1. Consider an economy two infinitely lived consumers, each of whom has the same utility function,

\[ u(c_0^i, c_1^i, \ldots) = \sum_{i=0}^{\infty} \beta^i \log c_i^i, \]

where \( 0 < \beta < 1 \). Suppose that consumer 1 has the endowments

\[ (w_0^1, w_1^1, w_2^1, w_3^1, \ldots) = (1, 5, 1, 5, \ldots) \]

and consumer 2 has the endowments

\[ (w_0^2, w_1^2, w_2^2, w_3^2, \ldots) = (5, 1, 5, 1, \ldots) \]

a) Describe an Arrow-Debreu market structure for this economy, explaining when markets are open, who trades with whom, and so on. Define an Arrow-Debreu equilibrium.

b) Define a Pareto efficient allocation for this economy. Calculate a Pareto efficient allocation by maximizing a weighted sum of utilities, \( \alpha_1 u_1 + \alpha_2 u_2 \).

c) Define an Arrow-Debreu equilibrium with transfers. Find the transfer payments necessary to implement the Pareto efficient allocation in part b as equilibrium with transfers. Demonstrate that the transfer payments are homogeneous of degree one in \((\alpha_1, \alpha_2)\) and sum to 0.

d) Find the transfer payments necessary to implement the allocation \((c_0^1, c_2^2) = (3, 3)\) as an equilibrium with transfers.

e) Calculate the (unique) Arrow-Debreu equilibrium of this economy.

f) Define a sequential markets equilibrium. Calculate the unique sequential markets equilibrium of the economy.

2. Consider a simple overlapping generations economy in which the representative consumer born in period \( t \), \( t = 1, 2, \ldots \), has the utility function

\[ u(c_t^i, c_{t+1}^i) = c_t^i + \left[ (c_{t+1}^i)^b - 1 \right] / b, \]

where \( b < 1 \). Suppose that his endowment is \((w_t^i, w_{t+1}^i) = (w_i, w_2)\).

a) What is the utility function in the case where \( b = 0 \)? [Hint: use l’Hôpital’s rule.]
b) Write down the utility maximization problem in an environment with Arrow-Debreu markets. Derive the excess demand functions \( y(p_t, p_{t+1}) \) and \( y(p_t, p_{t+1}) \). Demonstrate that they are homogeneous of degree zero and that they satisfy Walras’s law.

c) Suppose that the representative consumer in the first generation has the utility function \( u^0(c^0_t) = \left( (c^0_t)^b - 1 \right) / b \). This consumer is endowed with \( w^0_1 = w_2 \) of the good in period 1 as well as \( m \) units of fiat money, where \( m \) can be positive, negative, or 0. Explain the role of \( m \). Define an Arrow-Debreu equilibrium of this model. Write down the equilibrium conditions using the excess demand functions.

d) Find an expression for the offer curve for this model. (Hint: you have to solve for \( y \) as a function of \( z \).)

e) Suppose that \( w_1 = 2 \) and \( w_2 = 0.75 \). Draw the offer curve for the three cases \( b = 0.5 \), \( b = 0 \), and \( b = -1 \).

f) Define a sequential market equilibrium for this economy. Suppose that you have calculated the Arrow-Debreu equilibrium in part d. Explain how you can use the Arrow-Debreu equilibrium to calculate the sequential market equilibrium.