1. Consider an economy two infinitely lived consumers, each of whom has the same utility function,
\[ u(c^1, c^2, \ldots) = \sum_{t=0}^{\infty} \beta^t \log c^t, \]
where \( 0 < \beta < 1 \). Suppose that consumer 1 has the endowments
\[ (w^1_0, w^1_1, w^1_2, w^1_3, \ldots) = (6, 4, 6, 4, \ldots), \]
and consumer 2 has the endowments
\[ (w^2_0, w^2_1, w^2_2, w^2_3, \ldots) = (4, 6, 4, 6, \ldots). \]

a) Describe an Arrow-Debreu market structure for this economy, explaining when markets are open, who trades with whom, and so on. Define an Arrow-Debreu equilibrium.

b) Define a Pareto efficient allocation for this economy. Calculate a Pareto efficient allocation by maximizing a weighted sum of utilities, \( \alpha_1 u_1 + \alpha_2 u_2 \).

c) Define an Arrow-Debreu equilibrium with transfers. Find the transfer payments necessary to implement the Pareto efficient allocation in part b as equilibrium with transfers. Demonstrate that the transfer payments are homogeneous of degree one in \((\alpha_1, \alpha_2)\) and sum to 0.

d) Find the transfer payments necessary to implement the allocation \((c^1_t, c^2_t) = (5, 5)\) as an equilibrium with transfers.

e) Calculate the (unique) Arrow-Debreu equilibrium of this economy.

f) Define a sequential markets equilibrium. Calculate the unique sequential markets equilibrium of the economy.
2. Consider a simple overlapping generations economy in which the representative consumer born in period $t$, $t = 1, 2, \ldots$, has the utility function

$$u(c_t^t, c_{t+1}^t) = c_t^t + \left[ \left( c_{t+1}^t \right)^b - 1 \right] / b,$$

where $b < 1$. Suppose that his endowment is $(w_t^t, w_{t+1}^t) = (w_1, w_2)$.

a) What is the utility function in the case where $b = 0$? [Hint: use l’Hôpital’s rule.]

b) Write down the utility maximization problem in an environment with Arrow-Debreu markets. Derive the excess demand functions $\eta(p_t, p_{t+1})$ and $\eta(p_t, p_{t+1})$. Demonstrate that they are homogeneous of degree zero and that they satisfy Walras’s law.

c) Suppose that the representative consumer in the first generation has the utility function $u^0(c_1^0) = \left[ (c_1^0)^b - 1 \right] / b$. This consumer is endowed with $w_1^0 = w_2$ of the good in period 1 as well as $m$ units of fiat money, where $m$ can be positive, negative, or 0. Explain the role of $m$. Define an Arrow-Debreu equilibrium of this model. Write down the equilibrium conditions using the excess demand functions.

d) Find an expression for the offer curve for this model. (Hint: you have to solve for $y$ as a function of $z$.)

e) Suppose that $w_1 = 1$ and $w_2 = 0.25$. Draw the offer curve for the three cases $b = 0.5$, $b = 0$, and $b = -1$.

f) Define a sequential market equilibrium for this economy. Suppose that you have calculated the Arrow-Debreu equilibrium in part d. Explain how you can use the Arrow-Debreu equilibrium to calculate the sequential market equilibrium.