1. Consider an economy that has the aggregate production function

\[ Y_t = A_t K_t^\alpha L_t^{1-\alpha} \]

and in which feasible consumption and investment plans \((C_t, I_t)\) satisfy

\[ C_t + I_t = Y_t \]
\[ K_{t+1} = (1 - \delta)K_t + I_t. \]

a) Discuss carefully how you would measure each of the variables \(Y_t, C_t, I_t, K_t\). How would you measure \(A_t\)? Discuss the significance of \(A_t\).

b) Define a balanced growth path for this economy.

2. Find annual time series data on real output, real investment, employment, working age population, and — if you can — hours worked for some country. If you have sufficient data for other variables, calibrate an annual depreciation rate \(\delta\) and a capital share \(\alpha\). Otherwise, use the values \(\delta = 0.05\) and \(\alpha = 0.30\) in what follows.

a) Use the data for real investment to construct a series for the capital stock following the rule

\[ K_{t+1} = (1 - \delta)K_t + I_t \]
\[ K_{t_0} = \bar{K}_{t_0}. \]

where \(T_0\) is the first year for which you have data on output and investment. Choose \(\bar{K}_{t_0}\) so that

\[ K_{T_0+1} / K_{T_0} = (K_{T_0+10} / K_{T_0})^{1/10}. \]

b) Repeat part a, but choose \(\bar{K}_{t_0}\) so that

\[ K_{T_0} / Y_{T_0} = \left( \sum_{t=T_0}^{T_0+9} K_t / Y_t \right) / 10. \]

c) Compare the two series constructed in parts a and b.

d) Perform a growth accounting exercise for this economy. That is, decompose the growth and fluctuations in real GDP per working-age person into three factors, one of which depends on total factor productivity, one of which depends on the capital/output ratio, and the third of which depends on hours worked per working-age person:
\[
\frac{Y_t}{N_t} = A_t^{\frac{1}{1-a}} \left( \frac{K_t}{Y_t} \right)^{\frac{a}{1-a}} \frac{L_t}{N_t}.
\]

Discuss what happens during different time periods.