1. Risk-sharing and net exports

Consider an economy composed of two countries $i = 1, 2$. Each date a discrete event $s_t$ occurs and $s^t = (s_0, \ldots, s^t)$ is the history of events. Let $\pi(s^t)$ denote the probability distribution over $s^t$ and $\pi(s^t|s^{t-1})$ the conditional distribution over $s^t = (s^{t-1}, s_t)$ given $s^{t-1}$. Each country has a representative consumer with preferences

\[ X_t = 0_{X(s^t)}^1 U_i(c_i(s_t)) \]

and endowments $\{y_i(s^t)\}_{t=0}^\infty$. Let $y(s^t) = y_1(s^t) + y_2(s^t)$.

a. Let $q(s^t, s_{t+1})$ denote the price of a one-period bond bought at $t$ in state $s^t$ that pays off one unit at $t + 1$ if and only if state $s^{t+1} = (s^t, s_{t+1})$ occurs and let $b_i(s^t, s_{t+1})$ denote the amount of one period contingent bonds purchased by consumer $i$ at $t$. Define a competitive equilibrium is sequence form.

b. Let $p_t(s^t)$ denote the price in units of date 0 goods of one unit of date $t$ good. Define a competitive equilibrium in date 0 form.

c. Prove the equilibrium in parts a and b are the same. (Add the appropriate bound to debt and carefully use the transversality conditions in the sequence economy.)

d. Describe in words what happens when $\beta_1 < \beta_2$?

For the rest of the problem let $\beta_1 = \beta_2 = \beta$ and $U(c) = c^{1-\sigma}/1 - \sigma$. 
e. Compute the solution to the Pareto Problem of

$$\lambda \sum_{t=0}^{\infty} \sum_{s^t} \beta^t \pi(s^t)U(c_1(s^t)) + (1 - \lambda) \sum_{t=0}^{\infty} \sum_{s^t} \beta^t \pi(s^t)U(c_2(s^t)).$$

f. Find the Pareto weights (i.e. an explicit formula in terms of $\beta, \sigma, \pi(s^t)$, and $y_i(s^t)$) such that the solution to the Pareto problem coincides with that of the competitive equilibrium.

g. Define net exports of country $i$ as $nx_i = y_i - c_i$. What is the correlation of net exports and output? Are net export procyclical or countercyclical? What might explain the discrepancy between this model’s predictions and the data?

2. **A one country real business cycle model**

Calibrate the log-linearized model by hand in the Uhlig paper on the reading list and work out the impulse response of consumption, investment, labor and output to a one percent technology shock.

3. **A two country real business cycle model**

Consider a two country version of Gary Hansen’s model that is exposited in the Uhlig paper. Let the preferences be

$$\max E_0 \sum_{t=0}^{\infty} \beta^t (\log C_{1t} - AN_{1t})$$

the resource constraint is

$$\sum_i (C_{it} + K_{it}) = \sum_i (Y_{it} + (1 - \delta)K_{it-1})$$

where $Y_{it} = Z_{it}K_{it}^{\rho}N_{it}^{1-\rho}$ and

$$\log Z_{1t} = (1 - \psi) \log Z + a \log Z_{1t-1} + b \log Z_{2t-1} + E_{1t}$$
\[ \log Z_{2t} = (1 - \psi) \log \bar{Z} + a \log Z_{2t-1} + b \log Z_{1t-1} + E_{2t} \]

where \((E_{1t}, E_{2t})^{'}\) is distributed Normal with mean (0,0) and \(\text{var}(E_{1t}) = \sigma^2\), and \(\text{corr}(E_{1t}, E_{2t}) = \rho_e\).

a. Set \(A\) to zero. Define the Pareto problem and used the method of undetermined coefficients to solve for the equilibrium.

b. Let \(A\) be positive. Solve for the equilibrium.

c. Calibrate the log-linearized model and work out the impulse response of home and foreign consumption and investment and home net exports to a one percent shock to the productivity of the home country.

4. Segmented Markets


a. Prove proposition 1

b. Prove that with the long memory process a money injection can twist the yield curve

5. Time Varying Risk


a. Set up the model as described in appendix A and show that the planning problem can be written as in page 35.