1. Consider a static economy with a continuum of identical consumers. Consumer \( i \), \( i \in [0,1] \) is endowed with one unit of labor and one unit of capital. All consumers have the same utility function,

\[
\log c - \log(3\ell + 1).
\]

Individual labor supply can only take on the values \( \ell = 1 \) or \( \ell = 0 \). Output for consumption is produced according to the production function

\[
10K^{0.4}L^{0.6},
\]

where \( K \) is aggregate capital and \( L \) is aggregate labor.

a) Define a competitive equilibrium for this economy.

b) Prove that there is no equilibrium in which all consumers make the same labor supply choice. (Hint: It is easy to show that, if all consumers supply \( \ell(i) = 1 \), any individual consumer would prefer to set \( \ell(i) = 0 \). The case where all consumers supply \( \ell(i) = 0 \) is more difficult because of discontinuities at \( L = 0 \). Suppose instead that a measure \( \varepsilon > 0 \) of consumers supply \( \ell(i) = 1 \) and a measure \( 1 - \varepsilon \) supply \( \ell(i) = 0 \) and argue that, if \( \varepsilon \) is small enough, all consumers prefer to set \( \ell(i) = 1 \).)

c) Define the economic environment of an economy with lotteries and define a competitive equilibrium.

d) Consider an alternative economy in which the representative consumer has the utility function

\[
\log C - (\log 4)L
\]

and the endowments \( L = 1, K = 1 \). Labor is now perfectly divisible. Output is still produced according to the production function

\[
10K^{0.4}L^{0.6}.
\]

Define a competitive equilibrium for this economy. Define a Pareto efficient allocation. Set up and solve a social planner’s problem to find the unique Pareto efficient allocation for this economy. Use the solution to this social planner’s problem to find the equilibrium.
e) Use your results in part d to carefully specify the competitive equilibrium for the economy with lotteries in part c.

2. Find annual time series data on real output, real investment, hours worked, and working age population for some country. If you have sufficient data for other variables, calibrate an annual depreciation rate $\delta$ and a capital share $\alpha$. Otherwise, use the values $\delta = 0.05$ and $\alpha = 0.30$ in what follows.

a) Use the data for real investment to construct a series for the capital stock following the rule

$$K_{t+1} = (1 - \delta)K_t + I_t$$

$$K_{t_0} = \bar{K}_{t_0}.$$

where $T_0$ is the first year for which you have data on output and investment. Choose $\bar{K}_{t_0}$ so that

$$K_{t_0} / \bar{K}_{t_0} = (K_{T_0+10} / K_{T_0})^{1/10}.$$

b) Repeat part a, but choose $\bar{K}_{t_0}$ so that

$$K_{t_0} / Y_{t_0} = \left(\sum_{t=T_0}^{T_0+9} K_t / Y_t\right) / 10.$$

c) Compare the two series constructed in parts a and b.

d) Perform a growth accounting exercise for this economy. That is, decompose the growth and fluctuation in real GDP per working-age person into three factors, one of which depends on total factor productivity, one of which depends on the capital/output ratio, and the third of which depends on hours worked per working-age person. Discuss what happens during different time periods.