in Luttmer [2010, *Annual Review of Economics*]

Some Basic Facts

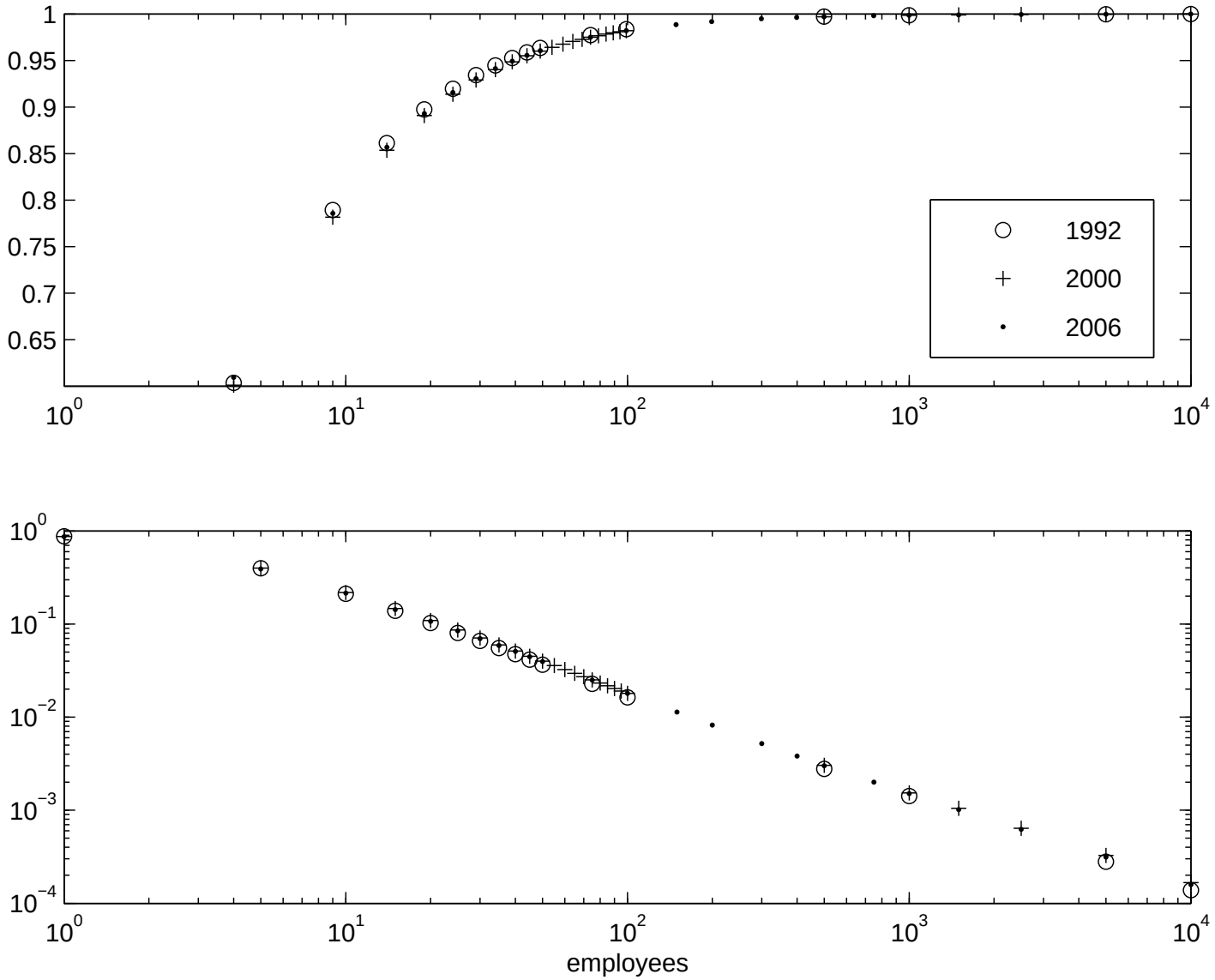
- the US population and the number of firms grow at about 1% per annum
- 10% of all firms exit in a given year, most of them very small
- 11% of all firms did not exist the year before
- 50% of all employees work for firms with more than 500 employees
- 25% of all employees work for firms with more than 10,000 employees
- 3% of all employees work for firms with fewer than 5 employees
- 50% of all firms with at least 10,000 employees are older than 70 years
- P&G, Ford, HP, WalMart, Microsoft, Google started out really small
- . . . and they did not grow at 1% per year.

The Number of Firms Grows with Population

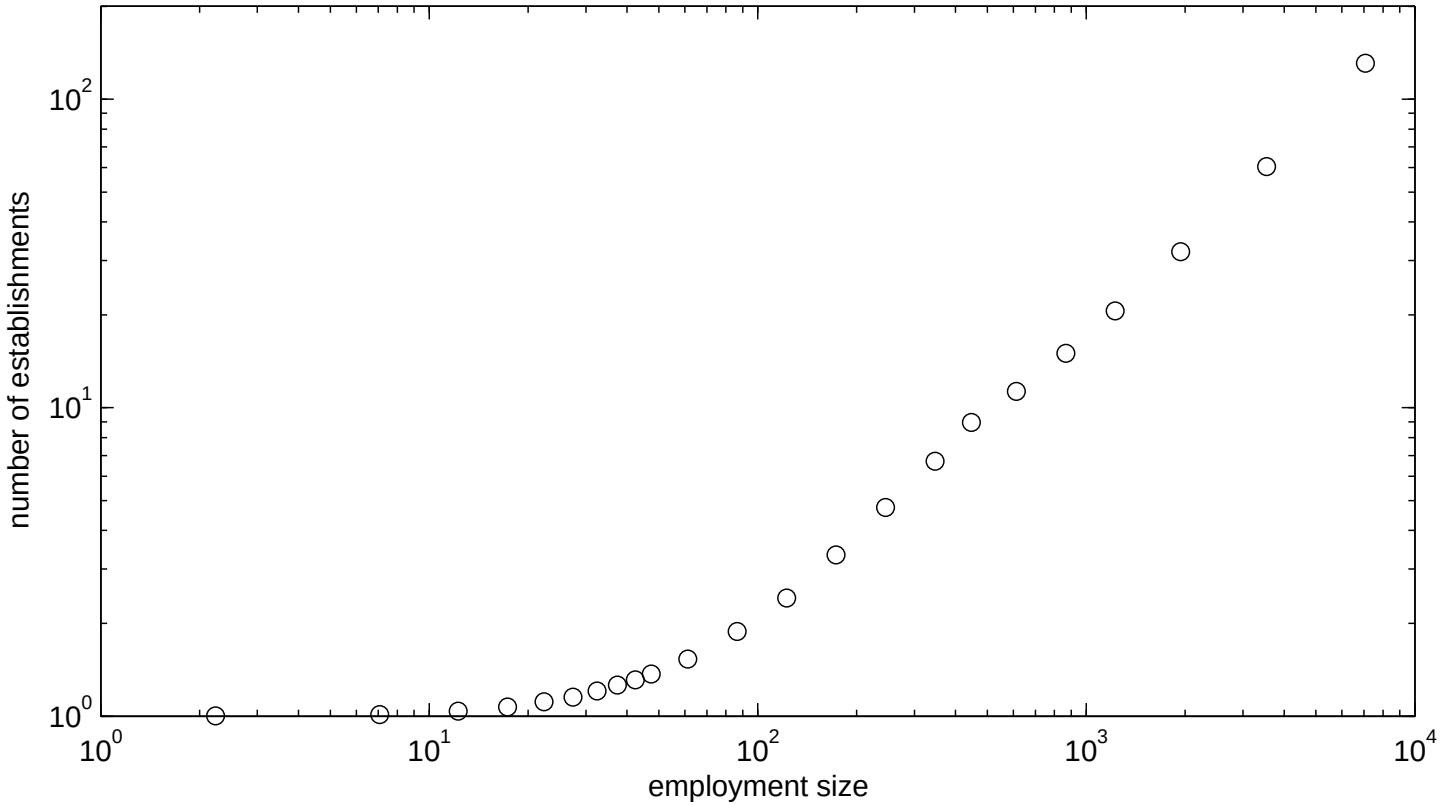


Where the Jobs Are

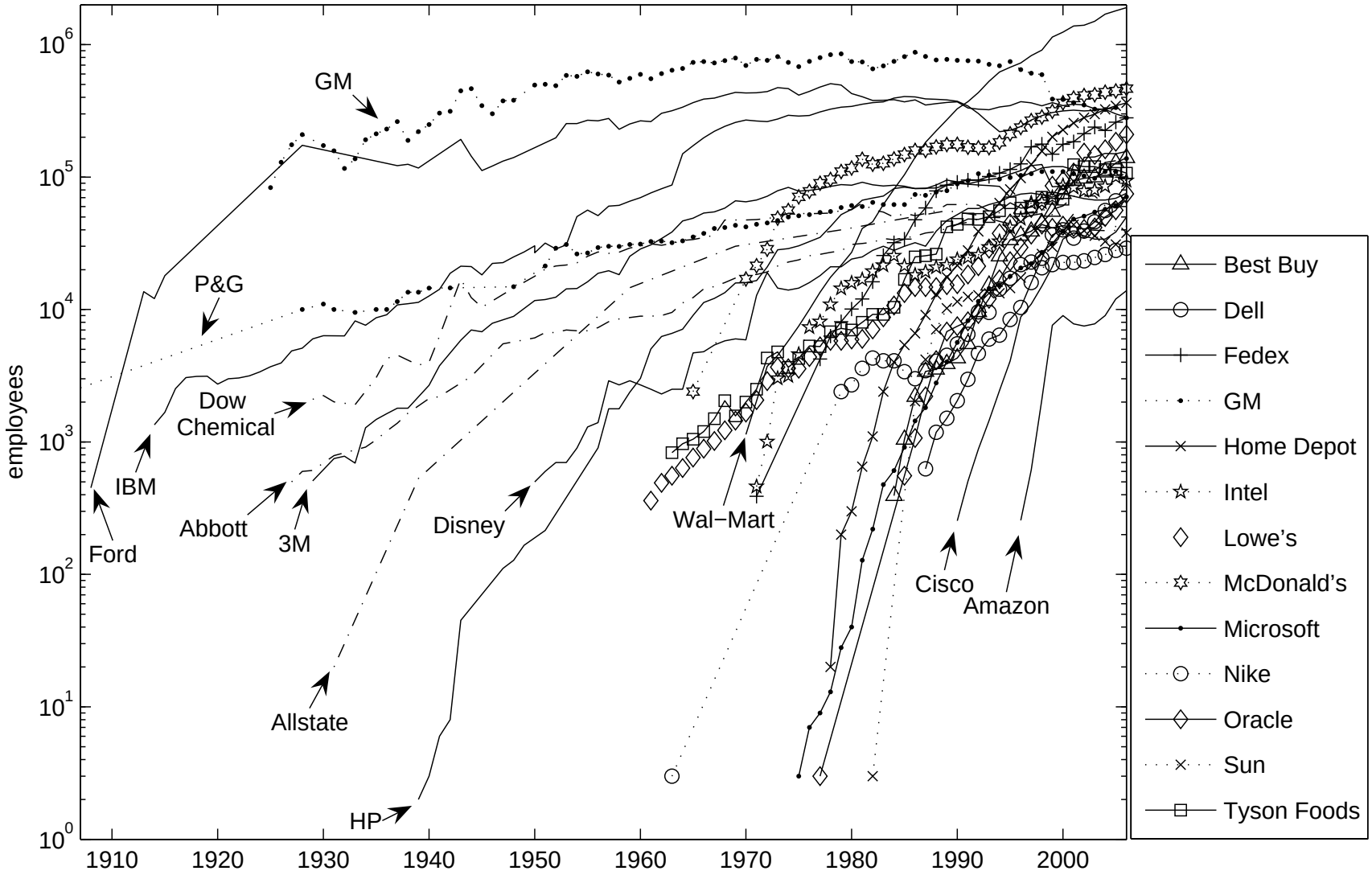
The Employment Size Distribution of Firms



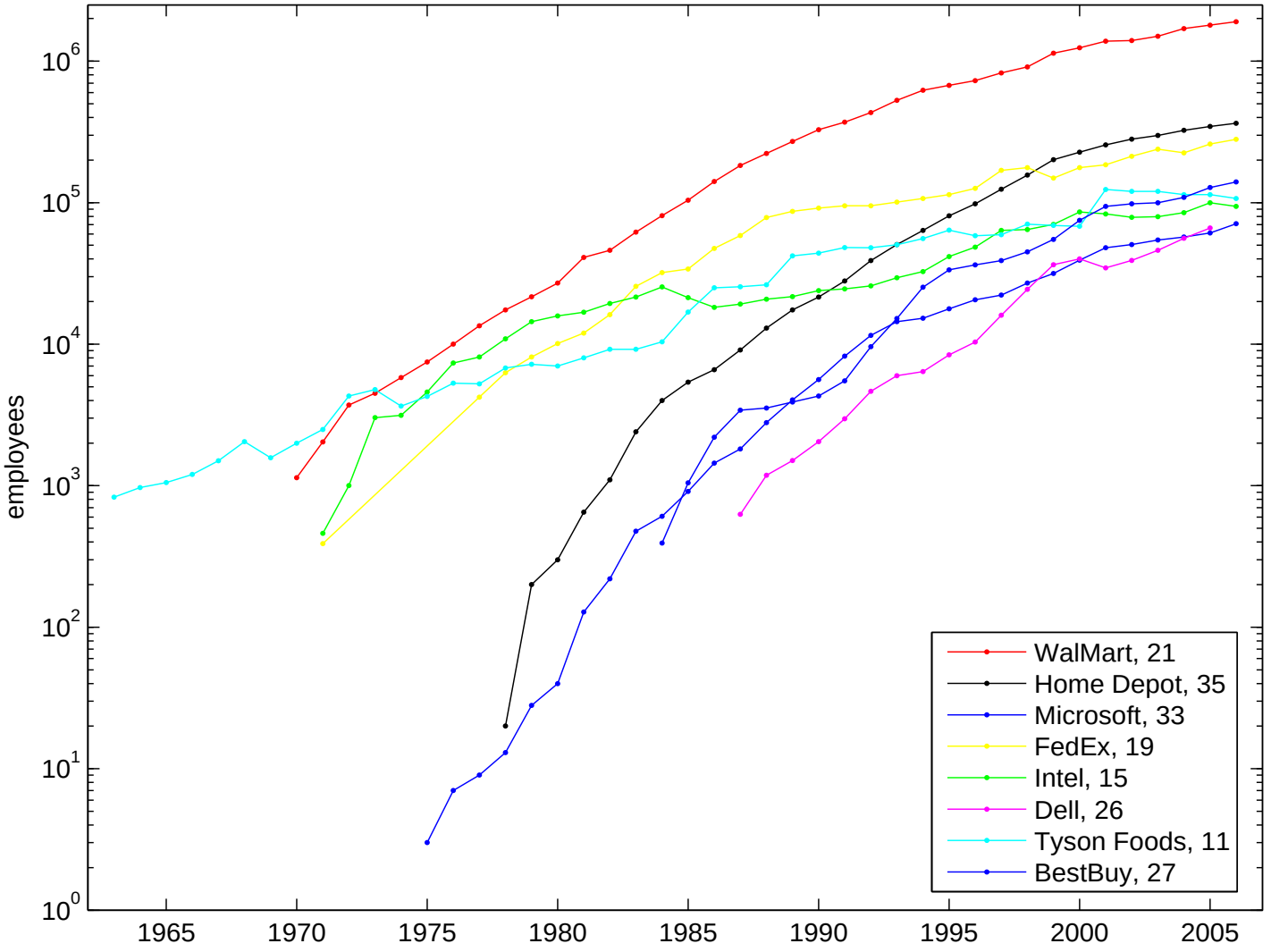
What Large Firms Are Like



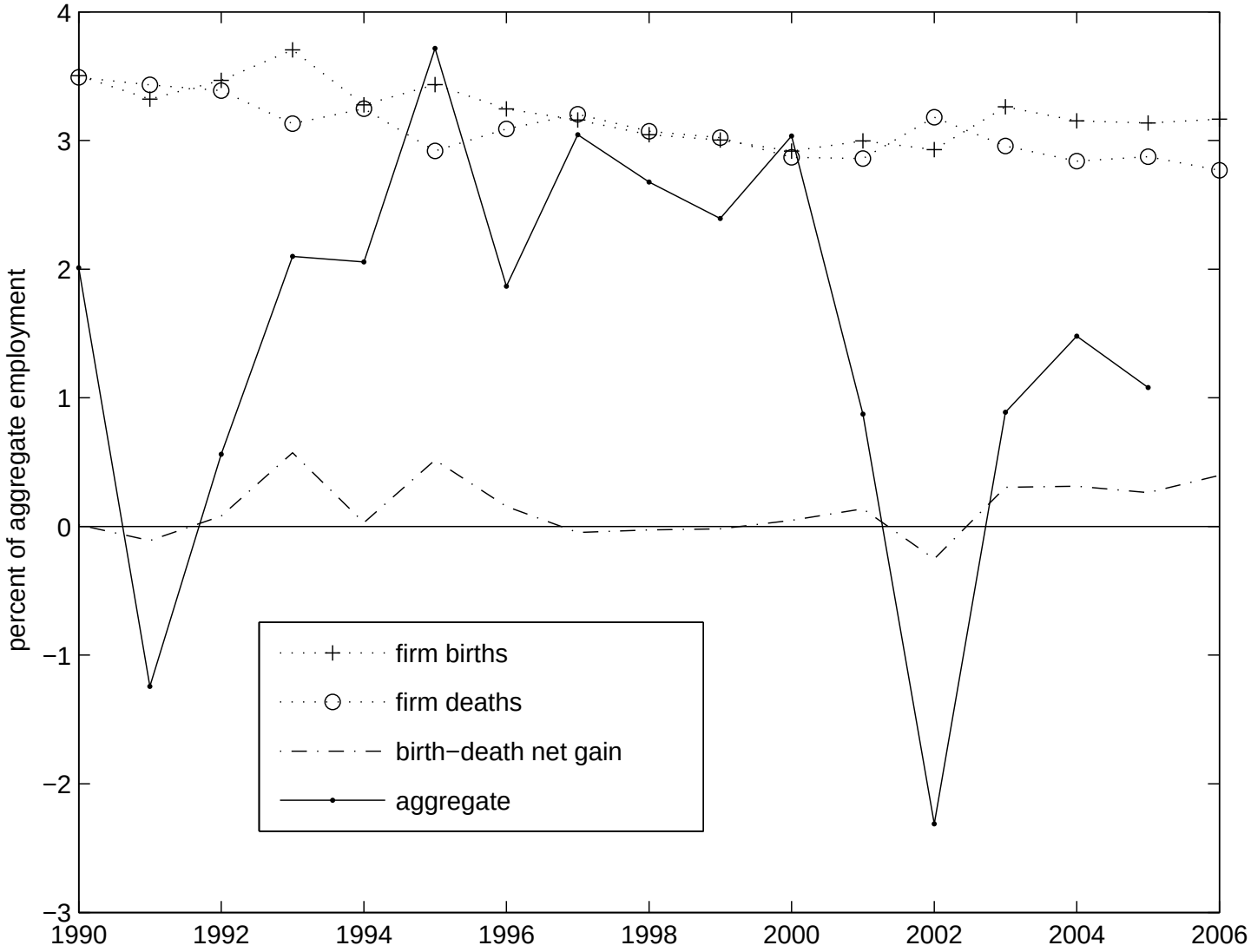
Luttmer, Review of Economic Studies, July 2011



Rapid Firm Growth

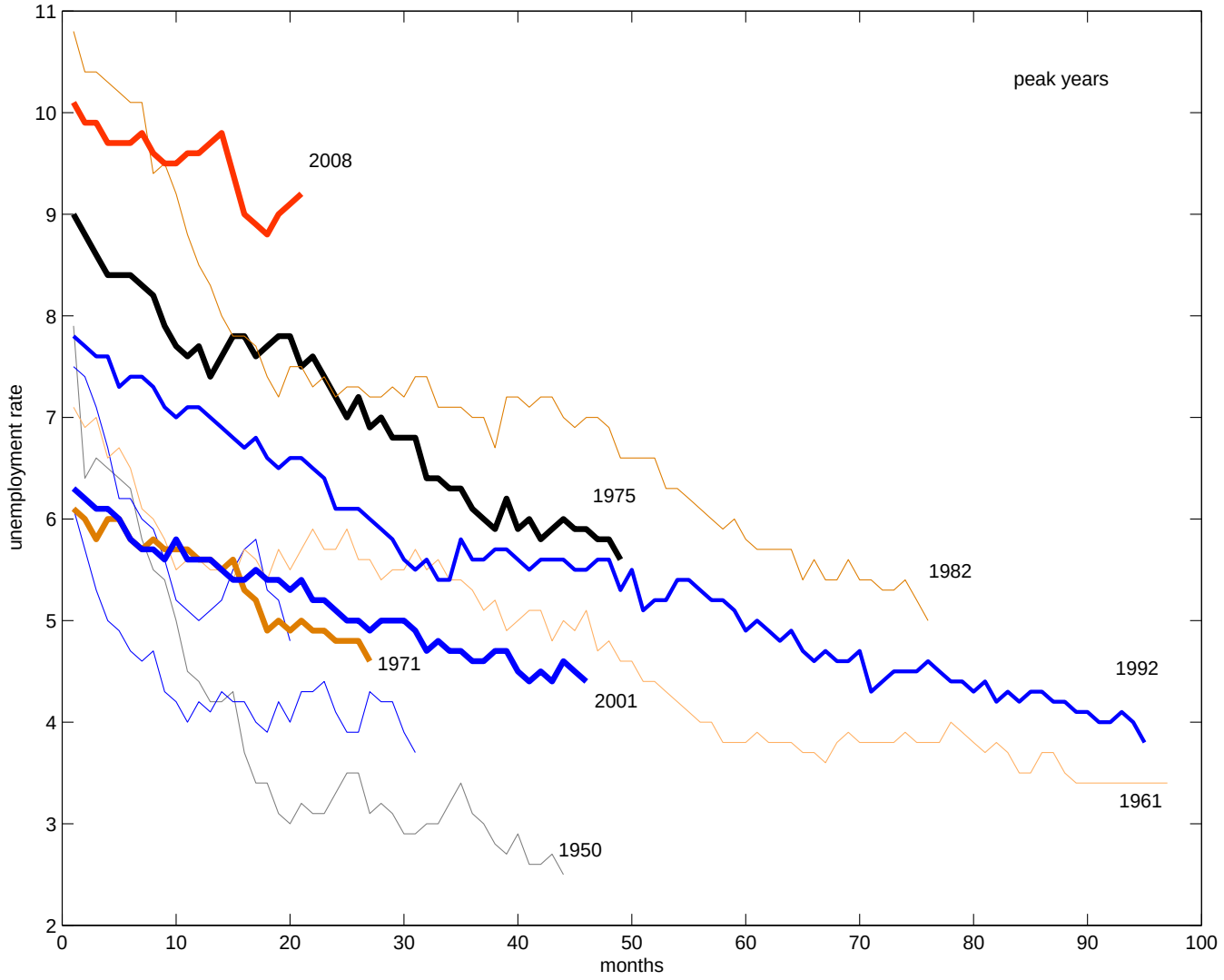


Contributions to Employment Growth



Average size of recent entrants and exiting firms: about 5.5 employees.

Peak to Trough Unemployment



Pareto Tails with Deterministic Growth

- Potential employees $H_t = He^{\eta t}$
- Firm entry $E_t = Ee^{\eta t}$, at size $n = 1$
- Firms grow at the rate μ and exit randomly at the rate δ
- Age density

$$f(a) \propto e^{-(\eta+\delta)a}$$

- Firms with n or more employees

$$\texttt{fraction} = \frac{\int_{\ln(n)/\mu}^{\infty} e^{-(\eta+\delta)a}\mathrm{d}a}{\int_0^{\infty} e^{-(\eta+\delta)a}\mathrm{d}a} = \frac{1}{n^{\zeta}}$$

$$\texttt{employment share} = \frac{\int_{\ln(n)/\mu}^{\infty} e^{-(\eta+\delta-\mu)a}\mathrm{d}a}{\int_0^{\infty} e^{-(\eta+\delta-\mu)a}\mathrm{d}a} = \frac{1}{n^{\zeta-1}}$$

where

$$\zeta = \frac{\eta+\delta}{\mu}$$

- Since firms larger that 500 employees account for half of employment

$$\frac{1}{500^{\zeta-1}} = \frac{1}{2}$$

or

$$\zeta = 1 + \frac{\ln(2)}{\ln(500)} = 1.1$$

- Population growth $= 1\%$ and large-firm exit rate is 2.5%

- Hence the average surviving incumbent must grow at the rate

$$\mu = \frac{\eta + \delta}{\zeta} = \frac{0.01 + 0.025}{1.06} = 0.033$$

Non-Stationary Firms & Aggregate Mean Reversion

- Employment

$$\mathrm{D}N_t = -(\delta - \mu)N_t + Ee^{\eta t}$$

- Employment-population ratio

$$\mathrm{D}\left[\frac{N_t}{H_t}\right] = -\left(\eta + \delta - \mu\right)\left[\frac{N_t}{H_t}\right] + \frac{E}{H} = -\left(1 - \frac{1}{\zeta}\right) \times (\eta + \delta) \times \left[\frac{N_t}{H_t}\right] + \frac{E}{H}$$

- Firm size distribution implies

$$\left(1 - \frac{1}{\zeta}\right) \times (\eta + \delta) = \left(1 - \frac{1}{1.1}\right) \times (0.01 + 0.025) = 0.003$$

or at most, if all exit is random,

$$\left(1 - \frac{1}{\zeta}\right) \times (\eta + \delta) = \left(1 - \frac{1}{1.1}\right) \times (0.01 + 0.10) = 0.01$$

- Respective half-lives $\ln(2)/.003 = 231$ and $\ln(2)/0.01 = 69$, in years...

Game Plan

- Replace competitive labor market in Luttmer [2011] with search friction
  - —joint account of firm employment dynamics and labor market flows
  - —$\mu_t = F(a_t, 1)$, managerial output $= (1 - a_t)y$, $a_t \in [0, 1]$
  - —a_t is high at start of recovery $\rightarrow$ low measured labor productivity
- Analytically tractable steady state used to identify most parameters
- Recession = one-time destruction of projects, or matches, or both
- Try to account for postwar recoveries
  - —unemployment
  - —vacancies
  - —measured labor productivity

Population and Utility

$$H_t = H_0 e^{\eta t}$$

$$\int_0^\infty e^{-\rho t} H_t U(C_t/H_t) \mathrm{d}t$$

$$U(c) = \ln(c)$$

$$\rho > \eta$$

Firms, Projects and Matches

- Entrepreneurs set up firms by creating startup projects at the rate α
- Projects must be assigned to managers, one per manager

  —recruited instantaneously from population of employed workers
- Managers can replicate projects at the rate $\mu_t = F(a_t, 1)$, $a_t \in [0, 1]$

  — projects stay within the firm, no internal labor markets
- Managers search unemployed population for workers to team up with

  — workers hired at rate $\beta_t = M(u_t, v_t)/v_t$

  — workers quit into unemployment at the rate θ
- Unemployed produce h, workers x, and managers $(1 - a_t)y$
- Projects fail at the rate λ, firms at the rate δ