1. Consider an economy with the following input-output matrix:

<table>
<thead>
<tr>
<th></th>
<th>Agr.</th>
<th>Mfg.</th>
<th>Con.</th>
<th>Inv.</th>
<th>Exp.</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Agriculture</td>
<td>2</td>
<td>3</td>
<td>8</td>
<td>2</td>
<td>5</td>
<td>20</td>
</tr>
<tr>
<td>Manufacturing</td>
<td>6</td>
<td>6</td>
<td>10</td>
<td>4</td>
<td>4</td>
<td>30</td>
</tr>
<tr>
<td>Imports</td>
<td>4</td>
<td>5</td>
<td></td>
<td></td>
<td></td>
<td>9</td>
</tr>
<tr>
<td>Tariff Revenue</td>
<td>2</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td>3</td>
</tr>
<tr>
<td>Labor Compensation</td>
<td>3</td>
<td>10</td>
<td></td>
<td></td>
<td></td>
<td>13</td>
</tr>
<tr>
<td>Returns to Capital</td>
<td>3</td>
<td>5</td>
<td></td>
<td></td>
<td></td>
<td>8</td>
</tr>
<tr>
<td>Total</td>
<td>20</td>
<td>30</td>
<td>18</td>
<td>6</td>
<td>9</td>
<td></td>
</tr>
</tbody>
</table>

a) What are the national income and product accounts for this economy?

b) Suppose that consumers and producers regard domestic goods and imports of goods as imperfect substitutes and that the Armington aggregators are Cobb-Douglas:

\[ y_{j} = \gamma_j y_{j, \text{d}}^{\delta_j} y_{j, \text{f}}^{1-\delta_j}, \quad j = \text{agr, man}. \]

Calibrate these Armington aggregators. Calibrate the tariff rates \( \tau_{\text{agr}} \), \( \tau_{\text{man}} \).

c) Suppose that all tariff revenues are transferred in lump-sum fashion to a representative consumer. Suppose that this consumer’s utility function is Cobb-Douglas:

\[ \theta_{\text{agr}} \log c_{\text{agr}} + \theta_{\text{man}} \log c_{\text{man}} + \theta_{\text{inv}} \log c_{\text{inv}}. \]

Calibrate the consumer’s utility function and endowments \( \bar{c}, \bar{k} \).

d) Suppose that net domestic production of each good is governed by a nested production function that produces value added by combining labor and capital using a Cobb-Douglas function and combines intermediate inputs of the other good and value-added in fixed proportions.

\[ y_{j,d} = \min \left[ x_{\text{agr}, j} / a_{\text{agr}, j}, x_{\text{man}, j} / a_{\text{man}, j}, \beta_j k_j^{\alpha_j} \ell_j^{1-\alpha_j} \right], \quad j = \text{agr, man}. \]
Calibrate the two production functions.

e) Suppose that there is a production function that produces the investment good using agriculture and manufactured goods in fixed proportions:

\[ y_{inv} = \min \left( \frac{x_{agr,inv}}{a_{agr,inv}}, \frac{x_{man,inv}}{a_{man,inv}} \right). \]

Calibrate this production function.

f) Suppose that the representative consumer in the rest of the world has income 100 and a Cobb-Douglas utility function.

\[ \theta_{agr,f} \log x_{agr,f} + \theta_{man,f} \log x_{man,f} + \theta_{j,f} \log x_{j,f}. \]

Suppose too the tariffs on imports in the rest of the world are \( \tau_{agr,f} = 0.10 \) and \( \tau_{man,f} = 0.05 \). Calibrate this utility function.

g) Suppose that the Armington elasticity of substitution between domestic goods and foreign goods is 5 in the Armington aggregators in part b,

\[ y_j = \frac{1}{\rho_j} \left( \delta_j y_{j,dl} + (1 - \delta_j) y_{j,f} \right)^{1/\rho_j}, \quad j = agr, \ man. \]

and 10 in the foreign utility function in part f,

\[ \left( \theta_{agr,f} x_{agr,f}^{\rho_f} + \theta_{man,f} x_{man,f}^{\rho_f} + \theta_{j,f} x_{j,f}^{\rho_f} - 1 \right) / \rho_x. \]

That is, \( \rho_m = 0.8 \) and \( \rho_x = 0.9 \). Recalibrate these functions.

2. Download data on bilateral trade by sector at the 4 digit SITC level from the OECD web site, http://oberon.sourceoecd.org. Follow the methodology in Kehoe and Ruhl, “How Important is the New Goods Margin in International Trade?” to create a set of least traded goods and carry out one of the two following exercises:

a) Consider trade between two countries over time. Construct diagrams with fractions of trade at the end of the period by deciles of sets of goods at the beginning of the period. Graph of the fraction of trade accounted for by the least traded decile over time. Do imports and exports separately.

b) Consider exports of one country to a number of trading partners during one year. Compare the sets of least traded goods. Do you see any patterns?
3. Find data to calculate the bilateral real exchange rate between two countries who have a bilateral trade relation that is important to at least one of the countries. Find data on the prices of traded goods in these two countries. Calculate a decomposition of the bilateral real exchange rate of the form

\[ \text{rer}_t = \text{rer}^T_t + \text{rer}^N_t, \]

where \( \text{rer}_t \) is the natural logarithm of the bilateral real exchange rate and \( \text{rer}^T_t \) is the logarithm of the bilateral real exchange rate for traded goods. Calculate the correlation between \( \text{rer}_t \) and \( \text{rer}^N_t \) in levels, in 1 year differences, and in 4 year differences. Calculate ratio of the standard deviations of \( \text{rer}_t \) and \( \text{rer}^N_t \) in levels, in 1 year differences, and in 4 year differences. Calculate a variance decomposition of \( \text{rer}_t \) in terms of \( \text{rer}^T_t \) and \( \text{rer}^N_t \) in levels, in 1 year differences, and in 4 year differences.