1. Consider the problem faced by an unemployed worker searching for a job. Every period that the worker searches, she receives a job offer with the wage $w$ drawn independently from the time invariant probability distribution $F(v) = \text{prob}(w \leq v)$, $v \in [0, B]$, $B > 0$. After receiving the wage offer $w$ the worker faces the choice (1) to accept it or (2) to reject it, receive unemployment benefit $b$, and search again next period. That is,

$$y_t = \begin{cases} w & \text{if job offer has been accepted} \\ b & \text{if searching} \end{cases}$$

The worker solves

$$\max E \sum_{t=0}^{\infty} \beta^t y_t$$

where $1 > \beta > 0$. Once a job offer has been accepted, there are no fires or quits.

a) Formulate the worker’s problem as a dynamic programming problem by writing down Bellman’s equation.

b) Using Bellman’s equation from part a, characterize the value function $V(w)$ in a graph and argue that the worker’s problem reduces to determining a reservation wage $\bar{w}$ such that she accepts any wage offer $w \geq \bar{w}$ and rejects any wage offer $w < \bar{w}$.

c) Consider two economies with different unemployment benefits $b_1$ and $b_2$ but otherwise identical. Let $\bar{w}_1$ and $\bar{w}_2$ be the reservation wages in these two economies. Suppose that that $b_2 > b_1$. Prove that $\bar{w}_2 > \bar{w}_1$. Provide some intuition for this result.

d) Consider two economies with different wage distributions $F_1$ and $F_2$ but otherwise identical. Let $\bar{w}_1$ and $\bar{w}_2$ be the reservation wages in these two economies. Define a mean preserving spread. Suppose that $F_2$ is a mean preserving spread of $F_1$. Prove that $\bar{w}_2 > \bar{w}_1$. Provide some intuition for this result. Explain why expected utility is higher in the economy with wage distribution $F_2$ than it is in the economy with wage distribution $F_1$. 