ECON 8401 **T. J. KEHOE**
INTERNATIONAL TRADE AND PAYMENTS THEORY **FALL 2016**

PROBLEM SET #2

1. Consider an economy in which there are two types of goods, agriculture and manufactured goods. Agricultural goods are homogeneous and are produced using labor according to the constant returns to scale production function

$$y_0 = \ell_0.$$

Manufactured goods are differentiated by firm. The production function for firm j is

$$y_j = (1/b)\max[\ell_j - f, 0].$$

Here f is the fixed cost, in terms of labor, necessary to operate the firm and b is the unit labor requirement. Suppose that there is a representative consumer with preferences

$$\log c_0 + (1/\rho)\log \sum_{j=1}^{n} c_j^{\rho},$$

where $1 \geq \rho > 0$. There is an endowment of $\bar{\ell}$ units of labor.

a) Define a monopolistically competitive equilibrium for this economy in which firms follow Cournot pricing rules and there is free entry and exit.

b) Suppose that $b = 2$, $f = 4$, $\rho = 1/2$, and $\bar{\ell} = 36$. Calculate the autarky equilibrium.

c) Suppose now that $\bar{\ell} = 180$. Calculate the equilibrium.

d) Interpret the equilibrium in part c as a trading equilibrium among two countries, one with $\bar{\ell}^1 = 36$ and the second with $\bar{\ell}^2 = 144$. Assume that production of the homogeneous good is distributed proportionally across the two countries. What impact does trade have on the number of manufacturing firms in each country? The average output of firms? The total number of products available? Consumer utility and real income? Illustrate the efficiency gains using an average cost curve diagram.

2. Repeat the analysis of question 1 for two variants of the model. Compare the gains from trade in these two alternative models with those in the model in question 1.

a) Suppose that there are again a finite number of differentiated goods but that firms are now Bertrand competitors, rather than Cournot competitors.

b) Suppose that consumers have the utility function

$$\log c_0 + (1/\rho)\log \int_0^n c(j)^\rho \, dj.$$

Here there is a continuum $[0, n]$ of differentiated goods. (Hint: You need to be very careful in taking derivatives when solving the firms' profit maximization problems. In particular, the answers change drastically.)

c) Compare the gains in real income in parts a and b with each other and with those in question 1, part d.

3. Consider an economy in which the consumption space is the set of functions $c: R_+ \times R_+ \to R_+$. In $c(x,t)$ the index x denotes the type of good and the index t denotes the date at which it is consumed. An individual consumer has preferences given by the functional

$$u(c) = \int_0^\infty e^{-\rho t} \left[\int_0^\infty \log(c(x,t)+1)dx \right] dt.$$

Goods are produced using a single factor of production, labor:

$$y(x,t) = \ell(x,t)/a(x,t).$$

Each consumer has an endowment of labor equal to 1, and the total number of consumers is fixed at $\bar{\ell}$. The unit labor requirement $a(x,t)$ is bounded from below, $a(x,t) > \bar{a}(x)$, where

$$\bar{a}(x) = e^{-x}.$$

At $t = 0$ there is a $z(0) > 0$ such that $a(x,0) = e^{-x}$ for all $x < z(0)$ and that $a(x,0) = e^{x-2z(0)}$ for all $x \geq z(0)$. There is learning by doing of the form

$$\frac{\dot{a}(x,t)}{a(x,t)} = \begin{cases} -\int_0^\infty b(v,t)\ell(v,t)dv & \text{if } a(x,t) > \bar{a}(x) \\ 0 & \text{if } a(x,t) = \bar{a}(x) \end{cases}.$$

Here $\dot{a}(x,t)$ denotes the partial derivative of $a(x,t)$ with respect to t and $b(v,t) = b > 0$ if $a(v,t) > \bar{a}(v)$ and $b(v,t) = 0$ if $a(v,t) = \bar{a}(v)$. There is no borrowing or lending, and there is no storage.

a) Provide a motivation for the production technology described above.

b) Define an equilibrium for this economy. Characterize the equilibrium as much as possible.

c) Consider now a two country world in which the two countries are identical except in their endowments of labor and their initial technology levels. In particular, $z^1(0) > z^2(0)$. Define an equilibrium for this economy.

d) Describe the environment of a static Ricardian model whose equilibrium has the same values of prices and quantities as $p(x,0)$, $w^1(0)$, $w^2(0)$, $y^1(x,0)$, $y^2(x,0)$, $c^1(x,0)$, $c^2(x,0)$ in the economy of part c. Illustrate and explain the (five) different possibilities for patterns of production and consumption in this model. (To make things easy assume that $z^1(0)$ and $z^2(0)$ are sufficiently large so that good $x = 0$ is not produced in equilibrium.)

e) Describe the dynamics of the model, explaining the crucial role played by the sizes of the two countries, $\bar{\ell}^1$ and $\bar{\ell}^2$.

4. Consider an economy with two goods that enter both consumption and investment. The utility function of the representative consumer is

$$\sum_{t=0}^{\infty} \beta^t \log(c_{1t}^{a_1} c_{2t}^{a_2}).$$

Here $0 < \beta < 1$, $a_1 \geq 0$, $a_2 \geq 0$, and $a_1 + a_2 = 1$. Investment goods are produced according to

$$k_{t+1} - (1-\delta)k_t = d x_{1t}^{a_1} x_{2t}^{a_2}.$$

Feasible consumption/investment plans satisfy the feasibility conditions

$$c_{1t} + x_{1t} = \phi_1(k_{1t}, \ell_{1t}) = k_{1t}$$

$$c_{2t} + x_{2t} = \phi_2(k_{2t}, \ell_{2t}) = \ell_{2t}$$

where

$$k_{1t} + k_{2t} = k_t$$

$$\ell_{1t} + \ell_{2t} = \ell_t.$$

The initial endowment of k_t is $\bar{k}_0$. ℓ_t is equal to 1. (In other words, all variables are expressed in per capita terms.)

a) Carefully define a competitive equilibrium for this economy.

b) Reduce the equilibrium conditions to two difference equations in k_t and c_t and a transversality condition. Here $c_t = d c_{1t}^{a_1} c_{2t}^{a_2}$ is aggregate consumption. [Here is one possible approach: Prove a version of the first and second welfare theorems for this economy. Show that the two-sector social planer's problem is equivalent to a one-sector social planner's problem. Derive the difference equations and transversality conditions from the one-sector social planner's problem.]

c) Suppose now that there is a world composed of n different countries, all with the same preferences and technologies, but with different initial endowments of capital per worker, $\bar{k}_0^i$ The countries also have different population sizes, L^i, which are constant over time. (In other words, there is a continuum of identical consumers/workers of measure L^i in country i.) Suppose that there is no international borrowing or lending and no and no international capital flows. Define an equilibrium for this world economy. Prove that in this equilibrium the variables $c_{jt} = \sum_{i=1}^{n} L^i c_{jt}^i \Big/ \sum_{j=1}^{n} L^i$, $k_t = \sum_{i=1}^{n} L^i k_t^i \Big/ \sum_{i=1}^{n} L^i$, p_{it}, r_t, and w_t satisfy the equilibrium conditions of the economy in part a when $\bar{k}_0 = \sum_{i=1}^{n} L^i \bar{k}_0^i \Big/ \sum_{i=1}^{n} L^i$.

d) State and prove versions of the factor price equalization theorem, the Stolper-Samuelson theorem, the Rybczynski theorem, and the Heckscher-Ohlin theorem for this particular world economy.

e) What effects do the assumptions of no international borrowing and lending and no international capital flows have on your analysis? Try to be precise about which variables are uniquely determined in the equilibrium with borrowing and lending and/or capital flows are which would not be uniquely determined.

f) Consider the case where $\delta = 1$. Set $z_0 = c_0 / (\beta r_0 k_0)$ and $z_t = c_{t-1} / k_t$, $t = 1, 2, \ldots$. Transform the two difference equations in part b into two difference equations in k_t and z_t. Prove that

$$\frac{k_t^i - k_t}{k_t} = \frac{z_t}{z_{t-1}} \left(\frac{k_{t-1}^i - k_{t-1}}{k_{t-1}} \right) = \frac{z_t}{z_0} \left(\frac{\bar{k}_0^i - \bar{k}_0}{\bar{k}_0} \right).$$

g) Consider again the case where $\delta = 1$. Let $s_t = c_t / y_t$ where

$$y_t = p_{1t} k_t + p_{2t} = d k_t^{a_1} = r_t k_t + w_t .$$

Transform the two difference equations in part b into two difference equations in k_t and s_t. Prove that

$$\frac{y_t^i - y_t}{y_t} = \frac{s_t}{s_{t-1}} \left(\frac{y_{t-1}^i - y_{t-1}}{y_{t-1}} \right) = \frac{s_t}{s_0} \left(\frac{y_0^i - y_0}{y_0} \right)$$

where $y_t^i = p_{1t} y_{1t}^i + p_{2t} y_{2t}^i = r_t k_t^i + w_t$. Calculate an expression for s_t and discuss the significance of this result.