ECON 8401 **T. J. KEHOE**
INTERNATIONAL TRADE AND PAYMENTS THEORY **FALL 2022**

PROBLEM SET #5

Consider a small open economy whose government borrows from international bankers. In every period, the value of output is

$$y(z) = Z^{1-z}\bar{y}$$

where $1 > Z > 0$ is a constant and $z = 0$ if the government defaults in that period or has defaulted in the past and $\bar{y}$ is a constant. The government's tax revenue is $\theta y(z)$ where the tax rate $1 > \theta > 0$ is constant. The consumers in the economy consume $c = (1-\theta)y(z)$. The government is benevolent and makes choices to maximize the expected discounted value of

$$u(c, g) = \log c + \gamma \log g$$

where $\gamma > 0$ and $1 > \beta > 0$ is the discount factor. At the beginning of every period, the state of the economy is $s = (B, z_{-1}, \zeta)$ where B is the level of government debt; $z_{-1} = 0$ if the government has defaulted in the past, and $z_{-1} = 1$ if not, and $\zeta \sim U[0,1]$ is the realization of a sunspot variable. The government first offers B' to international bankers. The intentional bankers have the same discount factor β as the government. They are also risk neutral and have deep pockets. These international bankers buy the bonds at a competitive auction that determines a price for B', $q(B', s)$. The government finally chooses to default or not, which determines private consumption c. Government spending g is determined by the government's budget constraint

$$g + zB = \theta y(z) + q(B', s)B'.$$

If the government defaults, setting $z = 0$, then assume that $z_{-1} = 0$ implies $z = 0$ thereafter; that is, the economy suffers from the default penalty $1 - Z$ forever. Furthermore, $z_{-1} = 0$ implies $q(B', s) = 0$; that is, the government is permanently excluded form credit markets.

a) Define a recursive equilibrium.

b) Assume that the bankers expect the government to default if $\zeta > 1 - \pi$ and if such an expectation would be self-fulfilling, where $1 > \pi \geq 0$ is an arbitrary constant. Find a level of debt $\bar{b}$ such that, if $B \leq \bar{b}$, no default occurs in equilibrium, but that, if $B > \bar{b}$, default occurs in equilibrium.

c) Suppose that $B_0 > \bar{b}$, and the government chooses to run down its debt to $B_T \leq \bar{b}$ in T periods. Prove that it cannot be optimal to set $B_T < \bar{b}$. Prove that it is optimal for the government to set g_t constant as long as $B_t > \bar{b}$ and no crisis occurs. Find expressions for g_t and B_t that depend on B_0 and T. Find an expression for the expected discounted value of the utility of running down the debt that starts at B_0 to $\bar{b}$ in T periods. Find the limit of these expressions when $T = \infty$.

d) Using the answers to part c, write down a formula that determines a value of debt $\bar{B}(\pi)$ such that the government would choose to default if $B > \bar{B}(\pi)$ even if international bankers do not expect a default.

e) Using the answers to parts a–d, construct a recursive equilibrium.

f) Use this model to interpret events of the Mexican financial crisis of December 1994 through January 1995.

g) Assume that $Z = 0.9$, $\bar{y} = 100$, $\theta = 0.4$, $\gamma = 0.5$, $\beta = 0.95$, and $\pi = 0.05$. Calculate $\bar{b}$. Calculate the expected discounted value of the utility of running down the debt that starts at B_0 to $\bar{b}$ in T periods for $T = 1, 2, 3, 4, 5, 6, 7$. Calculate $\bar{B}(0.05)$. Graph a policy function for government debt $B'(B)$. Graph a policy function for government spending $g(B)$.