MACROECONOMIC THEORY **T. J. KEHOE**
ECON 8105 **FALL 2022**

FINAL EXAMINATION

Answer ***two*** of the following four questions.

1. Consider an overlapping generations economy in which there is one good in each period and each generation, except the initial one, lives for two periods. The representative consumer in generation t, $t = 1, 2, \ldots,$ has the utility function

$$\log\ c_t^t + 2\log\ c_{t+1}^t$$

and the endowment $(w_t^t, w_{t+1}^t) = (3, 3)$. The representative consumer in generation 0 lives only in period 1, has the utility function $2\log\ c_1^0$, and has the endowment $w_1^0 = 3$. There is no fiat money.

a) Define an Arrow-Debreu equilibrium for this economy. Calculate the unique Arrow-Debreu equilibrium.

b) Define a sequential markets equilibrium for this economy. Calculate the unique sequential markets equilibrium.

c) Define a Pareto efficient allocation. Prove either that the equilibrium allocation in part a is Pareto efficient or prove that it is not.

d) Suppose now that there are two consumers in each generation t, $t = 1, 2, \ldots$. Both consumers have the utility function

$$\log\ c_t^{it} + 2\log\ c_{t+1}^{it},\ i = 1, 2.$$

Consumers of type 1 have the endowment $(w_t^{1t}, w_{t+1}^{1t}) = (4, 2)$, while consumers of type 2 have the endowment $(w_t^{2t},\ w_{t+1}^{2t}) = (2, 4)$. The two representative consumers in generation 0 live only in period 1, have the utility function $2\log\ c_1^{i0}$, $i = 1, 2$, and have the endowments $w_1^{10} = 2$ and $w_1^{20} = 4$. There is no fiat money. Define a sequential markets equilibrium for this economy.

e) Calculate the sequential market equilibrium in part d.

2. Consider the optimal growth problem

$$\begin{aligned}
\max \textstyle\sum_{t=0}^{\infty} \beta^t \left[\gamma \log c_t + (1-\gamma)\log(\bar{h} - \ell_t)\right] \\
\text{s.t.} \quad c_t + k_{t+1} - (1-\delta)k_t \le \theta k_t^{\alpha} \ell_t^{1-\alpha} \\
c_t, k_t \ge 0 \\
\bar{h} \ge \ell_t \ge 0 \\
k_0 = \bar{k}_0 .
\end{aligned}$$

Here $\bar{h} > 0$ is the endowment of time and the parameters satisfy $1 > \beta > 0$, $1 \ge \delta > 0$, $1 > \alpha > 0$ $1 > \alpha > 0$, $\theta > 0$.

(a) Write down the Euler conditions and the transversality condition for this problem. Calculate the nontrivial steady state values of *c* and *k*. (The trivial steady state is $\hat{c} = \hat{k} = 0$.)

(b) Let $B(K)$ be the set of bounded functions on $K \subset R_+$. Define a contraction mapping $T : B(K) \to B(K)$. State Blackwell's sufficient conditions for the mapping T to be a contraction mapping.

(c) Let

$$\begin{aligned}
T(V)(k) = \max \gamma \log c + (1-\gamma)\log(\bar{h} - \ell) + \beta V(k') \\
\text{s.t.} \quad c + k' - (1-\delta)k \le \theta k^{\alpha} \ell^{1-\alpha} \\
c, k' \ge 0 \\
\bar{h} \ge \ell \ge 0 .
\end{aligned}$$

Explain how you can choose the set $K \subset R_+$ so that $T(V)$ is bounded above without restricting the set of solutions to the optimal growth problem when $V = T(V)$. Ignore the fact that $T(V)$ is not bounded below. Prove that T satisfies Blackwell's sufficient conditions.

(d) Suppose now that $\delta = 1$. Guess that the value function has the form $a_0 + a_1 \log k$ and that the optimal level of ℓ is constant. Calculate the policy functions $k' = g_k(k)$, $c = g_c(c)$, and $\ell = g_\ell(k)$.

(e) Define a sequential markets equilibrium for which the solution to the optimal growth problem in part d is an equilibrium allocation and production plan. Explain carefully how to use the policy functions in part d to calculate this equilibrium.

3. Consider an economy with two types of consumers. There are equal measures of each type. The representative consumer of type i, $i = 1, 2$, has the utility function

$$\sum_{t=0}^{\infty} \beta^t \log c_t^i$$

where $0 < \beta < 1$. This consumer has an endowment of $\bar{k}_0^i$ units of capital in period 0. Labor endowments are

$$(\bar{\ell}_0^1, \bar{\ell}_1^1, \bar{\ell}_2^1, \bar{\ell}_3^1, \ldots) = (2,\ 1,\ 2,\ 1, \ldots)$$

and

$$(\bar{\ell}_0^2, \bar{\ell}_1^2, \bar{\ell}_2^2, \bar{\ell}_3^2, \ldots) = (1,\ 2,\ 1,\ 2, \ldots).$$

Feasible allocation/production plans satisfy

$$c_t + k_{t+1} - (1-\delta)k_t \leq \theta k_t^{\alpha} \ell_t^{1-\alpha},$$

where variables without superscripts denote aggregates.

(a) Describe an Arrow-Debreu market structure for this economy, explaining when markets are open, who trades with whom, and so on. Define an Arrow-Debreu equilibrium.

(b) Describe a sequential markets structure for this economy, explaining when markets are open, who trades with whom, and so on. Define a sequential markets equilibrium.

(c) Carefully state a proposition or propositions that establish the essential equivalence of the equilibrium concept in part a with that in part b. Be sure to specify the relationships between the objects in the Arrow-Debreu equilibrium and those in the sequential markets equilibrium.

(d) Define a Pareto efficient allocation and production plan for this environment.

(e) Does the equilibrium allocation and production plan in part a solve a dynamic programming problem? If so, write down Bellman's equation for this problem. If not, explain why not.

4. Consider the problem faced by an unemployed worker searching for a job. Every period that the worker searches, she receives a job offer with the wage w drawn independently from the time invariant probability distribution $F(v) = \text{prob}(w \le v)$, $v \in [0, B]$, $B > 0$. After receiving the wage offer w the worker faces the choice (1) to accept it or (2) to reject it, receive unemployment benefit b, and search again next period. That is,

$$y_t = \begin{cases} w & \text{if job offer has been accepted} \\ b & \text{if searching} \end{cases}.$$

The worker solves

$$\max E \sum_{t=0}^{\infty} \beta^t y_t$$

where $1 > \beta > 0$. Once a job offer has been accepted, there are no fires or quits.

(a) Formulate the worker's problem as a dynamic programming problem by writing down Bellman's equation.

(b) Using Bellman's equation from part a, characterize the value function $V(w)$ in a graph and argue that the worker's problem reduces to determining a reservation wage $\bar{w}$ such that she accepts any wage offer $w \ge \bar{w}$ and rejects any wage offer $w < \bar{w}$.

(c) Consider two economies with different unemployment benefits b_1 and b_2 but otherwise identical. Let $\bar{w}_1$ and $\bar{w}_2$ be the reservation wages in these two economies. Suppose that $b_2 > b_1$. Prove that $\bar{w}_2 > \bar{w}_1$. Provide some intuition for this result.

(d) Consider two economies with different wage distributions F_1 and F_2 but otherwise identical. Let $\bar{w}_1$ and $\bar{w}_2$ be the reservation wages in these two economies. Define a mean preserving spread. Suppose that F_2 is a mean preserving spread of F_1. Prove that $\bar{w}_2 > \bar{w}_1$. Provide some intuition for this result.