1. Consider an economy like that in question 3 on problem set #4 in which the equilibrium allocation is the solution to the optimal growth problem

\[
\begin{align*}
\max & \ E \sum_{t=0}^{\infty} \beta^t \left[ \theta \log C_t + (1 - \theta) \log (N_t \bar{h} - L_t) \right] \\
\text{s.t.} & \quad C_t + K_{t+1} - (1 - \delta)K_t \leq e^{\zeta} \left( \gamma^{1-\alpha} \right)^{\frac{1}{\gamma}} A_0 K_t^{\alpha} L_t^{1-\alpha} \\
& \quad C_t, K_t \geq 0 \\
& \quad K_0 = \bar{K}_0 \\
& \quad N_t = \eta N_0.
\end{align*}
\]

Here \( z_t \) is a random variable that takes on two values \( \bar{z}_t = -\zeta, \bar{z}_2 = \zeta \), and whose evolution is governed by the stationary, first order Markov chain with transition matrix \( \Pi = \begin{bmatrix} 1 - \pi & \pi \\ \pi & 1 - \pi \end{bmatrix} \).

Assume, for the sake of specificity, that, at \( t = 0 \), \( z_0 = \bar{z}_1 = -\zeta \).

(a) Define an Arrow-Debreu equilibrium for this economy.

(b) Define a sequential markets equilibrium for this economy.

Redefine variables \( C_t \) and \( K_t \) by dividing by the number of effective working age persons \( \bar{N}_t = \gamma^t N_t = (\gamma \eta)^t N_0 \). Divide \( L_t \) by \( N_t \):

\[
\begin{align*}
c_t &= C_t / \bar{N}_t = \gamma^{\bar{N}_t} (C_t / N_t) \\
k_t &= K_t / \bar{N}_t = \gamma^{\bar{N}_t} (K_t / N_t) \\
\ell_t &= L_t / \bar{N}_t.
\end{align*}
\]

Consider the social planner’s problem

\[
\begin{align*}
\max & \ E \sum_{t=0}^{\infty} \beta^t \left[ \theta \log c_t + (1 - \theta) \log (\bar{h} - \ell_t) \right] \\
\text{s.t.} & \quad c_t + \gamma \eta k_{t+1} - (1 - \delta)k_t \leq e^{\zeta} A_0 k_t^{\alpha} \ell_t^{1-\alpha} \\
& \quad c_t, k_t \geq 0, \bar{h} \geq \ell_t \geq 0 \\
& \quad k_0 = \bar{K}_0 / N_0,
\end{align*}
\]

with the associated Bellman’s equation.
\[ V(k, z) = \max \theta \log c + (1- \theta) \log(h - \ell) + \beta EV(k', z') \]
\[ \text{s. t. } c + \gamma \eta k' - (1- \delta) k \leq e^z A_0 k^\alpha \ell^{1-\alpha} \]
\[ c, k' \geq 0, \quad h \geq \ell \geq 0 \]
\[ k, z \text{ given.} \]

c) Suppose that you have solved this dynamic programming problem and have found the policy functions \( k' = g(k, z), c = c(k, z), \text{ and } \ell = \ell(k, z) \). Explain how you can use these policy functions to calculate the Arrow-Debreu equilibrium. Explain how you can use these policy functions to calculate the sequential markets equilibrium.

2. Consider an economy like that in question 1 in which the equilibrium allocation is the solution to the optimal growth problem with the Bellman’s equation
\[ V(k, z) = \max \theta \log c + (1- \theta) \log(h - \ell) + \beta EV(k', z') \]
\[ \text{s. t. } c + \gamma \eta k' - (1- \delta) k \leq e^z A_0 k^\alpha \ell^{1-\alpha} \]
\[ c, k' \geq 0, \quad h \geq \ell \geq 0 \]
\[ k, z \text{ given.} \]

That is, \( \delta = 1 \). Guess that the value function \( V(k, z) \) has the form
\[ V_i(k) = V(k, z_i) = a_{0i} + a_{1i} \log k, \quad i = 1, 2 \]

for the yet-to-be-determined coefficients \( a_{01}, a_{11}, a_{02}, a_{12} \).

a) Solve for the value functions \( V_i(k) \), \( i = 1, 2 \).

b) Solve for the policy functions \( k' = g_i(k), \quad c = c_i(k), \text{ and } \ell = \ell_i(k) \).

3. Consider the problem faced by an unemployed worker searching for a job. Every period that the worker searches, she receives a job offer with the wage \( w \) drawn independently from the time invariant probability distribution \( F(v) = \text{prob}(w \leq v), \quad v \in [0, B], \quad B > 0 \). After receiving the wage offer \( w \) the worker faces the choice (1) to accept it or (2) to reject it, receive unemployment benefit \( b \), and search again next period. That is,
\[ y_i = \begin{cases} w & \text{if job offer has been accepted} \\ b & \text{if searching} \end{cases} \]

The worker solves
\[ \max E \sum_{t=0}^{\infty} \beta^t y_t \]
where \( 1 > \beta > 0 \). Once a job offer has been accepted, there are no fires or quits.

a) Formulate the worker’s problem as a dynamic programming problem by writing down Bellman’s equation.
b) Using Bellman’s equation from part a, characterize the value function $V(w)$ in a graph and argue that the worker’s problem reduces to determining a reservation wage $\bar{w}$ such that she accepts any wage offer $w \geq \bar{w}$ and rejects any wage offer $w < \bar{w}$.

c) Consider two economies with different unemployment benefits $b_1$ and $b_2$ but otherwise identical. Let $\bar{w}_1$ and $\bar{w}_2$ be the reservation wages in these two economies. Suppose that $b_2 > b_1$. Prove that $\bar{w}_2 > \bar{w}_1$. Provide some intuition for this result.

d) Consider two economies with different wage distributions $F_1$ and $F_2$ but otherwise identical. Let $\bar{w}_1$ and $\bar{w}_2$ be the reservation wages in these two economies. Define a mean preserving spread. Suppose that $F_2$ is a mean preserving spread of $F_1$. Prove that $\bar{w}_2 > \bar{w}_1$. Provide some intuition for this result.