Consider a small open economy whose government borrows from international bankers. In every period, the value of output is
\[ y(z) = Z^{1-z} \bar{y} \]
where \( 1 > Z > 0 \) is a constant and \( z = 0 \) if the government defaults in that period or has defaulted in the past and \( \bar{y} \) is a constant. The government’s tax revenue is \( \theta y(z) \) where the tax rate \( 1 > \theta > 0 \) is constant. The consumers in the economy consume
\[ c = (1 - \theta) y(z) \].
The government is benevolent and makes choices to maximize the expected discounted value of
\[ u(c, g) = \log c + \gamma \log g \]
where \( \gamma > 0 \) and \( 1 > \beta > 0 \) is the discount factor. At the beginning of every period, the state of the economy is \( s = (B, z_{-1}, \zeta) \) where \( B \) is the level of government debt; \( z_{-1} = 0 \) if the government has defaulted in the past, and \( z_{-1} = 1 \) if not, and \( \zeta \sim U[0, 1] \) is the realization of a sunspot variable. The government first offers \( B' \) to international bankers. The intentional bankers have the same discount factor \( \beta \) as the government. They are also risk neutral and have deep pockets. These international bankers buy the bonds at a competitive auction that determines a price for \( B' \), \( q(B', s) \). The government finally chooses to default or not, which determines private consumption \( c \). Government spending \( g \) is determined by the government’s budget constraint
\[ g + zB = \theta y(z) + q(B', s)B'. \]
If the government defaults, setting \( z = 0 \), then assume that \( z_{-1} = 0 \) implies \( z = 0 \) thereafter; that is, the economy suffers from the default penalty \( 1 - Z \) forever. Furthermore, \( z_{-1} = 0 \) implies \( q(B', s) = 0 \); that is, the government is permanently excluded form credit markets.

a) Define a recursive equilibrium.

b) Assume that the bankers expect the government to default if \( \zeta > 1 - \pi \) and if such an expectation would be self-fulfilling, where \( 1 > \pi \geq 0 \) is an arbitrary constant. Find a level of debt \( \bar{b} \) such that, if \( B \leq \bar{b} \), no default occurs in equilibrium, but that, if \( B > \bar{b} \), default occurs in equilibrium.

c) Suppose that \( B_0 > \bar{b} \), and the government chooses to run down its debt to \( B_T \leq \bar{b} \) in \( T \) periods. Prove that it cannot be optimal to set \( B_T < \bar{b} \). Prove that it is optimal for the
government to set $g_t$ constant as long as $B_t > \bar{b}$ and no crisis occurs. Find expressions for $g_t$ and $B_t$ that depend on $B_0$ and $T$. Find an expression for the expected discounted value of the utility of running down the debt that starts at $B_0$ to $\bar{b}$ in $T$ periods. Find the limit of these expressions when $T = \infty$.

d) Using the answers to part c, write down a formula that determines a value of debt $\bar{B}(\pi)$ such that the government would choose to default if $B > \bar{B}(\pi)$ even if international bankers do not expect a default.

e) Using the answers to parts a–d, construct a recursive equilibrium.

f) Use this model to interpret events of the Mexican financial crisis of December 1994 through January 1995.

g) Assume that $Z = 0.9$, $\bar{y} = 100$, $\theta = 0.4$, $\gamma = 0.5$, $\beta = 0.95$, and $\pi = 0.05$. Calculate $\bar{b}$. Calculate the expected discounted value of the utility of running down the debt that starts at $B_0$ to $\bar{b}$ in $T$ periods for $T = 1, 2, 3, 4, 5, 6, 7$. Calculate $\bar{B}(0.05)$. Graph a policy function for government debt $B'(B)$. Graph a policy function for government spending $g(B)$. 