PROBLEM SET #1

1. Consider an economy two infinitely lived consumers, each of whom has the same utility function,

\[ u(c_i^0, c_i^1, \ldots) = \sum_{t=0}^{\infty} \beta^t \log c_i^t, \]

where \(0 < \beta < 1\). Suppose that consumer 1 has the endowments

\( (w_0^1, w_1^1, w_2^1, w_3^1, \ldots) = (1,5,1,5,\ldots) \)

and consumer 2 has the endowments

\( (w_0^2, w_1^2, w_2^2, w_3^2, \ldots) = (5,1,5,1,\ldots) \)

a) Describe an Arrow-Debreu market structure for this economy, explaining when markets are open, who trades with whom, and so on. Define an Arrow-Debreu equilibrium.

b) Define a Pareto efficient allocation for this economy. Calculate a Pareto efficient allocation by maximizing a weighted sum of utilities, \( \alpha_1 u_1 + \alpha_2 u_2 \).

c) Define an Arrow-Debreu equilibrium with transfers. Find the transfer payments necessary to implement the Pareto efficient allocation in part b as equilibrium with transfers. Demonstrate that the transfer payments are homogeneous of degree one in \( (\alpha_1, \alpha_2) \) and sum to 0.

d) Find the transfer payments necessary to implement the allocation \( (c_1^1, c_2^2) = (3,3) \) as an equilibrium with transfers.

e) Calculate the (unique) Arrow-Debreu equilibrium of this economy.

f) Define a sequential markets equilibrium. Calculate the unique sequential markets equilibrium of the economy.

2. Consider a simple overlapping generations economy in which the representative consumer born in period \( t \), \( t = 1, 2, \ldots, \) has the utility function

\[ u(c_i^t, c_{i+1}^t) = c_i^t + \left( c_{i+1}^t \right)^b - 1 \]

where \( b < 1 \). Suppose that his endowment is \( (w_i^t, w_{i+1}^t) = (w_i, w_2) \).

a) What is the utility function in the case where \( b = 0 \)? [Hint: use l’Hôpital’s rule.]
b) Write down the utility maximization problem in an environment with Arrow-Debreu markets. Derive the excess demand functions $y(p_t, p_{t+1})$ and $y(p_t, p_{t+1})$. Demonstrate that they are homogeneous of degree zero and that they satisfy Walras’s law.

c) Suppose that the representative consumer in the first generation has the utility function $u^0(c_t^0) = \left[\left(c_t^0\right)^{\beta} - 1\right] / \beta$. This consumer is endowed with $w_1^0 = w_2$ of the good in period 1 as well as $m$ units of fiat money, where $m$ can be positive, negative, or 0. Explain the role of $m$. Define an Arrow-Debreu equilibrium of this model. Write down the equilibrium conditions using the excess demand functions.

d) Find an expression for the offer curve for this model. (Hint: you have to solve for $y$ as a function of $z$.)

e) Suppose that $w_1 = 2$ and $w_2 = 0.75$. Draw the offer curve for the three cases $b = 0.5$, $b = 0$, and $b = -1$.

f) Define a sequential market equilibrium for this economy. Suppose that you have calculated the Arrow-Debreu equilibrium in part d. Explain how you can use the Arrow-Debreu equilibrium to calculate the sequential market equilibrium.