

Econ 8403, Spring 2007,

Instructor: Patrick Kehoe

Problem set 1

(Due in class in one week)

(Let's start with a warm-up/review question)

1. Risk-sharing and net exports

Consider an economy composed of two countries $i = 1, 2$. Each date a discrete event s_t occurs and $s^t = (s_0, \dots, s^t)$ is the history of events. Let $\pi(s^t)$ denote the probability distribution over s^t and $\pi(s^t|s^{t-1})$ the conditional distribution over $s^t = (s^{t-1}, s_t)$ given s^{t-1} . Each country has a representative consumer with preferences

$$\sum_{t=0}^{\infty} \sum_{s^t} \beta_i^t \pi(s^t) U^i(c_i(s^t))$$

and endowments $\{y_i(s^t)\}_{t=0}^{\infty}$. Let $y(s^t) = y_1(s^t) + y_2(s^t)$.

a. Let $q(s^t, s_{t+1})$ denote the price of a one-period bond bought at t in state s^t that pays off one unit at $t + 1$ if and only if state $s^{t+1} = (s^t, s_{t+1})$ occurs and let $b_i(s^t, s_{t+1})$ denote the amount of one period contingent bonds purchased by consumer i at t . Define a competitive equilibrium in sequence form.

b. Let $p_t(s^t)$ denote the price in units of date 0 goods of one unit of date t good. Define a competitive equilibrium in date 0 form.

c. Prove the equilibrium in parts a and b are the same. (Add the appropriate bound to debt and carefully use the transversality conditions in the sequence economy.)

d. Describe in words what happens when $\beta_1 < \beta_2$?

For the rest of the problem let $\beta_1 = \beta_2 = \beta$ and $U(c) = c^{1-\sigma}/1 - \sigma$.

e. Compute the solution to the Pareto Problem of

$$\lambda \sum_{t=0}^{\infty} \sum_{s^t} \beta^t \pi(s^t) U(c_1(s^t)) + (1 - \lambda) \sum_{t=0}^{\infty} \sum_{s^t} \beta^t \pi(s^t) U(c_2(s^t)).$$

f. Find the Pareto weights (i.e. an explicit formula in terms of β , σ , $\pi(s^t)$, and $y_i(s^t)$) such that the solution to the Pareto problem coincides with that of the competitive equilibrium.

g. Define net exports of country i as $nx_i = y_i - c_i$. What is the correlation of net exports and output? Are net export procyclical or countercyclical? What might explain the discrepancy between this model's predictions and the data?

2. A one country real business cycle model

Calibrate the log-linearized model by hand in the Uhlig paper on the reading list and work out the impulse response of consumption, investment, labor and output to a one percent technology shock.

3. A two country real business cycle model

Consider a two country version of Gary Hansen's model that is exposted in the Uhlig paper. Let the preferences be

$$\max E_0 \sum_{t=0}^{\infty} \beta^t (\log C_{1t} - AN_{1t})$$

the resource constraint is

$$\sum_i (C_{it} + K_{it}) = \sum_i (Y_{it} + (1 - \delta)K_{it-1})$$

where $Y_{it} = Z_{it} K_{it-1}^\rho N_{it}^{1-\rho}$ and

$$\log Z_{1t} = (1 - \psi) \log \bar{Z} + a \log Z_{1t-1} + b \log Z_{2t-1} + E_{1t}$$

$$\log Z_{2t} = (1 - \psi) \log \bar{Z} + a \log Z_{2t-1} + b \log Z_{1t-1} + E_{2t}$$

where $(E_{1t}, E_{2t})'$ is distributed Normal with mean $(0,0)$ and $var(E_{it}) = \sigma^2$, and $corr(E_{1t}, E_{2t}) = \rho_e$.

- a. Set A to zero. Define the Pareto problem and used the method of undetermined coefficients to solve for the equilibrium.
- b. Let A be positive. Solve for the equilibrium.
- c. Calibrate the log-linearized model and work out the impulse response of home and foreign consumption and investment and home net exports to a one percent shock to the productivity of the home country.

4. Segmented Markets

Alvarez, Atkeson and Kehoe paper on Segmented Markets, download from <http://www.econ.umn.edu/>

- a. Prove proposition 1
- b. Prove that with the long memory process a money injection can twist the yield curve

5. Time Varying Risk

Alvarez, Atkeson and Kehoe paper on Time-varying risk, download from <http://www.econ.umn.edu/>

- a. Set up the model as described in appendix A and show that the planning problem can be written as in page 35.