

PROBLEM SET #2

1. Consider an economy in which there are two types of goods, agriculture and manufactured goods. Agricultural goods are homogeneous and are produced using labor according to the constant returns to scale production function

$$y_0 = \ell_0.$$

Manufactured goods are differentiated by firm. The production function for firm  $j$  is

$$y_j = (1/b) \max[\ell_j - f, 0].$$

Here  $f$  is the fixed cost, in terms of labor, necessary to operate the firm and  $b$  is the unit labor requirement. Suppose that there is a representative consumer with preferences

$$\log c_0 + (1/\rho) \log \sum_{j=1}^n c_j^\rho,$$

where  $1 \geq \rho > 0$ . There is an endowment of  $\bar{\ell}$  units of labor.

- a) Define a monopolistically competitive equilibrium for this economy in which firms follow Cournot pricing rules and there is free entry and exit.
- b) Suppose that  $b = 2$ ,  $f = 4$ ,  $\rho = 1/2$ , and  $\bar{\ell} = 36$ . Calculate the autarky equilibrium.
- c) Suppose now that  $\bar{\ell} = 180$ . Calculate the equilibrium.
- d) Interpret the equilibrium in part c as a trading equilibrium among two countries, one with  $\bar{\ell}^1 = 36$  and the second with  $\bar{\ell}^2 = 144$ . Assume that production of the homogeneous good is distributed proportionally across the two countries. What impact does trade have on the number of manufacturing firms in each country? The average output of firms? The total number of products available? Consumer utility and real income? Illustrate the efficiency gains using an average cost curve diagram.

2. Repeat the analysis of question 1 for two variants of the model. Compare the gains from trade in these two alternative models with those in the model in question 1.

- a) Suppose that there are again a finite number of differentiated goods but that firms are now Bertrand competitors, rather than Cournot competitors.
- b) Suppose that consumers have the utility function

$$\log c_0 + (1/\rho) \log \int_0^n c(j)^\rho dj.$$

Here there is a continuum  $[0, n]$  of differentiated goods. (Hint: You need to be very careful in taking derivatives when solving the firms' profit maximization problems. In particular, the answers change drastically.)

c) Compare the gains in real income in parts a and b with each other and with those in question 1, part d.