

# Topics in International Trade

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## **Outline:**

1. Standard theory (hybrid Heckscher-Ohlin/New Trade Theory) does not well when matched with the data on the growth and composition of trade.
2. Applied general equilibrium models that put the standard theory to work do not well in predicting the impact of trade liberalization experiences like NAFTA.
3. Much of the growth of trade after a trade liberalization experience is growth on the extensive margin. Models need to allow for corner solutions or fixed costs.
4. Fixed costs seem better than Ricardian corner solutions for reconciling time series data on real exchange rate fluctuations with data on trade growth after liberalization experiences.

5. Models of trade with heterogeneous firms typically impose fixed costs on firms that decide to export. The focus is on the decision to export. The theory and the data indicate that there is a lot of room for focusing on the decision to import.
6. Models with uniform fixed cost across firms with heterogeneous productivity have implications that are sharply at odds with micro data. A model with increasing costs of accessing a fraction of a market has many of features of models with fixed costs without these undesirable properties.
7. Growth theory needs to be reconsidered in the light of trade theory. In particular, a growth model that includes trade can have the opposite convergence properties from a model of closed economies.

8. Favorable changes in the terms of trade and/or reductions tariffs make it easier to import intermediate goods. Although this is often interpreted as an increase in productivity, it does not show up as such in productivity measures that use real GDP as a measure of output.
9. In models with heterogeneous firms (for example, Melitz, Chaney), trade liberalization can cause resources to shift from less productive firms to more productive firms. Although this is often interpreted as an increase in productivity, it does not show up as such in productivity measures that use real GDP as a measure of output.

# **TRADE THEORY AND TRADE FACTS**

- Some recent trade facts
- A “New Trade Theory” model
- Accounting for the facts
- Intermediate goods?
- Policy?

**How important is the quantitative failure of the New Trade Theory?**

**Where should trade theory and applications go from here?**

## **SOME RECENT TRADE FACTS**

- **The ratio of trade to product has increased.**

World trade/world GDP increased by 59.3 percent 1961-1990.

OECD-OECD trade/OECD GDP increased by 111.5 percent 1961-1990.

- **Trade has become more concentrated among industrialized countries**

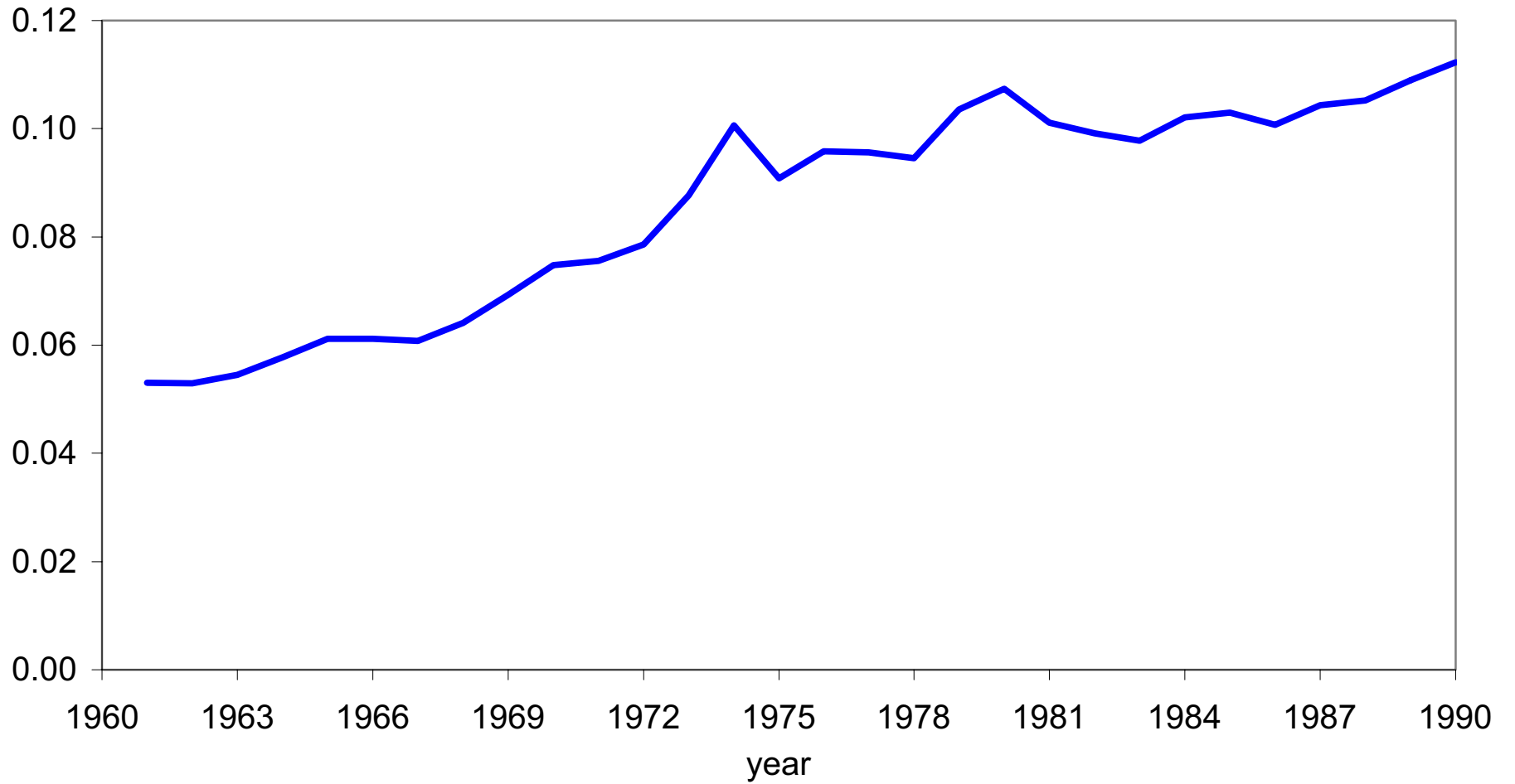
OECD-OECD trade/OECD-RW trade increased by 87.1 percent 1961-1990.

- **Trade among industrialized countries is mostly intraindustry trade**

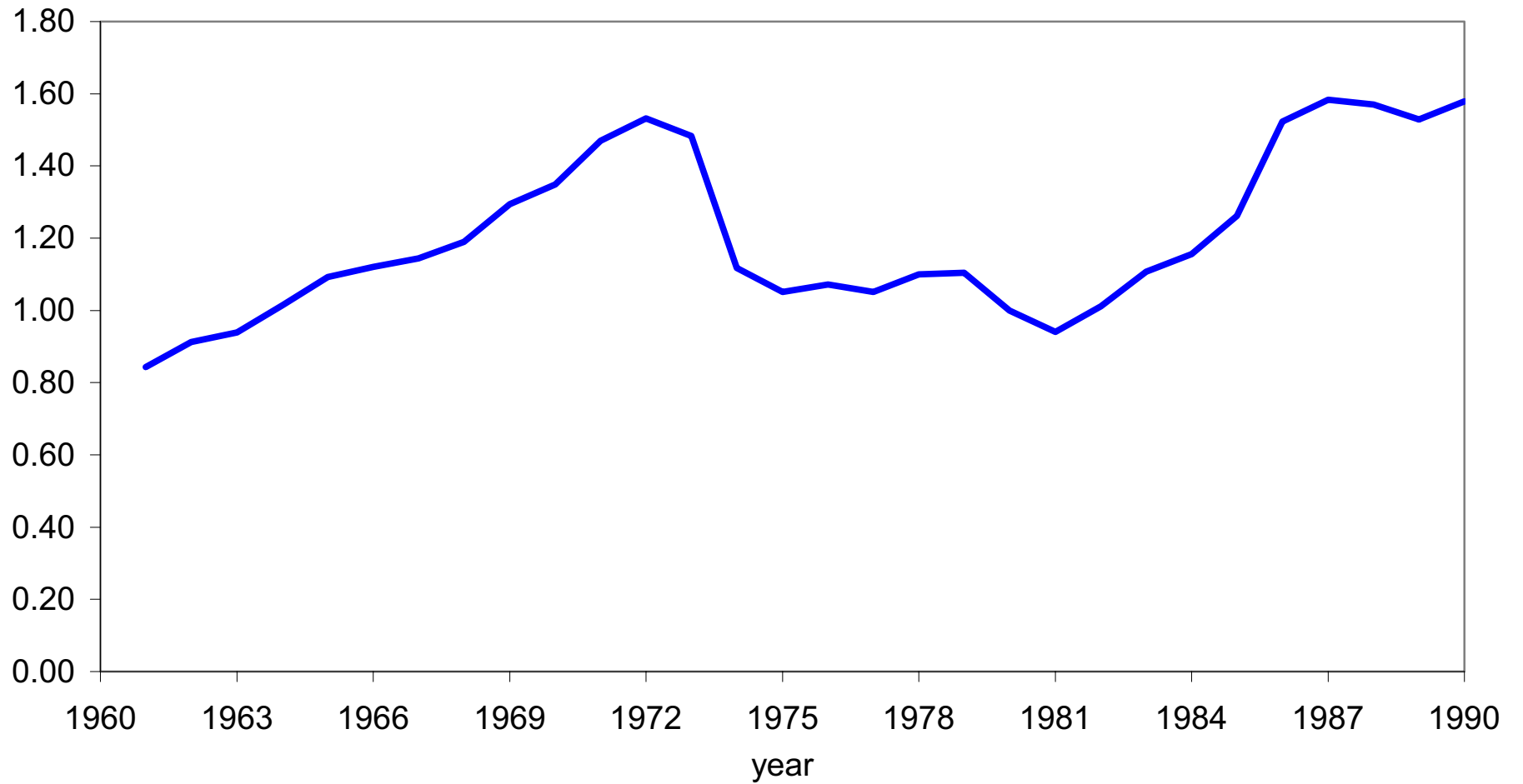
Grubel-Lloyd index for OECD-OECD trade in 1990 is 68.4.

Grubel-Lloyd index for OECD-RW trade in 1990 is 38.1.

# OECD-OECD Trade / OECD GDP



## OECD-OECD Trade / OECD-RW Trade





**Helpman and Krugman (1985):**

“These....empirical weaknesses of conventional trade theory...become understandable once economies of scale and imperfect competition are introduced into our analysis.”

**Markusen, Melvin, Kaempfer, and Maskus (1995):**

“Thus, nonhomogeneous demand leads to a decrease in North-South trade and to an increase in intraindustry trade among the northern industrialized countries. These are the stylized facts that were to be explained.”

**Goal: To measure how much of the increase in the ratio of trade to output in the OECD and of the concentration of world trade among OECD countries can be accounted for by the “New Trade Theory.”**

# **PUNCHLINE**

**In a calibrated general equilibrium model,  
the New Trade Theory cannot account for the  
increase in the ratio of trade to output in the  
OECD.**

## Back-of-the-envelope calculations:

Suppose that the world consists of the OECD and the only trade is manufactures.

With Dixit-Stiglitz preferences, country  $j$  exports all of its production of manufactures  $Y_m^j$  except for the fraction  $s^j = Y^j / Y^{oe}$  that it retains for domestic consumption.

World imports:

$$M = \sum_{j=1}^n (1 - s^j) Y_m^j.$$

World trade/GDP:

$$\frac{M}{Y^{oe}} = \frac{M}{Y_m^{oe}} \frac{Y_m^{oe}}{Y^{oe}} = \left( 1 - \sum_{j=1}^n (s^j)^2 \right) \frac{Y_m^{oe}}{Y^{oe}}.$$

World trade/GDP:

$$\frac{M}{Y^{oe}} = \frac{M}{Y_m^{oe}} \frac{Y_m^{oe}}{Y^{oe}} = \left(1 - \sum_{j=1}^n (s^j)^2\right) \frac{Y_m^{oe}}{Y^{oe}}.$$

$\left(1 - \sum_{j=1}^n (s^j)^2\right)$  goes from 0.663 in 1961 to 0.827 in 1990.

$Y_m^{oe} / Y^{oe}$  goes from 0.295 in 1961 to 0.222 in 1990.

$$0.663 \times 0.295 = 0.196 \approx 0.184 = 0.827 \times 0.222.$$

**Effects cancel!**

# A “NEW TRADE THEORY” MODEL

Environment:

- Static: endowments of factors are exogenous
- 2 regions: OECD and rest of world
- 2 traded goods: homogeneous — primaries (CRS) and differentiated — manufactures (IRS)
- 1 nontraded good — services (CRS)
- 2 factors: (effective) labor and capital
- Identical technologies and preferences (love for variety) across regions
- Primaries are inferior to manufactures

**We only consider merchandise trade in both the data and in the model.**

# Key Features of the Model

Consumers' problem:

$$\max \frac{\beta_p (c_p^j + \gamma_p)^\eta + \beta_m \left( \int_{D^w} c_m^j(z)^\rho dz_p \right)^{\eta/\rho} + \beta_s (c_s^j + \gamma_s)^\eta - 1}{\eta}$$

$$\text{s.t.} \quad q_p c_p^j + \int_{D^w} q_m(z) c_m^j(z) dz_p + q_s^j c_s^j \leq r^j k^j + w^j h^j.$$

## Firms' problems

Primaries and Services: Standard CRS problems.

$$Y_p^j = \theta_p (K_p^j)^{\alpha_p} (H_p^j)^{1-\alpha_p}$$

$$Y_s^j = \theta_s (K_s^j)^{\alpha_s} (H_s^j)^{1-\alpha_s}$$

Manufactures: Standard (Dixit-Stiglitz) monopolistically competitive problem:

- Fixed cost.

$$Y_m(z) = \max \left[ \theta_m K_m(z)^{\alpha_m} H_m(z)^{1-\alpha_m} - F, 0 \right]$$

- Firm  $z$  sets its price  $q_m(z)$  to max profits given all of the other prices.

$$Y_m(z) = \sum_{j=1}^n C_m^j(z) + C_m^{rw}(z).$$

$$C_m^j(z) = \frac{\beta_m^{\frac{1}{1-\eta}} (r^j K^j + w^j H^j + q_p \gamma_p N^j + q_s^j \gamma_s N^j)}{q_m(z)^{\frac{1}{1-\rho}} \left[ \int_{D^w} q_m(z')^{\frac{-\rho}{1-\rho}} dz' \right]^{\frac{\rho-\eta}{\rho(1-\eta)}} \Delta}$$

$$\Delta = \beta_p^{\frac{1}{1-\eta}} q_p^{\frac{-\eta}{1-\eta}} + \beta_m^{\frac{1}{1-\eta}} \left[ \left( \int_{D^w} q_m(z')^{\frac{-\rho}{1-\rho}} dz' \right)^{\frac{-(1-\rho)}{\rho}} \right]^{\frac{-\eta}{1-\eta}} + \beta_s^{\frac{1}{1-\eta}} q_s^{\frac{-\eta}{1-\eta}}$$

- Every firm is uniquely associated with only one variety (symmetry).
- Free entry.
- $D^w = [0, d^w]$  with  $d^w$  finite and endogenously determined.



## Volume of Trade

Let  $s^j$  be the share of country  $j$ ,  $j = 1, \dots, n, rw$ , in the world production of manufactures,

$$s^j = \int_{D^j} Y_m(z) dz / \int_{D^w} Y_m(z) dz = Y_m^j / Y_m^w.$$

The imports by country  $j$  from the OECD are

$$M_{oe}^j = (1 - s^{rw} - s^j) C_m^j$$
$$M_{oe}^{rw} = (1 - s^{rw}) C_m^{rw}.$$

Total imports in the OECD from the other OECD countries are

$$M_{oe}^{oe} = \sum_{j=1}^n M_{oe}^j (1 - s^{rw} - \sum_{j=1}^n (s^j)^2 / (1 - s^{rw})) C_m^{oe}.$$

## OECD in 1990

Country	Share of GDP %	Country	Share of GDP %
Australia	1.79	Japan	18.04
Austria	0.97	Netherlands	1.72
Belgium-Lux	1.26	New Zealand	0.26
Canada	3.45	Norway	0.70
Denmark	0.78	Portugal	0.41
Finland	0.81	Spain	3.00
France	7.26	Sweden	1.40
Germany	9.96	Switzerland	0.17
Greece	0.50	Turkey	0.91
Iceland	0.04	United Kingdom	5.92
Ireland	0.28	United States	33.72
Italy	6.64		

# ACCOUNTING FOR THE FACTS

Compare the changes that the model predicts for 1961-1990 with what actually took place.

Focus on key variables:

OECD-OECD Trade/OECD GDP

OECD-OECD Trade/OECD-RW Trade

OECD Manufacturing GDP/OECD GDP

Calibrate to 1990 data.

Backcast to 1961 by imposing changes in parameters:

relative sizes of countries in the OECD

populations

sectoral productivities

endowments

# ACCOUNTING FOR THE FACTS

Benchmark 1990 OECD Data Set  
(Billion U.S. dollars)

	<b>Primaries</b>	<b>Manufactures</b>	<b>Services</b>	<b>Total</b>
$H_i^{oe}$	228	2,884	8,644	11,756
$K_i^{oe}$	441	775	3,497	4,713
$Y_i^{oe}$	669	3,659	12,141	16,469
$C_i^{oe}$	862	3,466	12,141	16,469
$Y_i^{oe} - C_i^{oe}$	-193	193	0	0

# ACCOUNTING FOR THE FACTS

Benchmark 1990 Rest of the World Data Set  
(Billion U.S. dollars)

	<b>Primaries</b>	<b>Manufactures</b>	<b>Services</b>	<b>Total</b>
$Y_i^{rw}$	1,223	1,159	3,447	5,829
$C_i^{rw}$	1,030	1,352	3,447	5,829
$Y_i^{rw} - C_i^{rw}$	193	-193	0	0

# ACCOUNTING FOR THE FACTS

- $N^{oe} = 854$ ,  $N^{rw} = 4,428$ .
- $\sum_{i=p,m,s} Y_i^{rw} = \sum_{i=p,m,s} C_i^{rw} = 5,829$ .
- Set  $q_p = q_m(z) = q_s = w = r = 1$  (quantities are 1990 values).
- $\rho = 1/1.2$  (Morrison 1990, Martins, Scarpetta, and Pilat 1996).
- Normalize  $d^w = 100$ .
- Calibrate  $H^{rw}$ ,  $K^{rw}$  so that benchmark data set is an equilibrium.
- Alternative calibrations of utility parameters  $\gamma_p$ ,  $\gamma_s$ , and  $\eta$ .

## OECD in 1961

Country	Share of GDP %	Country	Share of GDP %
Austria	0.75	Netherlands	1.37
Belgium-Lux	1.25	Norway	0.60
Canada	4.22	Portugal	0.32
Denmark	0.70	Spain	1.38
France	6.99	Sweden	1.62
Germany	9.71	Switzerland	1.07
Greece	0.50	Turkey	0.83
Iceland	0.03	United Kingdom	8.08
Ireland	0.21	United States	55.74
Italy	4.64		

# Numerical Experiments

Calculate equilibrium in 1961:

$$\theta_{p,1961} = \theta_{p,1990}$$

$$\theta_{m,1961} = \theta_{m,1990} / 1.014^{29}, F_{1961} = F_{1990} / 1.014^{29}$$

$$\theta_{s,1961} = \theta_{s,1990} / 1.005^{29} \text{ (Echevarria 1997)}$$

$$N^{oe} = 536, N^{rw} = 2,545$$



# Numerical Experiments

Choose  $H_{1961}^{oe}$ ,  $K_{1961}^{oe}$ ,  $H_{1961}^{rw}$ ,  $K_{1961}^{rw}$  so that

$$\frac{\sum_{i=p,m,s} Y_{i,1990}^{oe} / N_{1990}^{oe}}{\sum_{i=p,m,s} Y_{i,1961}^{oe} / N_{1961}^{oe}} = 2.400$$

$$\frac{\sum_{i=p,m,s} Y_{i,1990}^{rw} / N_{1990}^{rw}}{\sum_{i=p,m,s} Y_{i,1961}^{rw} / N_{1961}^{rw}} = 2.055$$

$$\frac{K_{1961}^{oe}}{H_{1961}^{oe}} = \frac{K_{1990}^{oe}}{H_{1990}^{oe}}$$

$$\frac{q_{p,1961} (Y_{p,1961}^{rw} - C_{p,1961}^{rw})}{\sum_{i=p,m,s} q_{i,1961} Y_{i,1961}^{rw}} = 0.050$$

## **How Can the Model Work in Matching the Facts?**

- The ratio of trade to product has increased:

The size distribution of countries has become more equal (Helpman-Krugman).

- Trade has become more concentrated among industrialized countries:

OECD countries have comparative advantage in manufactures, while the RW has comparative advantage in primaries.

Because they are inferior to manufactures, primaries become less important in trade as the world becomes richer (Markusen).

## **How Can the Model Work in Matching the Facts?**

- Trade among industrialized countries is largely intraindustry trade:

OECD countries export manufactures. Because of taste for variety, every country consumes some manufactures from every other country (Dixit-Stiglitz).

- The different total factor productivity growth rates across sectors imply that the price of manufactures relative to primaries and services has fallen sharply between 1961 and 1990. If price elasticities of demand are not equal to one, a lot can happen.

# Experiment 1

$$\gamma_p = \gamma_p = \eta = 0$$

	1961	1990	Change
Data			
OECD-OECD Trade/OECD GDP	0.053	0.112	111.5%
OECD-OECD Trade/OECD-RW Trade	0.844	1.579	87.1%
OECD Manf GDP/OECD GDP	0.295	0.222	-24.6%
1. $\gamma_p = 0, \gamma_s = 0, \eta = 0$			
OECD-OECD Trade/OECD GDP	0.108	0.136	25.8%
OECD-OECD Trade/OECD-RW Trade	0.893	1.169	30.9%
OECD Manf GDP/OECD GDP	0.223	0.222	-0.4%

## Experiment 2

$\gamma_p = -169.5$ ,  $\gamma_s = 314.7$  to match consumption in RW in 1990,  
 $\eta = 0$

	1961	1990	Change
Data			
OECD-OECD Trade/OECD GDP	0.053	0.112	111.5%
OECD-OECD Trade/OECD-RW Trade	0.844	1.579	87.1%
OECD Manf GDP/OECD GDP	0.295	0.222	-24.6%
2. $\gamma_p = -169.5$ , $\gamma_s = 314.7$ , $\eta = 0$			
OECD-OECD Trade/OECD GDP	0.103	0.132	28.1%
OECD-OECD Trade/OECD-RW Trade	0.739	1.060	43.6%
OECD Manf GDP/OECD GDP	0.225	0.222	-1.4%

## Experiment 3

$$\gamma_p = -169.5, \gamma_s = 314.7,$$

$\eta = 0.559$  to match growth in OECD-OECD Trade/OECD GDP

	1961	1990	Change
Data			
OECD-OECD Trade/OECD GDP	0.053	0.112	111.5%
OECD-OECD Trade/OECD-RW Trade	0.844	1.579	87.1%
OECD Manf GDP/OECD GDP	0.295	0.222	-24.6%
3. $\gamma_p = -169.5, \gamma_s = 314.7, \eta = 0.559$			
OECD-OECD Trade/OECD GDP	0.063	0.132	111.5%
OECD-OECD Trade/OECD-RW Trade	0.738	1.060	43.7 %
OECD Manf GDP/OECD GDP	0.137	0.222	62.7%

## Experiments 4 and 5

$\gamma_p = -169.5$ ,  $\gamma_s = 314.7$ , reasonable values of  $\eta$  ( $0.5 \geq 1/(1-\eta) \geq 0.1$ )

	1961	1990	Change
Data			
OECD-OECD Trade/OECD GDP	0.053	0.112	111.5%
OECD-OECD Trade/OECD-RW Trade	0.844	1.579	87.1%
OECD Manf GDP/OECD GDP	0.295	0.222	-24.6%
4. $\gamma_p = -169.5$ , $\gamma_s = 314.7$ , $\eta = -1$			
OECD-OECD Trade/OECD GDP	0.118	0.132	11.7%
OECD-OECD Trade/OECD-RW Trade	0.739	1.060	43.5%
OECD Manf GDP/OECD GDP	0.259	0.222	-14.1%
5. $\gamma_p = -169.5$ , $\gamma_s = 314.7$ , $\eta = -9$			
OECD-OECD Trade/OECD GDP	0.118	0.132	1.6%
OECD-OECD Trade/OECD-RW Trade	0.739	1.060	43.5%
OECD Manf GDP/OECD GDP	0.284	0.222	-21.8%

## **Sensitivity Analysis: Alternative Calibration Methodologies**

- Alternative specifications of nonhomogeneity
- Gross imports calibration
- Alternative RW endowment calibration
- Alternative RW growth calibration
- Intermediate goods



## INTERMEDIATE GOODS?

$$Y_p^j = \min \left[ \frac{X_{pp}^j}{a_{pp}}, \frac{\int_{D^w} X_{mp}^j(z) dz}{a_{mp}}, \frac{X_{sp}^j}{a_{sp}}, \theta_p (K_p^j)^{\alpha_p} (H_p^j)^{1-\alpha_p} \right]$$

$$Y_m(z) = \min \left[ \frac{X_{pm}^j(z)}{a_{pm}}, \frac{\int_{D^w} X_{mm}^j(z, z') dz'}{a_{mm}}, \frac{X_{sm}^j(z)}{a_{sm}}, \theta_m (K_m(z))^{\alpha_m} (H_m(z))^{1-\alpha_m} - F \right]$$

$$Y_s^j = \min \left[ \frac{X_{ps}^j}{a_{ps}}, \frac{\int_{D^w} X_{ms}^j(z) dz}{a_{ms}}, \frac{X_{ss}^j}{a_{ss}}, \theta_s (K_s^j)^{\alpha_s} (H_s^j)^{1-\alpha_s} \right]$$

## Results for Model with Intermediate Goods

	1961	1990	Change
Data			
OECD-OECD Trade/OECD GDP	0.053	0.112	111.5%
OECD-OECD Trade/OECD-RW Trade	0.844	1.579	87.1%
OECD Manf GDP/OECD GDP	0.295	0.222	-24.6%
4. $\gamma_p = -307.8, \gamma_s = 262.2, \eta = -1$			
OECD-OECD Trade/OECD GDP	0.323	0.370	14.5%
OECD-OECD Trade/OECD-RW Trade	0.994	1.305	31.3%
OECD Manf GDP/OECD GDP	0.263	0.222	-15.6%
5. $\gamma_p = -307.8, \gamma_s = 262.2, \eta = -9$			
OECD-OECD Trade/OECD GDP	0.337	0.370	9.7%
OECD-OECD Trade/OECD-RW Trade	0.933	1.305	39.9%
OECD Manf GDP/OECD GDP	0.307	0.222	-27.5%

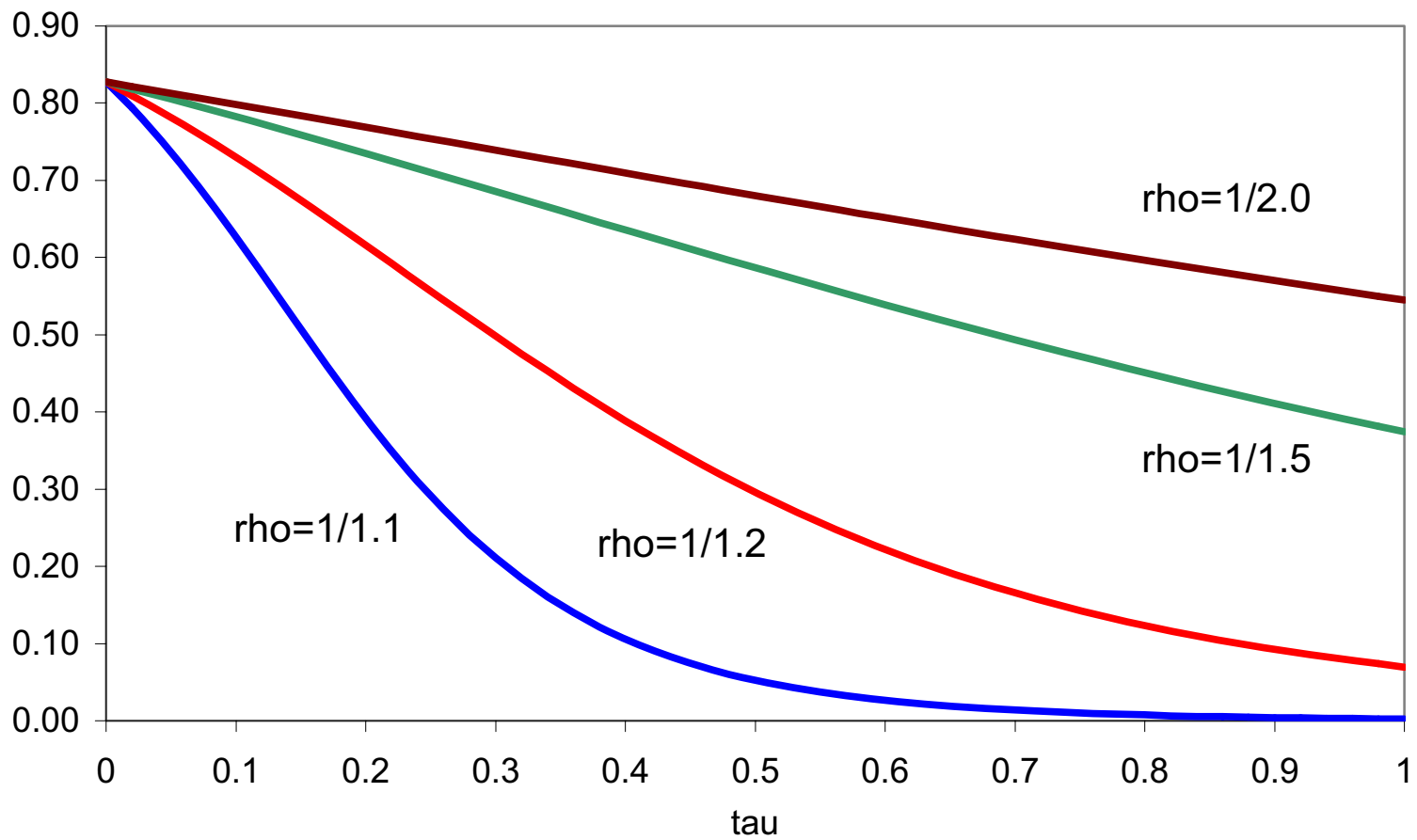
## POLICY?

In a version of our model with  $n$  OECD countries, a manufacturing sector, and a uniform ad valorem tariff  $\tau$ , the ratio of exports to income is given by

$$\frac{M}{Y} = \frac{(n-1)C_f}{Y} = \frac{n-1}{n-1 + (1+\tau)^{1/(1-\rho)}}$$

Fixing  $n$  to replicate the size distribution of national incomes in the OECD, and setting  $\rho = 1/1.2$ , a fall in  $\tau$  from 0.45 to 0.05 produces an increase in the ratio of trade to output as seen in the data.

# World Trade / World GDP



## **2. Applied general equilibrium models that put the standard theory to work do not well in predicting the impact of trade liberalization experiences like NAFTA.**

Applied general equilibrium models were the only analytical game in town when it came to analyzing the impact of NAFTA in 1992-1993.

Typical sort of model: Static applied general equilibrium model with large number of industries and imperfect competition (Dixit-Stiglitz or Eastman-Stykolt) and finite number of firms in some industries. In some numerical experiments, new capital is placed in Mexico owned by consumers in the rest of North America to account for capital flows.

Examples:

Brown-Deardorff-Stern model of Canada, Mexico, and the United States

Cox-Harris model of Canada

Sobarzo model of Mexico

T. J. Kehoe, “An Evaluation of the Performance of Applied General Equilibrium Models of the Impact of NAFTA,” in T. J. Kehoe, T. N. Srinivasan, and J. Whalley, editors, *Frontiers in Applied General Equilibrium Modeling: Essays in Honor of Herbert Scarf*, Cambridge University Press, 2005, 341-77.

### Research Agenda:

- Compare results of numerical experiments of models with data.
- Determine what shocks — besides NAFTA policies — were important.
- Construct a simple applied general equilibrium model and perform experiments with alternative specifications to determine what was wrong with the 1992-1993 models.

## **Applied GE Models Can Do a Good Job!**

**Spain:** Kehoe-Polo-Sancho (1992) evaluation of the performance of the Kehoe-Manresa-Noyola-Polo-Sancho-Serra MEGA model of the Spanish economy: A Shoven-Whalley type model with perfect competition, modified to allow government and trade deficits and unemployment (Kehoe-Serra). Spain's entry into the European Community in 1986 was accompanied by a fiscal reform that introduced a value-added tax (VAT) on consumption to replace a complex range of indirect taxes, including a turnover tax applied at every stage of the production process. What would happen to tax revenues? Trade reform was of secondary importance.

**Canada-U.S.:** Fox (1999) evaluation of the performance of the Brown-Stern (1989) model of the 1989 Canada-U.S. FTA.

**Other changes besides policy changes are important!**

## Changes in Consumer Prices in the Spanish Model (Percent)

sector	data 1985-1986	model policy only	model shocks only	model policy&shocks
food and nonalcoholic beverages	1.8	-2.3	4.0	1.7
tobacco and alcoholic beverages	3.9	2.5	3.1	5.8
clothing	2.1	5.6	0.9	6.6
housing	-3.3	-2.2	-2.7	-4.8
household articles	0.1	2.2	0.7	2.9
medical services	-0.7	-4.8	0.6	-4.2
transportation	-4.0	2.6	-8.8	-6.2
recreation	-1.4	-1.3	1.5	0.1
other services	2.9	1.1	1.7	2.8
weighted correlation with data		-0.08	0.87	0.94
variance decomposition of change		0.30	0.77	0.85
regression coefficient <i>a</i>		0.00	0.00	0.00
regression coefficient <i>b</i>		-0.08	0.54	0.67



# Measures of Accuracy of Model Results

1. Weighted correlation coefficient.
2. Variance decomposition of the (weighted) variance of the changes in the data:

$$\mathit{vardec}(y^{data}, y^{model}) = \frac{\mathit{var}(y^{model})}{\mathit{var}(y^{model}) + \mathit{var}(y^{data} - y^{model})}.$$

- 3, 4. Estimated coefficients  $a$  and  $b$  from the (weighted) regression

$$x_i^{data} = a + bx_i^{model} + e_i.$$

## Changes in Value of Gross Output/GDP in the Spanish Model (Percent)

sector	data 1985-1986	model policy only	model shocks only	model policy&shocks
agriculture	-0.4	-1.1	8.3	6.9
energy	-20.3	-3.5	-29.4	-32.0
basic industry	-9.0	1.6	-1.8	-0.1
machinery	3.7	3.8	1.0	5.0
automobile industry	1.1	3.9	4.7	8.6
food products	-1.8	-2.4	4.7	2.1
other manufacturing	0.5	-1.7	2.3	0.5
construction	5.7	8.5	1.4	10.3
commerce	6.6	-3.6	4.4	0.4
transportation	-18.4	-1.5	1.0	-0.7
services	8.7	-1.1	5.8	4.5
government services	7.6	3.4	0.9	4.3
weighted correlation with data		0.16	0.80	0.77
variance decomposition of change		0.11	0.73	0.71
regression coefficient <i>a</i>		-0.52	-0.52	-0.52
regression coefficient <i>b</i>		0.44	0.75	0.67

**Changes in Trade/GDP  
in the Spanish Model (Percent)**

	<b>data</b>	<b>model</b>	<b>model</b>	<b>model</b>
<b>direction of exports</b>	<b>1985-1986</b>	<b>policy only</b>	<b>shocks only</b>	<b>policy&amp;shocks</b>
<b>Spain to rest of E.C.</b>	<b>-6.7</b>	<b>-3.2</b>	<b>-4.9</b>	<b>-7.8</b>
<b>Spain to rest of world</b>	<b>-33.2</b>	<b>-3.6</b>	<b>-6.1</b>	<b>-9.3</b>
<b>rest of E.C. to Spain</b>	<b>14.7</b>	<b>4.4</b>	<b>-3.9</b>	<b>0.6</b>
<b>rest of world to Spain</b>	<b>-34.1</b>	<b>-1.8</b>	<b>-16.8</b>	<b>-17.7</b>
<b>weighted correlation with data</b>		<b>0.69</b>	<b>0.77</b>	<b>0.90</b>
<b>variance decomposition of change</b>		<b>0.02</b>	<b>0.17</b>	<b>0.24</b>
<b>regression coefficient <i>a</i></b>		<b>-12.46</b>	<b>2.06</b>	<b>5.68</b>
<b>regression coefficient <i>b</i></b>		<b>5.33</b>	<b>2.21</b>	<b>2.37</b>

## Changes in Composition of GDP in the Spanish Model (Percent of GDP)

variable	data 1985-1986	model policy only	model shocks only	model policy&shocks
wages and salaries	-0.53	-0.87	-0.02	-0.91
business income	-1.27	-1.63	0.45	-1.24
net indirect taxes and tariffs	1.80	2.50	-0.42	2.15
correlation with data		0.998	-0.94	0.99
variance decomposition of change		0.93	0.04	0.96
regression coefficient <i>a</i>		0.00	0.00	0.00
regression coefficient <i>b</i>		0.73	-3.45	0.85
private consumption	-0.81	-1.23	-0.51	-1.78
private investment	1.09	1.81	-0.58	1.32
government consumption	-0.02	-0.06	-0.38	-0.44
government investment	-0.06	-0.06	-0.07	-0.13
exports	-3.40	-0.42	-0.69	-1.07
-imports	3.20	-0.03	2.23	2.10
correlation with data		0.40	0.77	0.83
variance decomposition of change		0.20	0.35	0.58
regression coefficient <i>a</i>		0.00	0.00	0.00
regression coefficient <i>b</i>		0.87	1.49	1.24

## Public Finances in the Spanish Model (Percent of GDP)

variable	data 1985-1986	model policy only	model shocks only	model policy&shocks
indirect taxes and subsidies	2.38	3.32	-0.38	2.98
tariffs	-0.58	-0.82	-0.04	-0.83
social security payments	0.04	-0.19	-0.03	-0.22
direct taxes and transfers	-0.84	-0.66	0.93	0.26
government capital income	-0.13	-0.06	0.02	-0.04
correlation with data		0.99	-0.70	0.92
variance decomposition of change		0.93	0.08	0.86
regression coefficient $a$		-0.06	0.35	-0.17
regression coefficient $b$		0.74	-1.82	0.80

# **Models of NAFTA Did Not Do a Good Job!**

Ex-post evaluations of the performance of applied GE models are essential if policy makers are to have confidence in the results produced by this sort of model.

Just as importantly, they help make applied GE analysis a scientific discipline in which there are well-defined puzzles and clear successes and failures for alternative hypotheses.

For the past three decades, the tool of choice for analyzing the impact of trade policy has been the multisectoral applied GE model.

At a conference organized by the U.S. International Trade Commission, held in February 1992 at the request of the U.S. Congress, to which all economists studying the impact of NAFTA had been invited, 10 of the 12 studies presented used applied GE models.

This is still the sort of model used to analyze policies like the U.S.-Central America-Dominican Republic Free Trade Agreement and the U.S.-Korea Free Trade Agreement.

Typical sort of model: Static applied general equilibrium model with 20-40 industries, imperfect competition (Dixit-Stiglitz or Eastman-Stykolt) in some industries, perfect competition and Armington aggregators in others.

Comparison of predictions of model with changes that occurred over 1988–2006 (1988–2006 for Mexico-United States)

Brown-Deardorff-Stern model of Canada, Mexico, and the United States

Cox-Harris model of Canada

Sobarzo model of Mexico



Methodology for comparing results of models with data

Kehoe, Polo, and Sancho (1995), Kehoe (2005)

For exports by country  $i$  of industry  $j$  to country  $k$ ,  $x_{ij}^k$ ,

$$z_{ij}^k = 100 \left( \frac{x_{ijT_1}^k / y_{iT_1}}{x_{ijT_0}^k / y_{iT_0}} - 1 \right)$$

where  $y_{it}$  is the current price GDP in country  $i$  in year  $t$

To compare the predictions of the model with the changes in data, we calculate the weighted correlation coefficient and the coefficients  $a$  and  $b$  from the weighted regression

$$\min_{a,b} \sum_{j=1}^n \alpha_j \left( a + bz_j^{model} - z_j^{data} \right)^2$$

How to turn least traded products data into predictions:

Predict that least traded products will have their exports grow  $a + b$  percent faster than the exporting country's GDP and other products grow  $a$  percent faster.

For a particular industry, this prediction becomes

$$a(1 - s_i) + (a + b)s_i = a + bs_i,$$

where  $s_i$  is the fraction of the industry's exports made up of least traded products in the base year.

Notice that this method does not make use of the products with 0 recorded trade.

## Product-industry detail

Use concordance between the ISIC and the SITC based on those in the World Bank's Trade and Production Database (Nicita and Olarreaga, 2001, 2006)

Canadian exports of chemicals to the United States grew 113.5 percent while the BDS model predicted -3.1 percent.

88 complete 4-digit SITC categories — 80 least traded.

Parts of another 27 categories — 11 least traded.

Compared to Canadian GDP, the 22 percent of 1988 exports of chemicals that are least traded increase by 212 percent, while the other 78 percent increase by only 32 percent.

Compared to Canadian GDP

Exports of 5823 (Alkyds and Other Polyesters) increases by 1,285 percent

Exports of 5832 (Polypropylene) increases by 738 percent

Exports of 5121 (Acyclic Alcohols and their Halogenated and Derivatives) decreases by 37 percent.

Kehoe and Ruhl (2013) show that products that are traded very little or not at all account disproportionately for aggregate changes in bilateral trade following trade liberalization or rapid growth experiences, but not over the business cycle.

Hypothesis: Industries with more trade due to these little-traded and non-traded products should experience more growth following trade liberalization.

**Product:** A 5-digit SITC, rev. 2 code. There are 1,836 products.

**Industry:** A 3-digit ISIC code. There are 38 industries. (We are only interested in industries that produce goods in merchandise trade — agriculture, mining and extraction, and manufacturing.)

We map products into industries using concordance developed by Muendler (2009).

Notice that each industry, on average, consists of 48.3 products.

<b>ISIC code</b>	<b>industry name</b>
111	Agriculture and livestock production
113	Hunting, trapping and game propagation
121	Forestry
122	Logging
130	Fishing
210	Coal mining
220	Crude petroleum and natural gas production
230	Metal ore mining
290	Other mining
311–312	Food manufacturing
313	Beverage industries
314	Tobacco manufactures
321	Manufacture of textiles
322	Manufacture of wearing apparel, except footwear
323	Manufacture of leather and products of leather, leather substitutes and fur
324	Manufacture of footwear
331	Manufacture of wood and wood and cork products, except furniture
332	Manufacture of furniture and fixtures, except primarily of metal
341	Manufacture of paper and paper products

- 342 Printing, publishing and allied industries
- 351 Manufacture of industrial chemicals
- 352 Manufacture of other chemical products
- 353 Petroleum refineries
- 354 Manufacture of miscellaneous products of petroleum and coal
- 355 Manufacture of rubber products
- 356 Manufacture of plastic products not elsewhere classified
- 361 Manufacture of pottery, china and earthenware
- 362 Manufacture of glass and glass products
- 369 Manufacture of other non-metallic mineral products
- 371 Iron and steel basic industries
- 372 Non-ferrous metal basic industries
- 381 Manufacture of fabricated metal products
- 382 Manufacture of machinery except electrical
- 383 Manufacture of electrical machinery apparatus, appliances and supplies
- 384 Manufacture of transport equipment
- 385 Manufacture of professional and scientific equipment
- 390 Other manufacturing industries



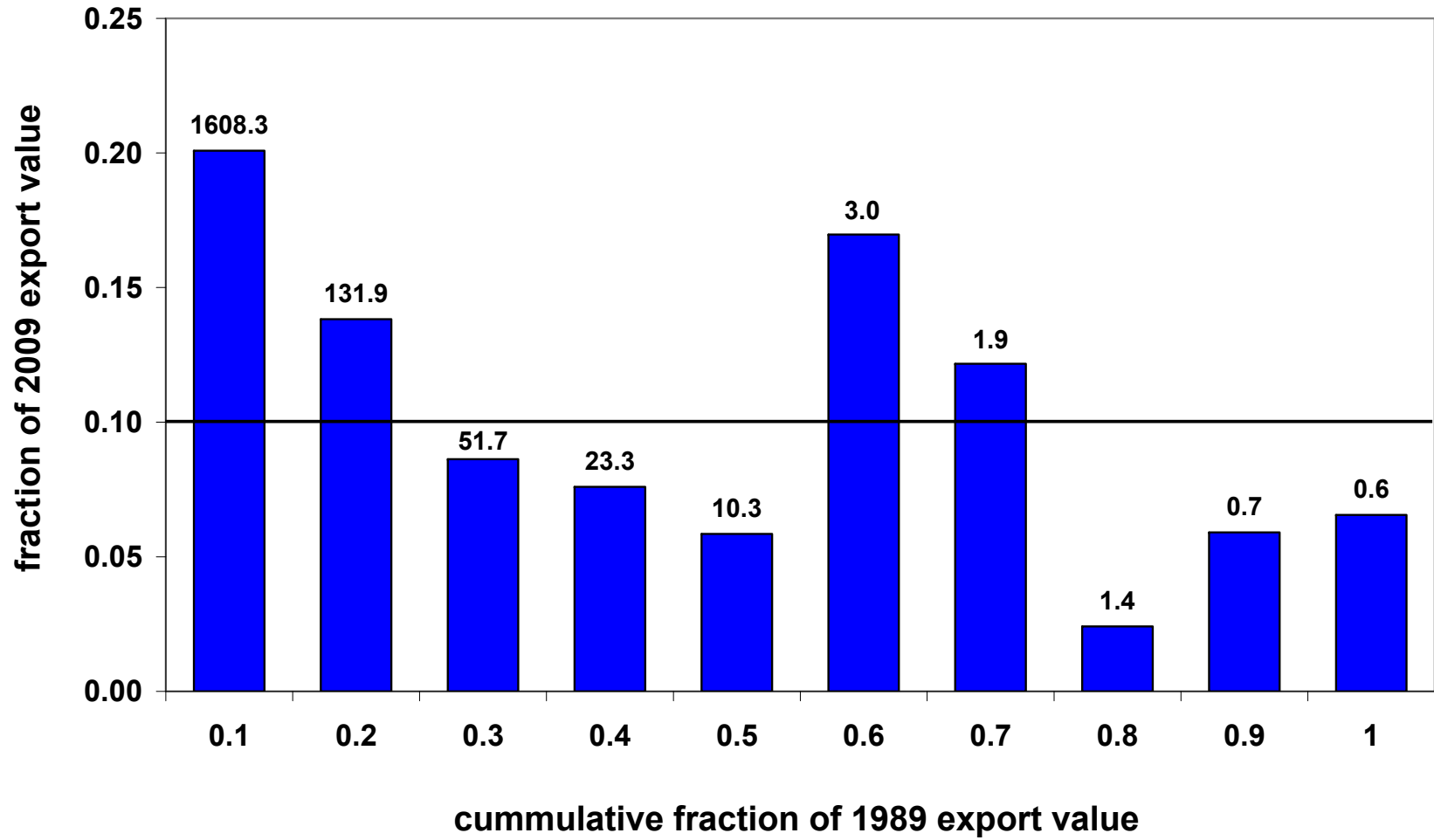
## **The New Product, or Extensive, Margin**

We sort each of the 1,836 products by average amount of trade over the first three years of our period

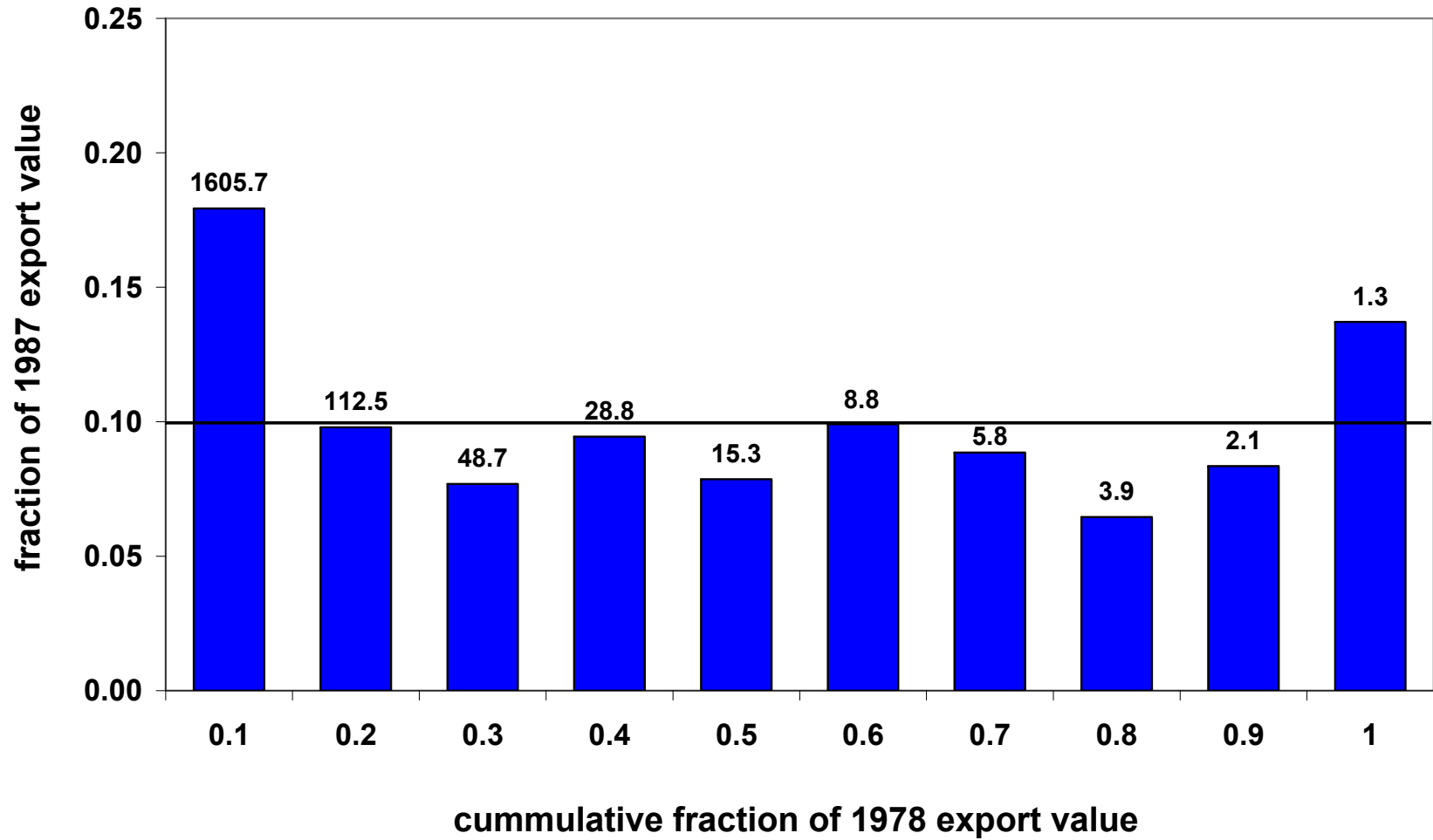
We then place each product into bins sequentially until each bin accounts for 10 percent of total trade in the base period.

We define Least Traded Products (LTP) to be the products in the final 10 percent bin, the products with the least amount of trade over the first three years.

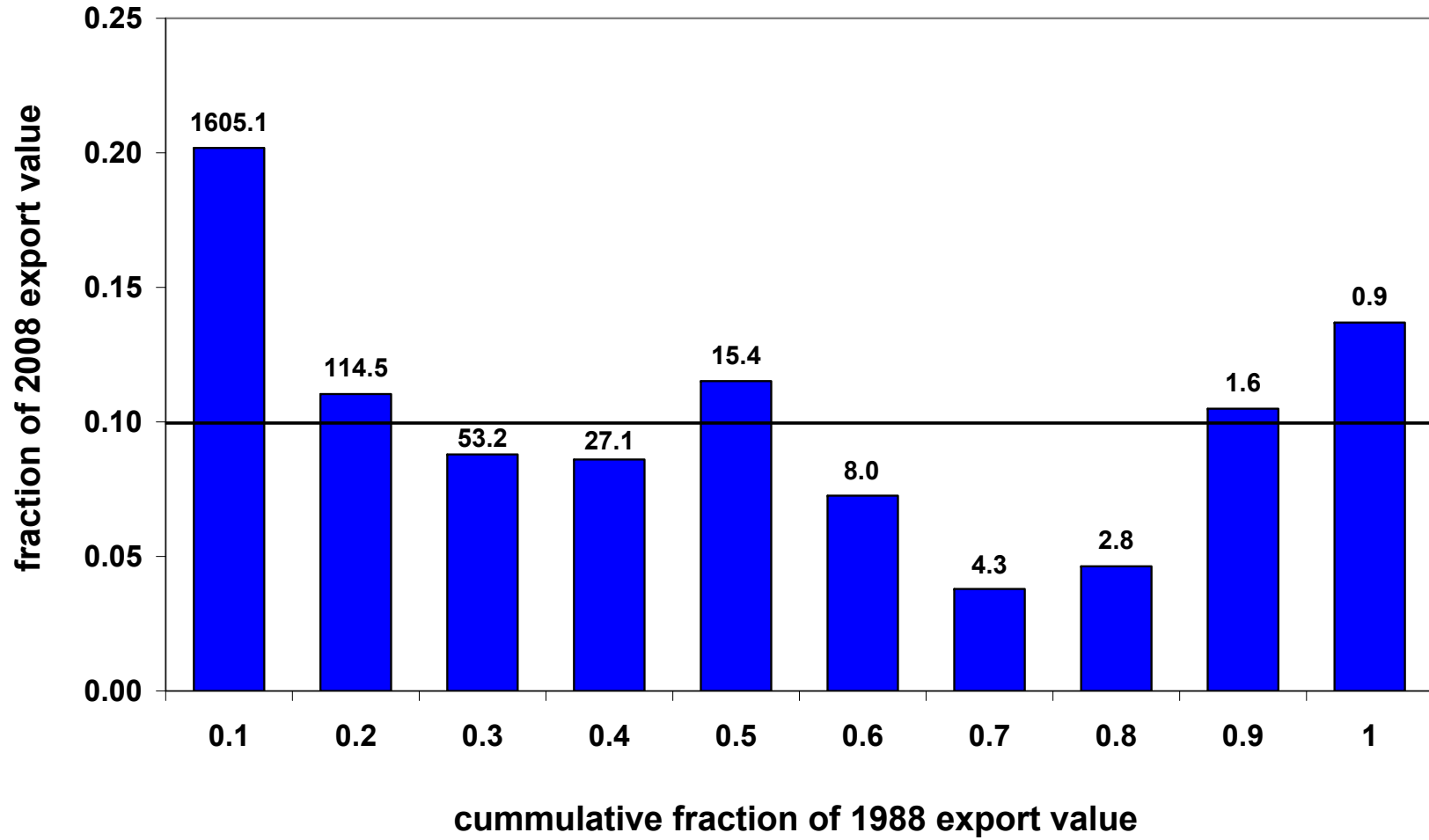
## Composition of Exports: Canada to United States 1988–2009



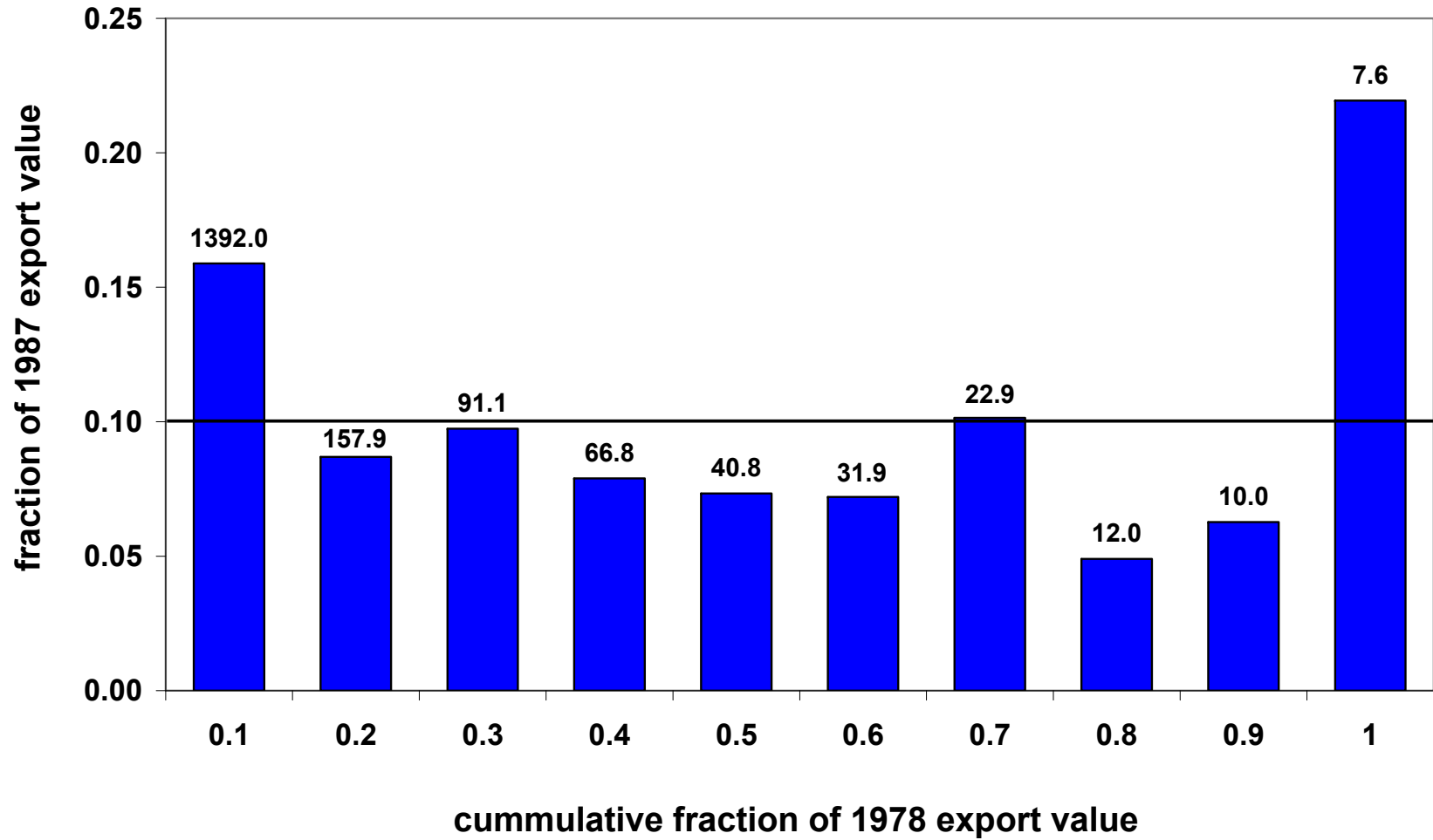
## Composition of Exports: Spain to Germany 1978–1987



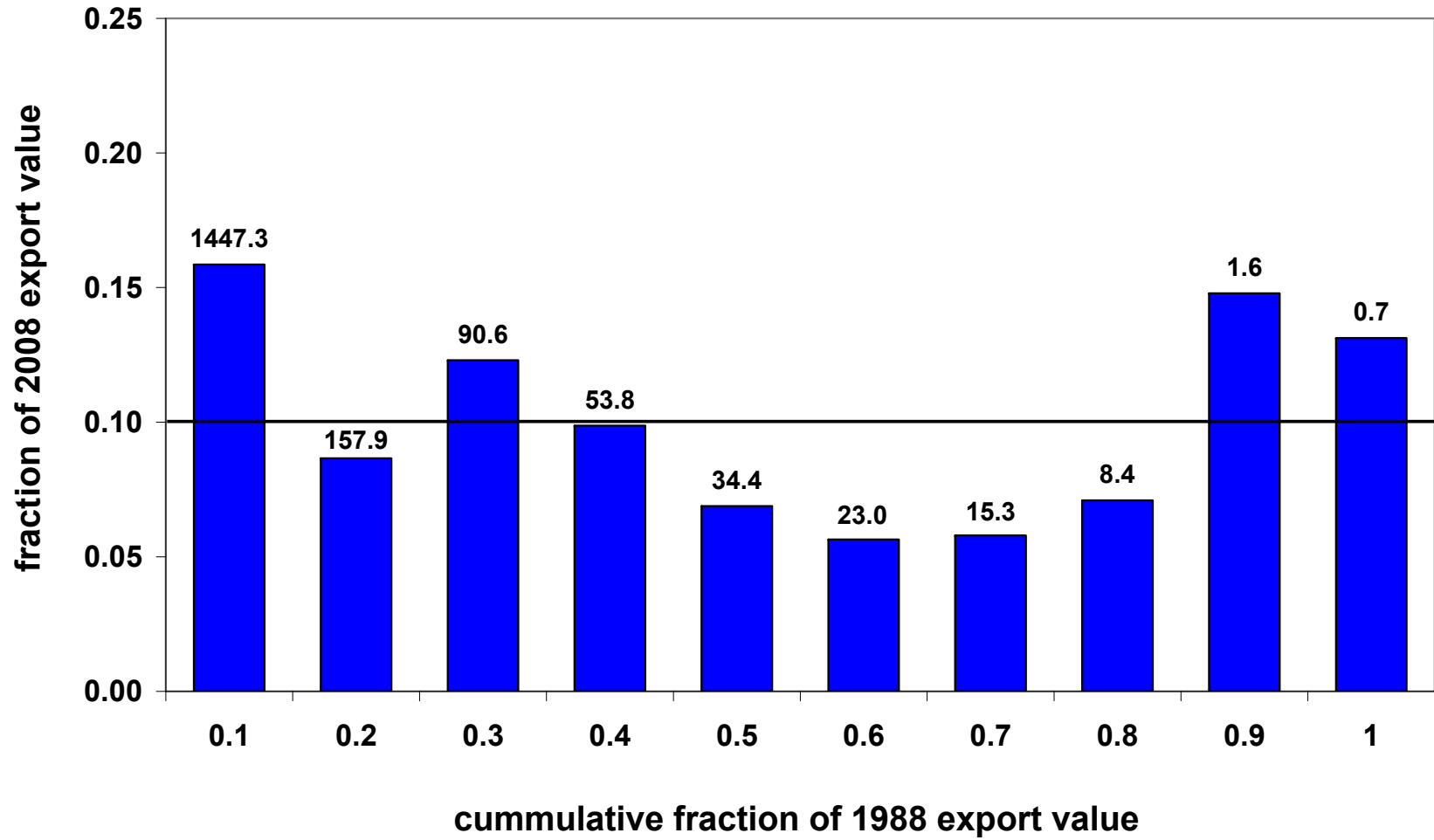
## Composition of Exports: Spain to Germany 1988–2008



## Composition of Exports: Germany to Spain 1978–1987



## Composition of Exports: Germany to Spain 1988–2008



## **Comparison to other extensive margins**

Most of the literature uses a fixed cutoff when deciding whether a product is part of the extensive margin, Feenstra (1994) uses a value of \$0, and Evenett and Venables (2002) use \$50,000.

In contrast, our measure varies by country pairs. The cutoff for Ecuador-Peru differs from the cutoff for U.S.-Canada.

We keep our set of extensive margin products fixed, as opposed to focusing on movement into and out of the extensive margin.

## Predicting changes in trade by industry

Compute the fraction of trade in each industry accounted for by LTP  $s_j$  in the base period  $t_0$ . Predict

$$z_j = \alpha + \beta s_j$$

$$z_j = \frac{X_{jit}^k / GDP_{it}}{X_{jit_0}^k / GDP_{it_0}} - 1$$

and  $X_{jit}^k$  are exports of industry  $j$  from country  $i$  to country  $k$  in year  $t$ .

We use experience from previous trade reforms (in this case NAFTA) to estimate  $\alpha$  and  $\beta$ . Our hypothesis is that  $\beta > 0$ .



**Least Traded Products predictions compared to those of Yaylaci-Shikher (forthcoming)**

industry	Korea to United States			United States to Korea		
	Yaylaci-Shikher predictions	LTP predictions	2005 fraction LTP	Yaylaci-Shikher predictions	LTP predictions	2005 fraction LTP
Chemicals	28.2	54.00	0.36	30.3	20.70	0.16
Electrical mach.	15.5	-0.44	0.02	41.0	-3.02	0.04
Food	70.1	86.03	0.56	422.3	26.63	0.19
Other machinery	8.9	9.17	0.08	31.9	6.86	0.09
Medical	9.9	74.82	0.49	45.0	-1.05	0.05
Metals	9.3	17.18	0.13	17.0	28.61	0.20
Nonmetals	20.5	39.59	0.27	38.7	80.00	0.46
Other	11.8	50.80	0.34	28.5	40.47	0.26
Paper	1.4	105.24	0.68	5.5	6.86	0.09
Petroleum	2.2	15.57	0.12	7.2	-5.00	0.03
Metal products	14.2	62.01	0.41	33.8	20.70	0.16
Rubber	19.8	10.77	0.09	48.0	22.68	0.17
Textile	56.3	58.81	0.39	63.5	117.56	0.65
Transport. equip.	23.3	-2.04	0.01	33.9	-5.00	0.03
Wood	7.9	29.99	0.21	21.1	38.49	0.25
Chemicals	28.2	54.00	0.36	30.3	20.70	0.16
<b>KS-LTP weighted correlation</b>			<b>0.43</b>			<b>0.19</b>

Kehoe (2005) showed that several of the leading models built to predict the industry level effects of NAFTA performed poorly

We confirm this finding for Brown-Deardorff-Stern (BDS), Cox-Harris, and Sobarzo models over the 1989-2009 period.

Focus on the BDS model since it has bilateral trade predictions for all importer-exporter pairs between Canada, Mexico, and the U.S.

## Methodology for evaluating the NAFTA models

We compute the weighted correlation coefficient between the model predictions and the results from the data

We also compute the weighted regression coefficients  $a$  and  $b$  from

$$\min_{a,b} \sum_{j=1}^{23} \omega_j \left( a + bz_j^{model} - z_j^{data} \right)^2$$

Here  $a$  indicates how well the models did in matching average change ( $a = 0$  is ideal) and  $b$  indicates how well the models did in matching the signs and magnitudes of the changes ( $b = 1$  is ideal)

### Changes in Canada-U.S. trade relative to exporter's GDP (percent)

industry	Canada to U.S.			U.S. to Canada		
	1989–2009 data	BDS model	1989 fraction LTP	1989–2009 data	BDS model	1989 fraction LTP
Agriculture	12.5	3.4	0.26	-6.4	5.1	0.19
Mining and quarrying	237.6	0.4	0.05	51.3	1.0	0.16
Food	101.2	8.9	0.24	124.1	12.7	0.25
Textiles	42.4	15.3	0.77	-35.9	44.0	0.52
Clothing	50.2	45.3	0.59	-3.0	56.7	1.00
Leather products	-67.7	11.3	1.00	-64.0	7.9	0.61
Footwear	-49.9	28.3	1.00	-67.2	45.7	0.34
Wood products	-54.5	0.1	0.01	-30.6	6.7	0.07
Furniture and fixtures	-46.6	12.5	0.00	22.5	35.6	0.00
Paper products	-65.9	-1.8	0.04	13.7	18.9	0.15
Printing and publishing	0.7	-1.6	0.12	-19.6	3.9	0.05
Rubber products	45.8	9.5	0.10	30.2	19.1	0.05
Chemicals	99.6	-3.1	0.38	50.2	21.8	0.24
Petroleum products	-79.8	0.5	0.07	-43.1	0.8	0.13
Glass products	-45.7	30.4	0.40	-20.0	4.4	0.23
Nonmetal mineral products	-0.4	1.2	0.38	-1.9	11.9	0.59
Iron and steel	-12.7	12.9	0.36	53.5	11.6	0.28

Nonferrous metals	-20.9	18.5	0.07	-20.8	-6.7	0.11
Metal products	17.7	15.2	0.20	-5.3	18.2	0.16
Nonelectrical machinery	-8.4	3.3	0.21	-38.9	9.9	0.08
Electrical machinery	-16.4	14.5	0.15	-42.6	14.9	0.05
Transportation equipment	-44.3	10.7	0.01	-37.8	-4.6	0.01
Misc. manufactures	56.1	-2.1	0.45	-19.2	11.5	0.15
<b>weighted corr. with data</b>		-0.28	0.30		0.39	0.54
<b>regression coeff. <math>a \setminus \alpha</math></b>		21.82	-20.42		-26.62	-34.54
<b>regression coeff. <math>b \setminus \beta</math></b>		-3.33	185.24		1.34	175.84
<b>BDS-LTP weighted corr.</b>			-0.11			0.70

**Results for the BDS model:** the BDS model fared poorly in predicting industry level changes in bilateral trade

<b>exporter</b>	<b>importer</b>	<b>correlation</b>	<b><i>a</i></b>	<b><i>b</i></b>
Canada	Mexico	-0.10	645.29	-7.94
Canada	United States	-0.28	21.82	-3.33
Mexico	Canada	0.06	135.79	0.16
Mexico	United States	-0.13	66.64	-0.11
United States	Canada	0.39	-26.62	1.34
United States	Mexico	-0.06	88.47	-0.24
weighted average		-0.00	19.83	-0.94
pooled regression		0.06	10.54	0.17

Correlation is the weighted correlation of predictions with the data.

**Results for the LTP exercise:** the LTP exercise fares much better in predicting industry level changes in bilateral trade

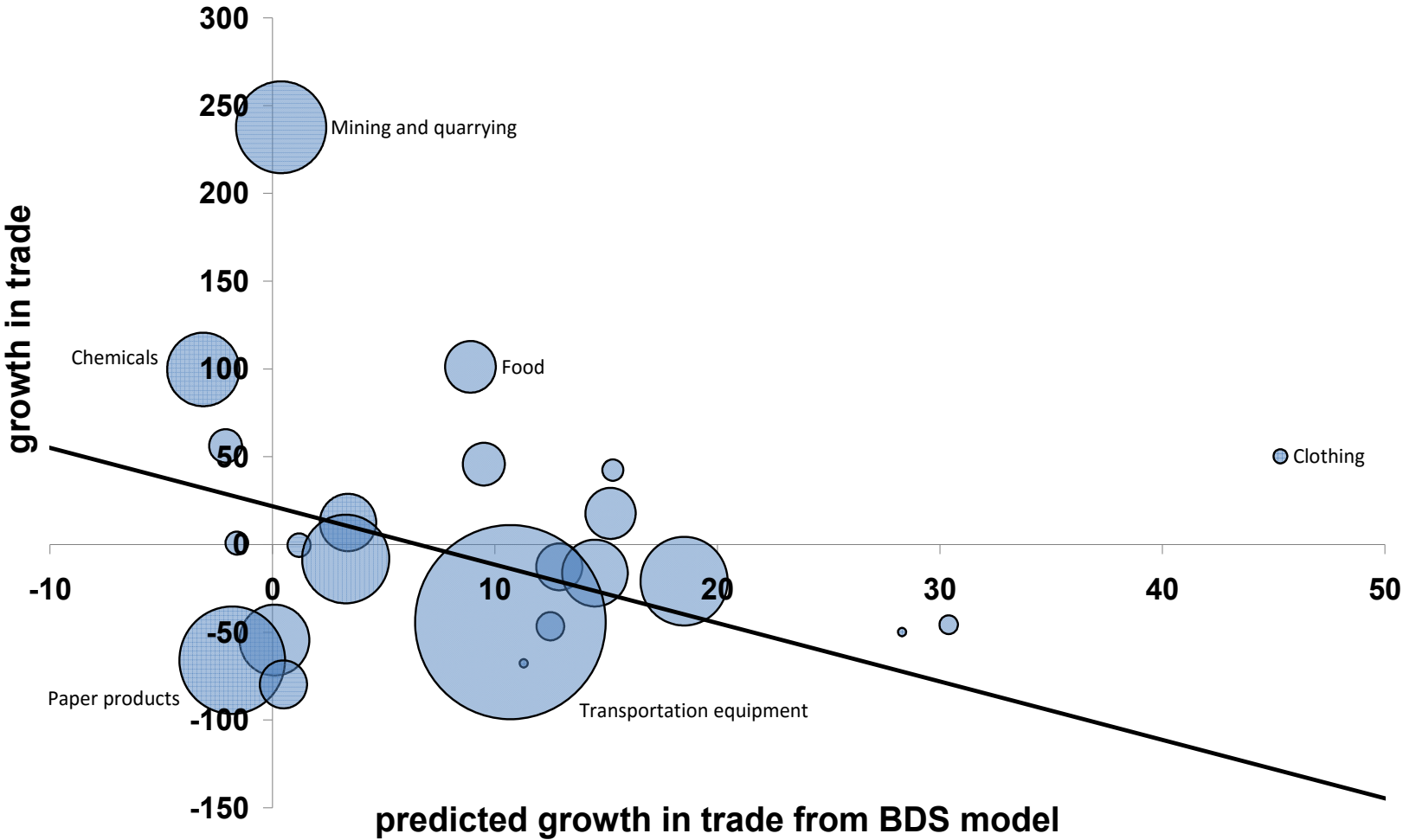
<b>exporter</b>	<b>importer</b>	<b>correlation</b>	<b><math>\alpha</math></b>	<b><math>\beta</math></b>
Canada	Mexico	0.55	254.23	4468.37
Canada	United States	0.30	-20.42	185.24
Mexico	Canada	0.33	115.16	286.39
Mexico	United States	0.19	51.52	77.54
United States	Canada	0.54	-34.54	175.84
United States	Mexico	0.47	62.31	265.44
weighted average		0.39	-5.74	87.29
pooled regression		0.24	-5.30	181.18

**Comparison of the BDS results and LTP exercise results: LTP exercise performs better the BDS model for every country pair.**

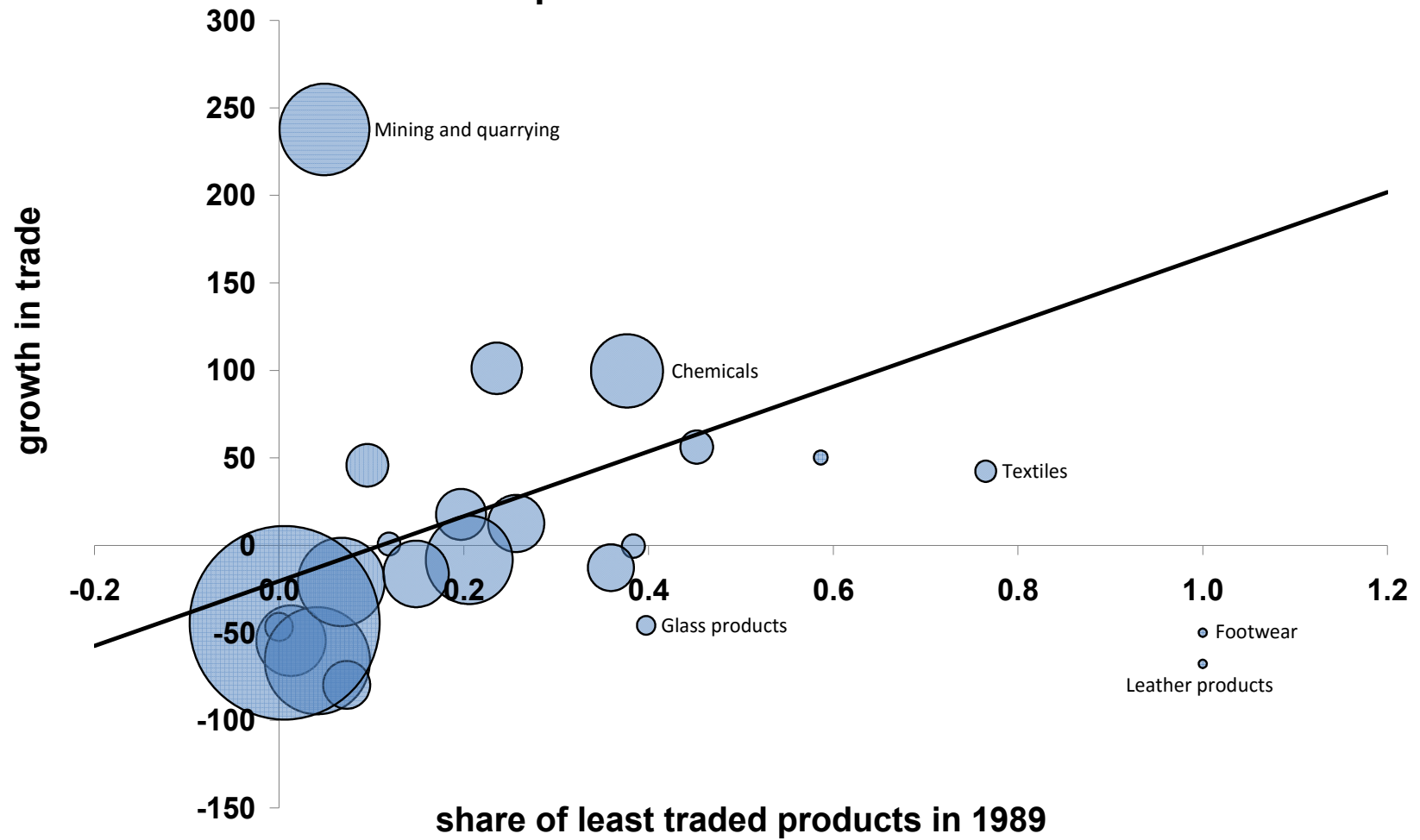
<b>exporter</b>	<b>importer</b>	<b>BDS correlation</b>	<b>LTP correlation</b>
Canada	Mexico	-0.10	0.55
Canada	United States	-0.28	0.30
Mexico	Canada	0.06	0.33
Mexico	United States	-0.13	0.19
United States	Canada	0.39	0.54
United States	Mexico	-0.06	0.47
weighted average		-0.00	0.39
pooled regression		0.06	0.24



# Growth in data versus BDS predicted growth Canadian exports to the United States



## Growth in data versus share of LTP by industry Canadian exports to the United States



Our exercise shows that looking at the share of least traded products in an industry is a useful predictor of which industries will experience the most growth following trade liberalization.

Major downside to our method: As of now it is atheoretical.

It is our hope that our results will spur the development of models able to account for the importance of the new product margin in trade.

## **Robustness:**

The  $\alpha$  and  $\beta$  computed from our industry-level regressions tell us how much LTP and non-LTP products grew on average

We compare these industry-based estimates to the average growth rates computed directly from the product data.

The industry level growth rates will not account for products with zero trade in 1989, while the product level growth rates will. If the estimated growth rates are similar, it indicates the important products are the ones with small, but positive trade.

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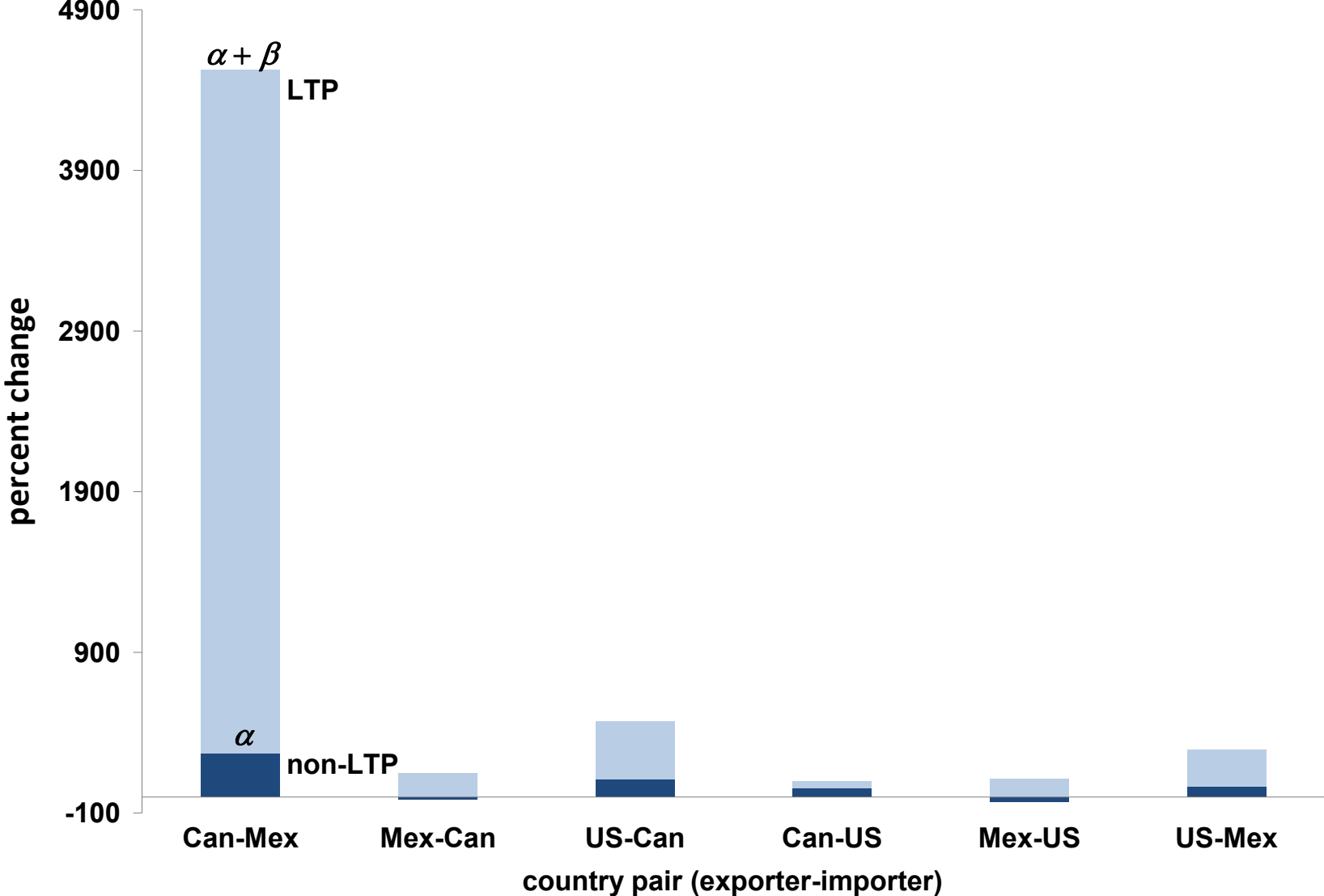
**We find a weighted correlation of 0.97 for  $\alpha$  and 0.91 for  $\beta$**

**Changes in North American trade relative to exporter's GDP:  
Estimates from industry data versus estimates from product data**

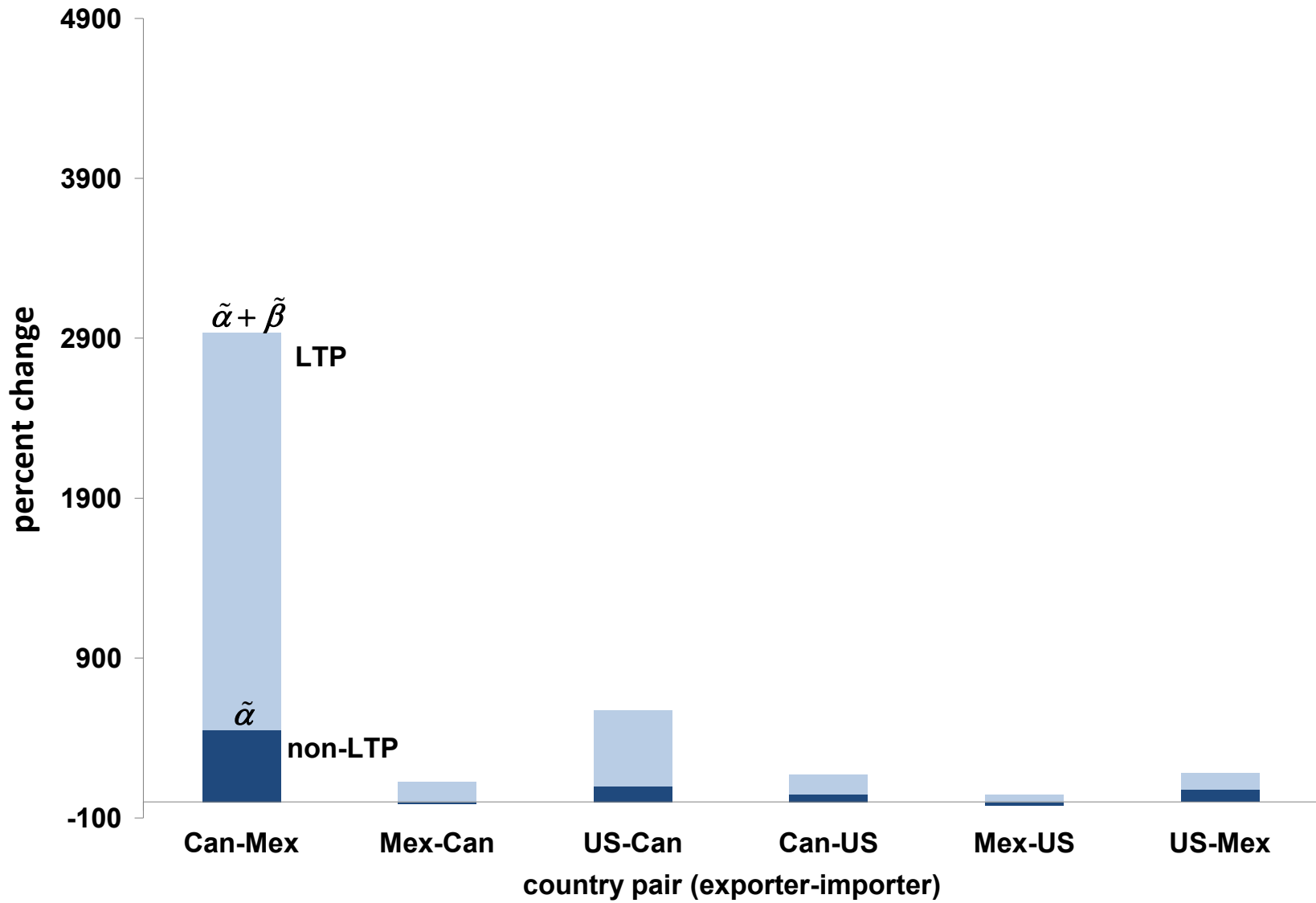
exporter	importer	period	industry data		product data	
			$\alpha$	$\beta$	$\tilde{\alpha}$	$\tilde{\beta}$
Canada	Mexico	89–09	273.01	4253.33	452.67	2483.99
Canada	United States	89–09	-16.89	149.92	-14.57	126.73
Mexico	Canada	89–09	107.47	363.23	96.13	476.67
Mexico	United States	89–09	54.92	43.54	46.89	123.86
United States	Canada	89–09	-28.22	112.55	-21.61	46.48
United States	Mexico	89–09	65.96	228.93	78.46	103.92
<b>weighted correlation <math>\alpha, \tilde{\alpha}</math></b>			0.97			
<b>weighted correlation <math>\beta, \tilde{\beta}</math></b>			0.91			

C. Arkolakis (2010), “Market Penetration Costs and the New Consumers Margin in International Trade.”

### Estimates from industry-level data



### Estimates from product-level data





## **Robustness**

We also find that our results hold when changing our end dates. For example if we use 1988(when available)–2007 to avoid the great recession.

We also find that, for goods for which we have both price and quantity data, after deflating by the exporter's PPI — most changes in value are driven by changes in quantity.

Our exercise similarly performs well when compared to alternative models used to predict the effects of NAFTA, for example Cox-Harris for Mexico and Sobarzo for Canada.

**Changes in North American trade deflated by Exporter's PPI:  
Growth due to quantities versus change due to prices**

<b>exporter</b>	<b>importer</b>	<b>average share of total growth</b>	
		<b>P</b>	<b>Q</b>
Canada	Mexico	2.3	97.7
Canada	United States	-2.5	102.5
Mexico	Canada	31.7	68.3
Mexico	United States	24.9	75.1
United States	Canada	-8.9	108.9
United States	Mexico	5.3	94.7
weighted average		-0.2	100.2
pooled		0.8	99.2

**Changes in Mexican trade relative to Mexican GDP in the Sobarzo Model (Percent)**

sector	Exports to North America			Imports from North America		
	1989– 2009 data	Sobarzo growth rate	1989 fraction least traded	1989– 2009 data	Sobarzo growth rate	1989 fraction least traded
Agriculture	-15.3	-11.1	0.07	3.2	3.4	0.10
Beverages	161.8	5.2	0.01	85.2	-1.8	0.32
Chemicals	34.1	-4.4	0.60	104.2	-2.7	0.23
Electrical Machinery	54.7	1.0	0.02	6.6	9.6	0.01
Food	100.8	-6.9	0.41	46.7	-5.0	0.15
Iron and Steel	19.6	-4.9	0.37	23.1	17.7	0.24
Leather	-64.6	12.4	0.53	2.5	-0.4	0.67
Metal Products	86.2	-4.4	0.30	24.8	9.5	0.14
Mining	27.7	-17.0	0.01	15.0	13.2	0.17
Nonelectrical Machinery	166.5	-7.4	0.12	38.3	20.7	0.09
Nonferrous Metals	36.8	-9.8	0.13	37.1	9.8	0.10
Nonmetallic Min. Prod.	-16.0	-6.2	0.26	5.3	10.9	0.49
Other Manufactures	88.4	-4.5	0.23	26.1	4.2	0.16
Paper	-35.9	-7.9	0.30	-4.1	-4.7	0.07
Petroleum	-98.0	-19.5	0.12	-81.6	-6.8	0.06
Rubber	158.9	12.8	0.43	78.3	-0.1	0.06
Textiles	69.5	1.9	0.76	48.3	-1.2	0.44
Tobacco	-61.3	2.8	1.00	333.0	-11.6	1.00
Transportation Equip.	126.1	-5.0	0.02	26.7	11.2	0.02
Wearing Apparel	197.2	30.0	0.23	-17.2	4.5	0.20
Wood	30.8	-8.5	0.04	-34.0	11.7	0.05
<b>weighted correlation with data</b>		0.43	0.02		-0.12	0.47
<b>regression coefficient <math>a \setminus \alpha</math></b>		62.91	81.13		30.91	9.61
<b>regression coefficient <math>b \setminus \beta</math></b>		7.92	3.06		-0.49	175.76

**Changes in Canadian trade relative to Canadian GDP  
in the Cox-Harris Model (Percent)**

sector	Exports to World			Imports from World		
	1989– 2009 data	C-H growth rate	1989 fraction least traded	1989– 2009 data	C-H growth rate	1989 fraction least traded
Agriculture	39.1	-4.1	0.13	13.4	7.2	0.18
Chem. & Misc. Man.	70.9	28.1	0.34	59.1	10.4	0.20
Fishing	-30.9	-5.4	0.05	32.9	9.5	0.22
Food, Bev., and Tobacco	95.5	18.6	0.22	86.6	3.8	0.19
Forestry	-24.8	-11.5	0.15	4.5	7.1	0.24
Machinery and Appl.	11.7	57.1	0.19	-6.6	13.3	0.06
Mining	117.0	-7.0	0.03	103.0	4.0	0.06
Nonmetallic Minerals	20.9	31.8	0.64	3.4	7.3	0.32
Refineries	-67.8	-2.7	0.06	-71.9	1.5	0.03
Rubber and Plastics	107.3	24.5	0.22	56.0	13.8	0.07
Steel and Metal Products	6.6	19.5	0.15	33.2	10.0	0.17
Textiles and Leather	18.4	108.8	0.86	-1.9	18.2	0.33
Transportation Equip.	-37.5	3.5	0.01	-19.7	3.0	0.01
Wood and Paper	-58.5	7.3	0.02	12.8	7.2	0.09
<b>weighted correlation with data</b>		0.06	0.40		0.04	0.48
<b>regression coefficient <math>a \setminus \alpha</math></b>		2.00	-13.73		9.77	-7.55
<b>regression coefficient <math>b \setminus \beta</math></b>		0.16	199.46		0.30	199.46

In 1993 Ross Perot said

The reason that most U.S. policymakers are so blind to the job shifting that will occur if NAFTA is ratified is that they rely on dozens of “reputable” academic studies that say it won’t happen. Yet these studies are based on unrealistic assumptions and flawed mathematical models...Let’s be clear about this: these studies certainly do not provide a basis on which Congress can make an informed decision about NAFTA.

Ross Perot with Pat Choate, *Save Your Job, Save Our Country: Why NAFTA Must Be Stopped — Now!* 1993.

## What Do We Learn from these Evaluations?

The Spanish model seems to have been far more successful in predicting the consequences of policy changes than the three models of NAFTA, but

- Kehoe, Polo, and Sancho (KPS) knew the structure of their model well enough to precisely identify the relationships between the variables in their model with those in the data;
- KPS were able to use the model to carry out numerical exercises to incorporate the impact of exogenous shocks.

KPS had an incentive to show their model in the best possible light.

## Armington aggregator

$$x_i^{mex} = \theta_i \left( \alpha_{i,can}^{mex} x_{i,can}^{mex \rho} + \alpha_{i,mex}^{mex} x_{i,mex}^{mex \rho} + \alpha_{i,us}^{mex} x_{i,us}^{mex \rho} + \alpha_{i,rw}^{mex} x_{i,rw}^{mex \rho} \right)^{1/\rho}$$

## Dixit-Stiglitz/Ethier aggregator

$$x_i^{mex} = \theta_i \left( \sum_{j=1}^{n_i} x_{i,j}^{mex \rho} \right)^{1/\rho}$$

modified to allow for home country bias

$$x_i^{mex} = \theta_i \left( \alpha_{i,can}^{mex} \sum_{j=1}^{n_{i,can}} x_{i,j,can}^{mex \rho} + \alpha_{i,mex}^{mex} \sum_{j=1}^{n_{i,mex}} x_{i,j,mex}^{mex \rho} + \alpha_{i,us}^{mex} \sum_{j=1}^{n_{i,us}} x_{i,j,us}^{mex \rho} + \alpha_{i,rw}^{mex} \sum_{j=1}^{n_{i,rw}} x_{i,j,rw}^{mex \rho} \right)^{1/\rho}$$

**3. Much of the growth of trade after a trade liberalization experience is growth on the extensive margin. Models need to allow for corner solutions or fixed costs.**

T. J. Kehoe and K. J. Ruhl, “How Important is the New Goods Margin in International Trade?” Federal Reserve Bank of Minneapolis, 2002.

What happens to the **least-traded** goods:

Over the business cycle?

During trade liberalization?

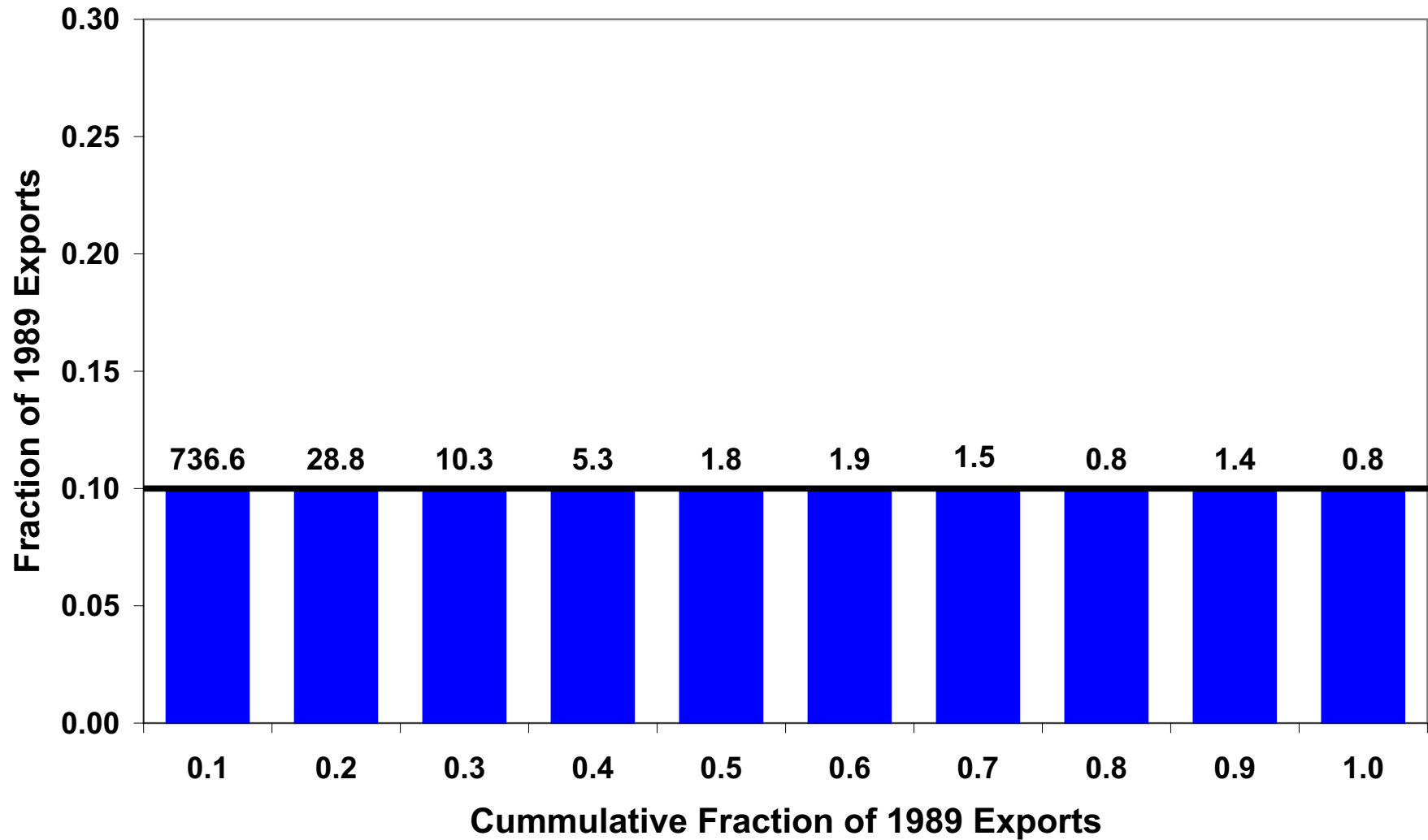
Indirect evidence on the extensive margin



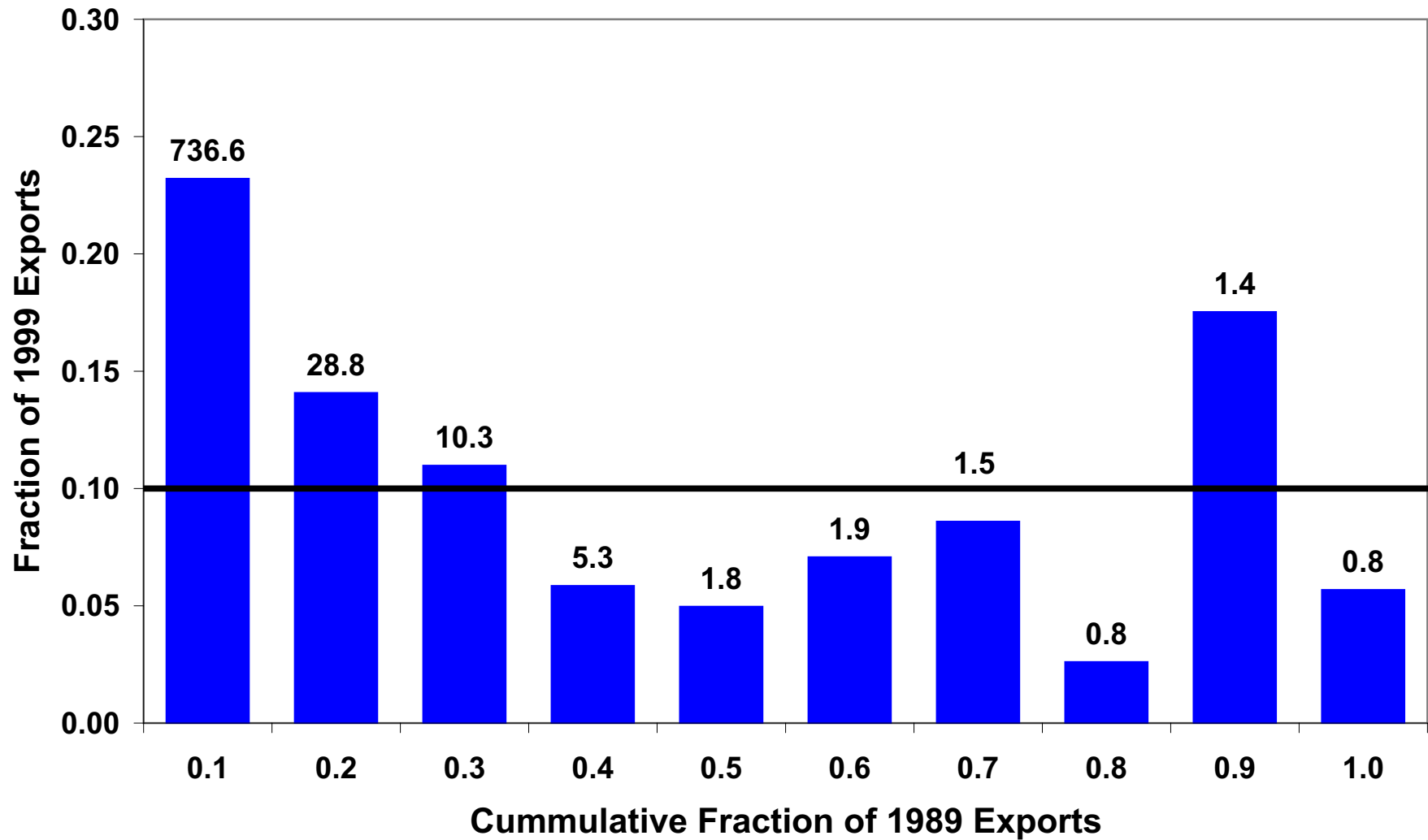
## Evidence on the Extensive Margin

- Data
  - 4 digit SITC bilateral trade data (OECD)
  - 789 codes in revision 2
- Least Traded Goods
  - Look 5 years before trade agreement
  - Rank codes from lowest value of exports to highest based on average of first 3 years in sample
  - Lowest decile of codes = least-traded goods
- Two Episodes
  - Canada-Mexico during NAFTA
  - United States-Germany in 1990s

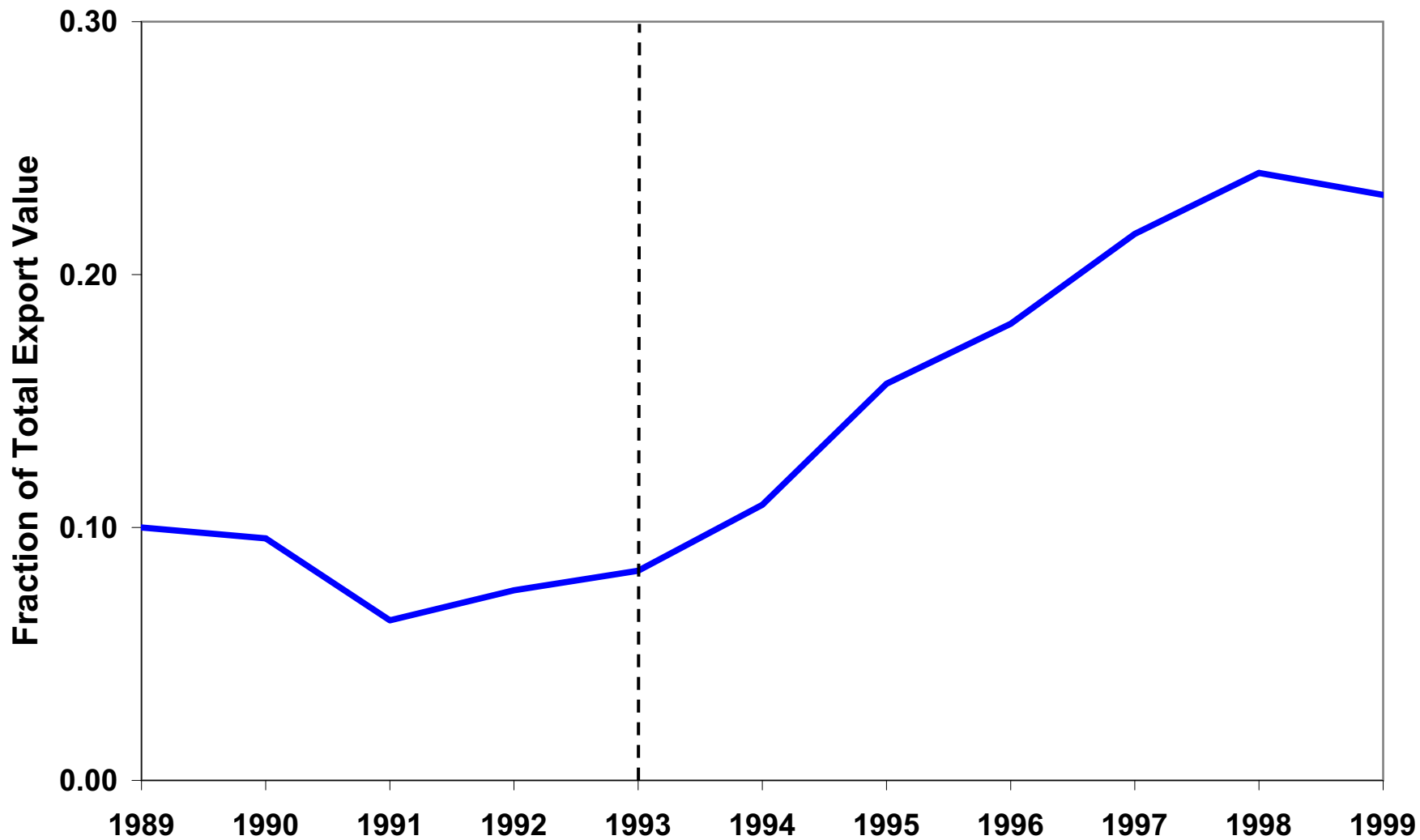
## Composition of Exports: Mexico to Canada



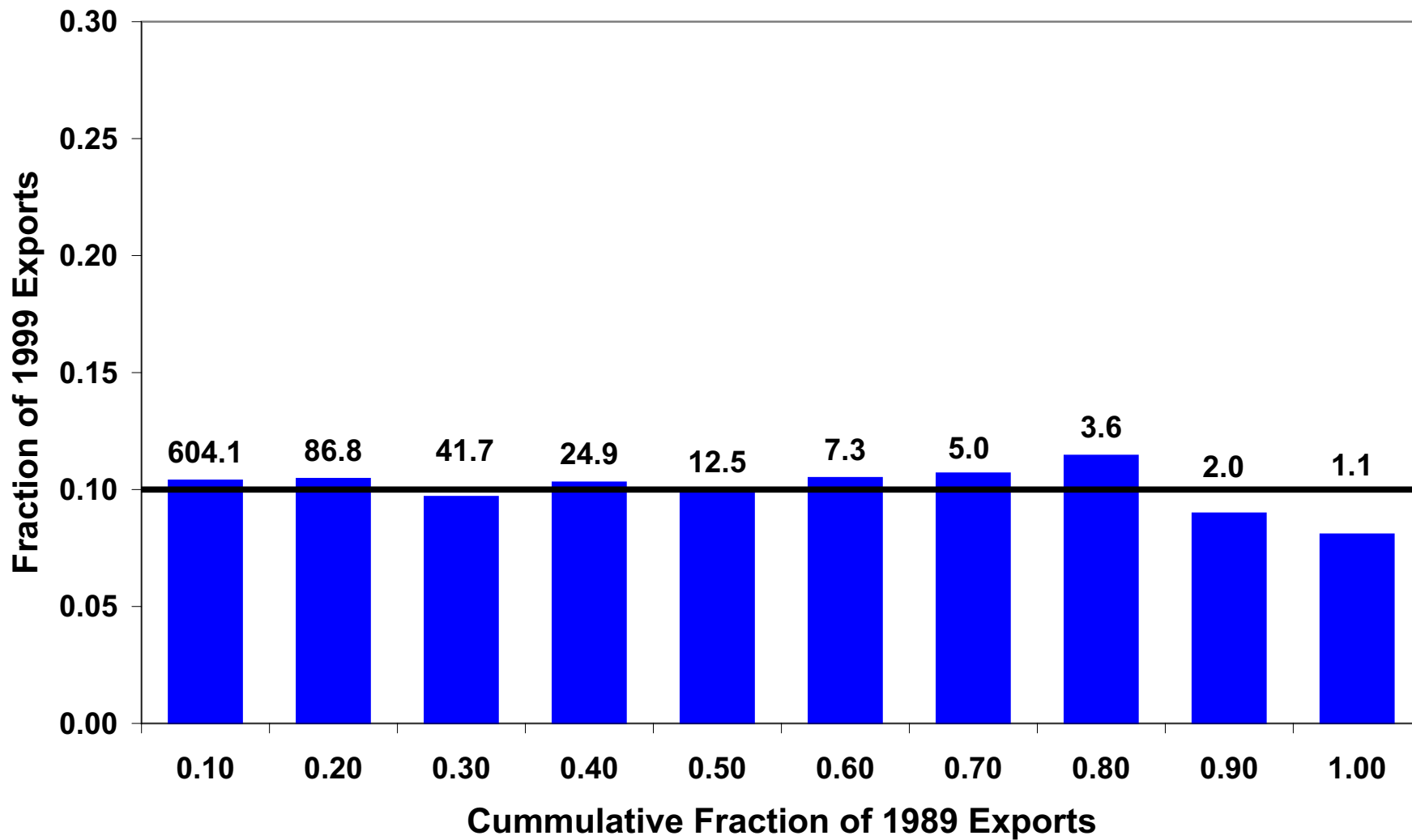
## Composition of Exports: Mexico to Canada



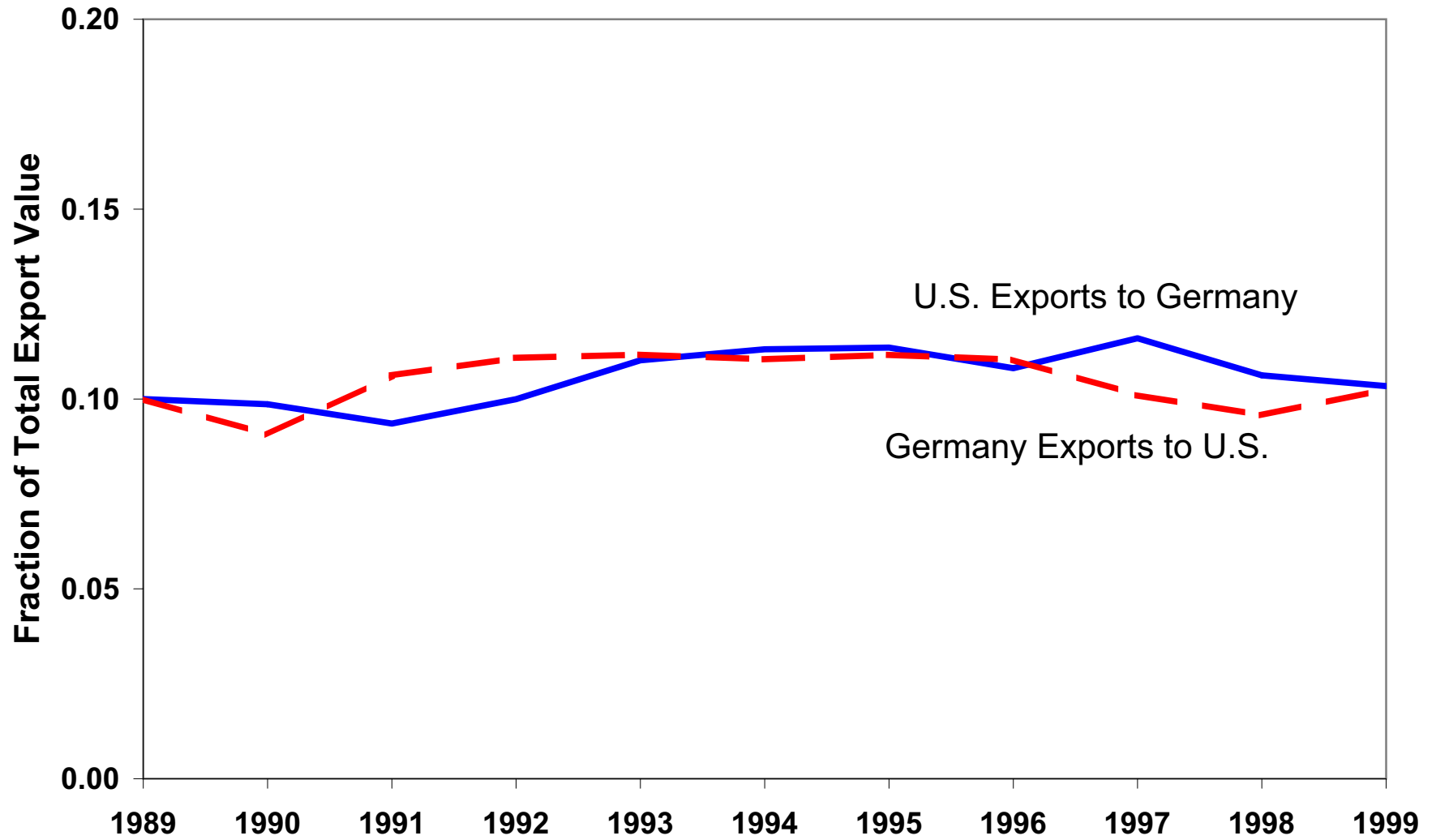
## Exports: Mexico to the Canada



## Composition of Exports: U.S. to Germany



## United States and Germany



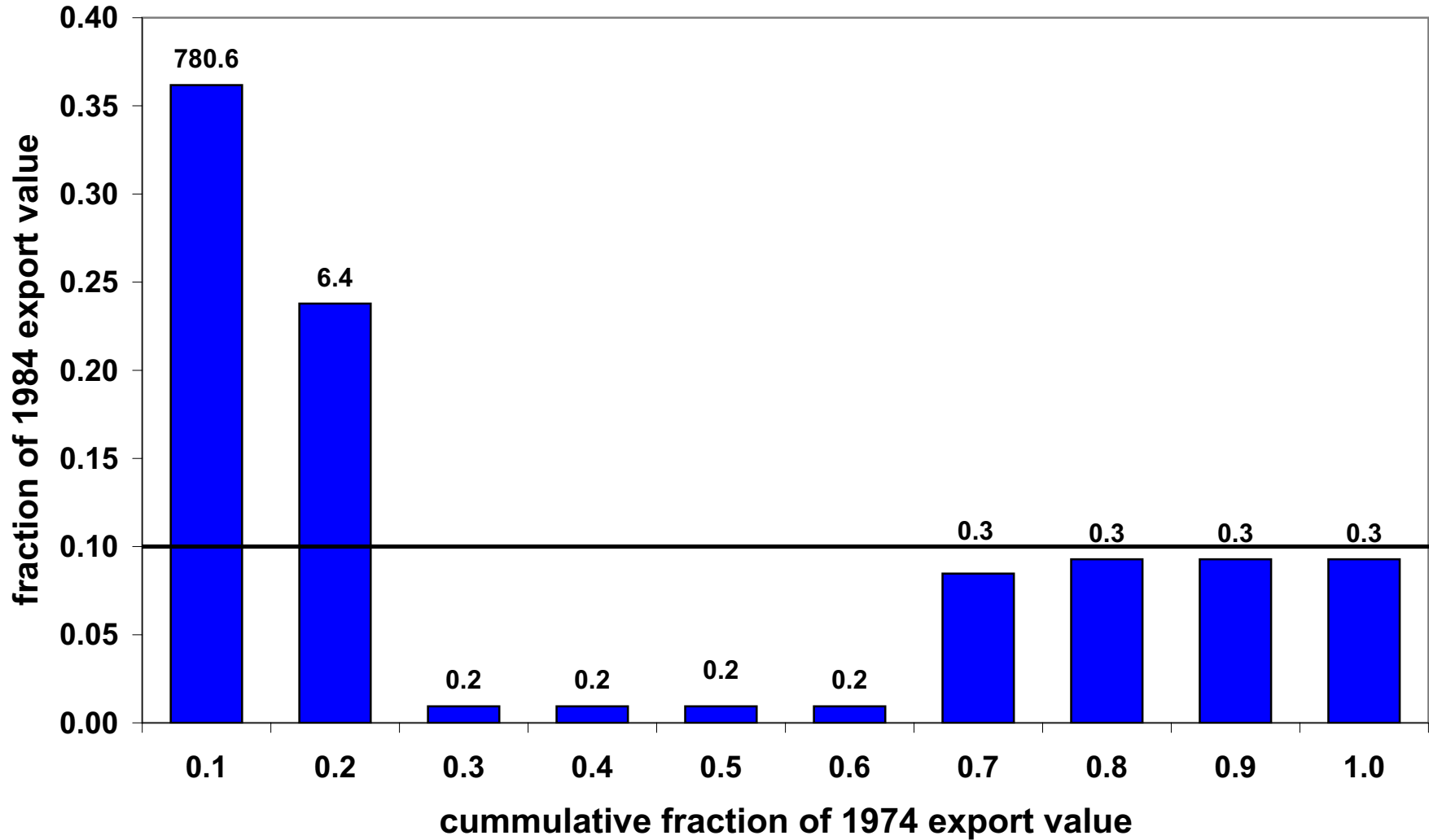
## **Lessons from data**

Trade liberalization increases trade on the extensive margin, business cycle fluctuations do not.

Structural changes may increase trade on the extensive margin.

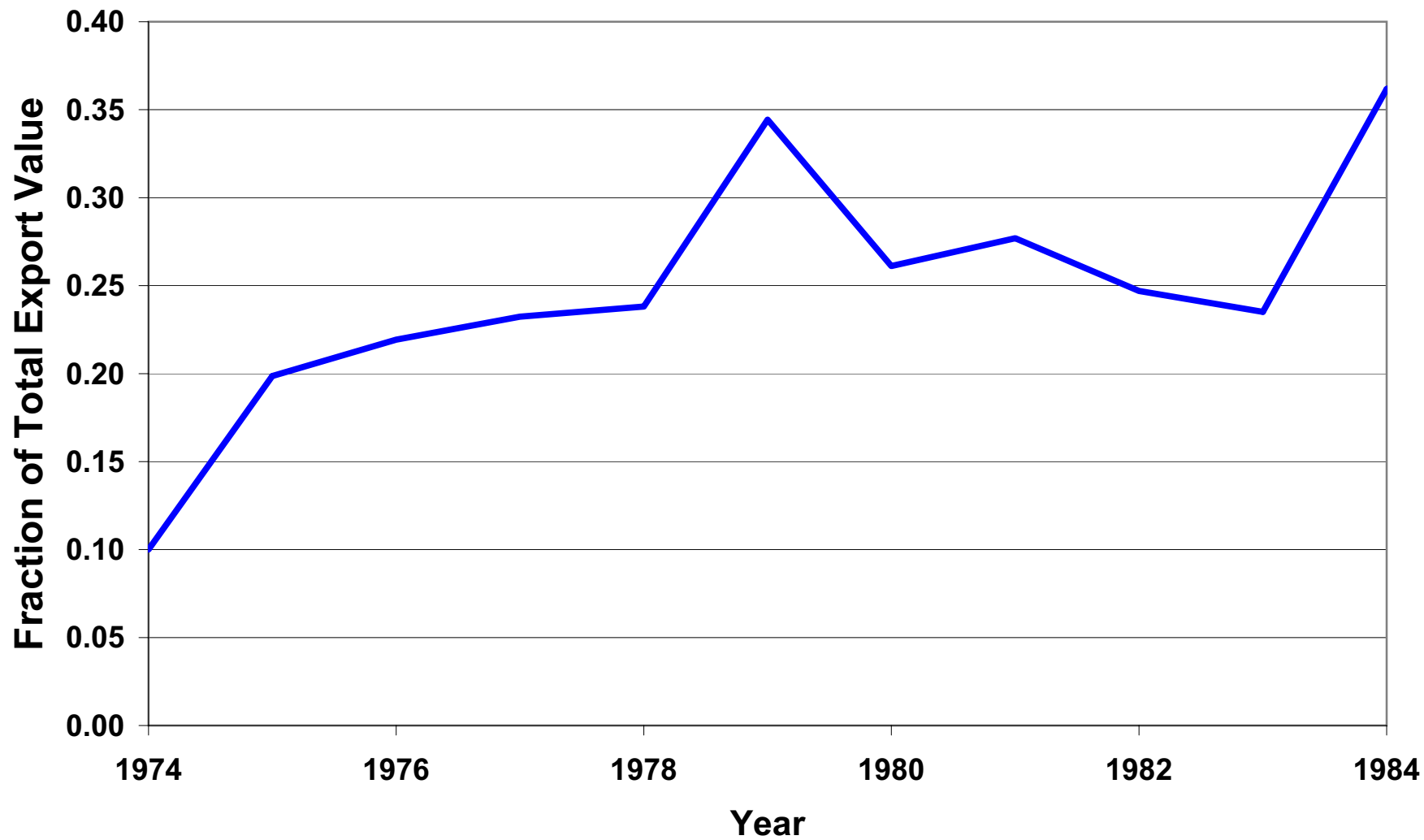
A country increasing its exports on the extensive margin because of trade liberalization may increase its exports on the extensive margin to other countries.

## Composition of Exports: Chile to the United States

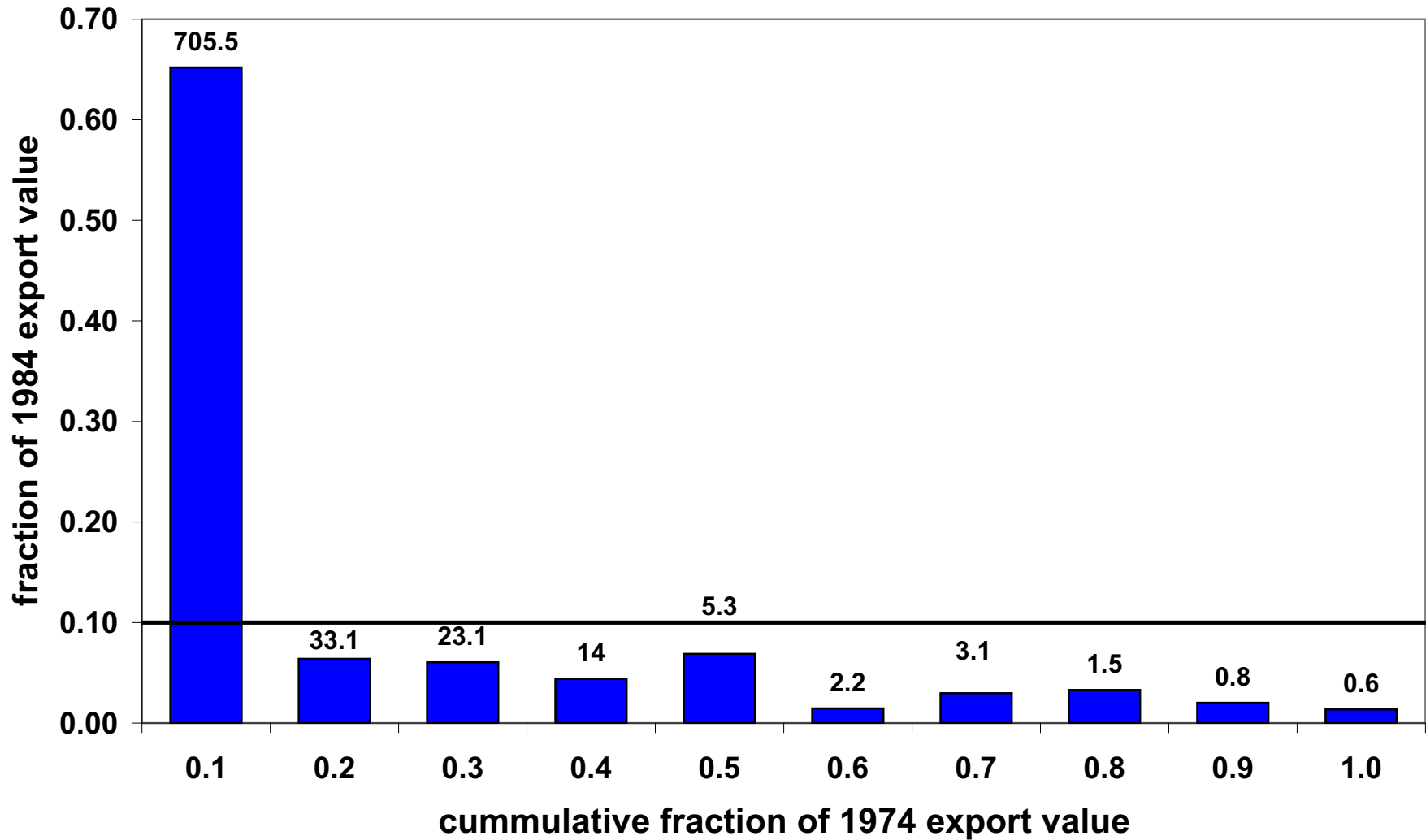




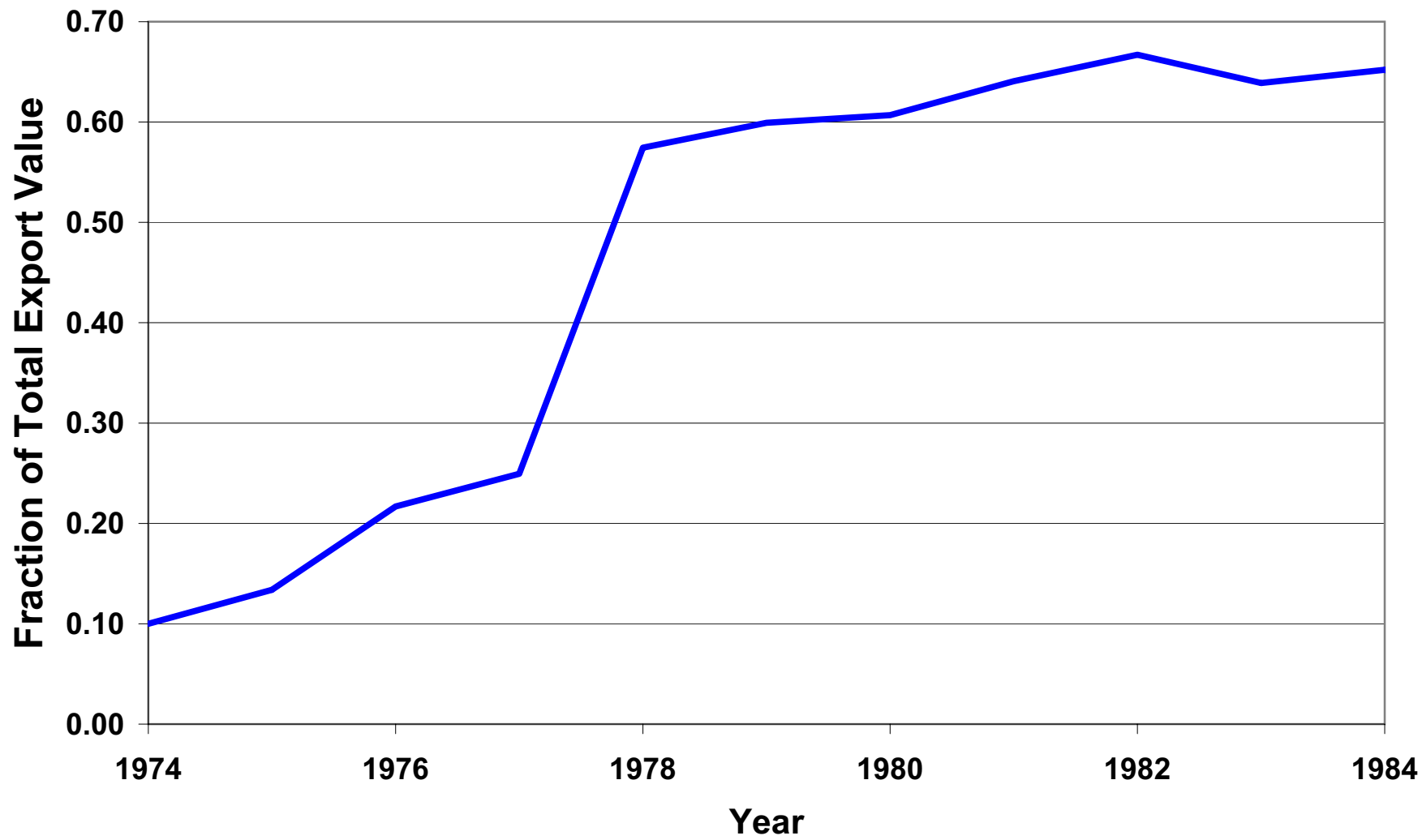
## Exports: Chile to the United States



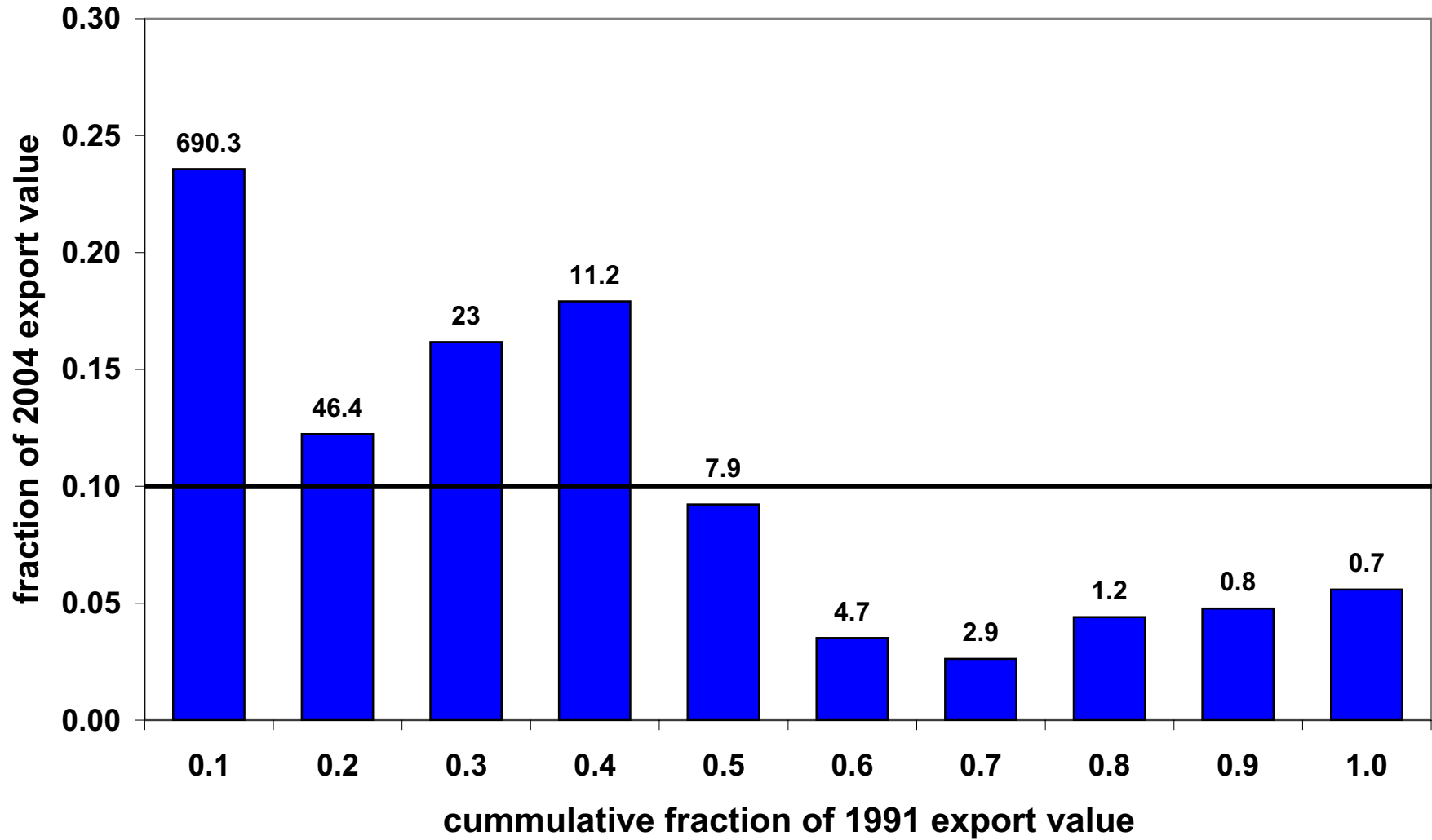
## Composition of Exports: United States to Chile



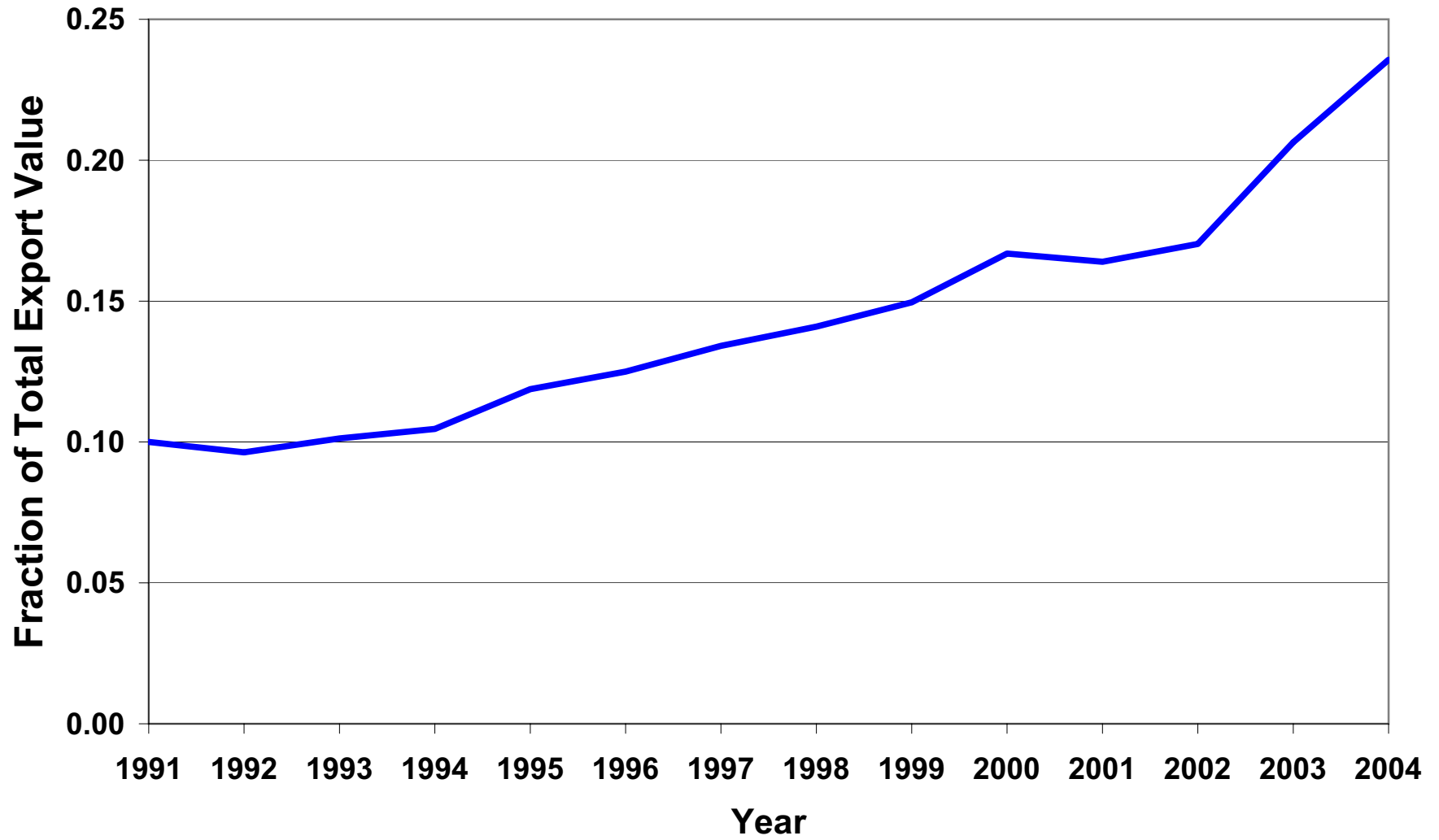
## Exports: United States to Chile



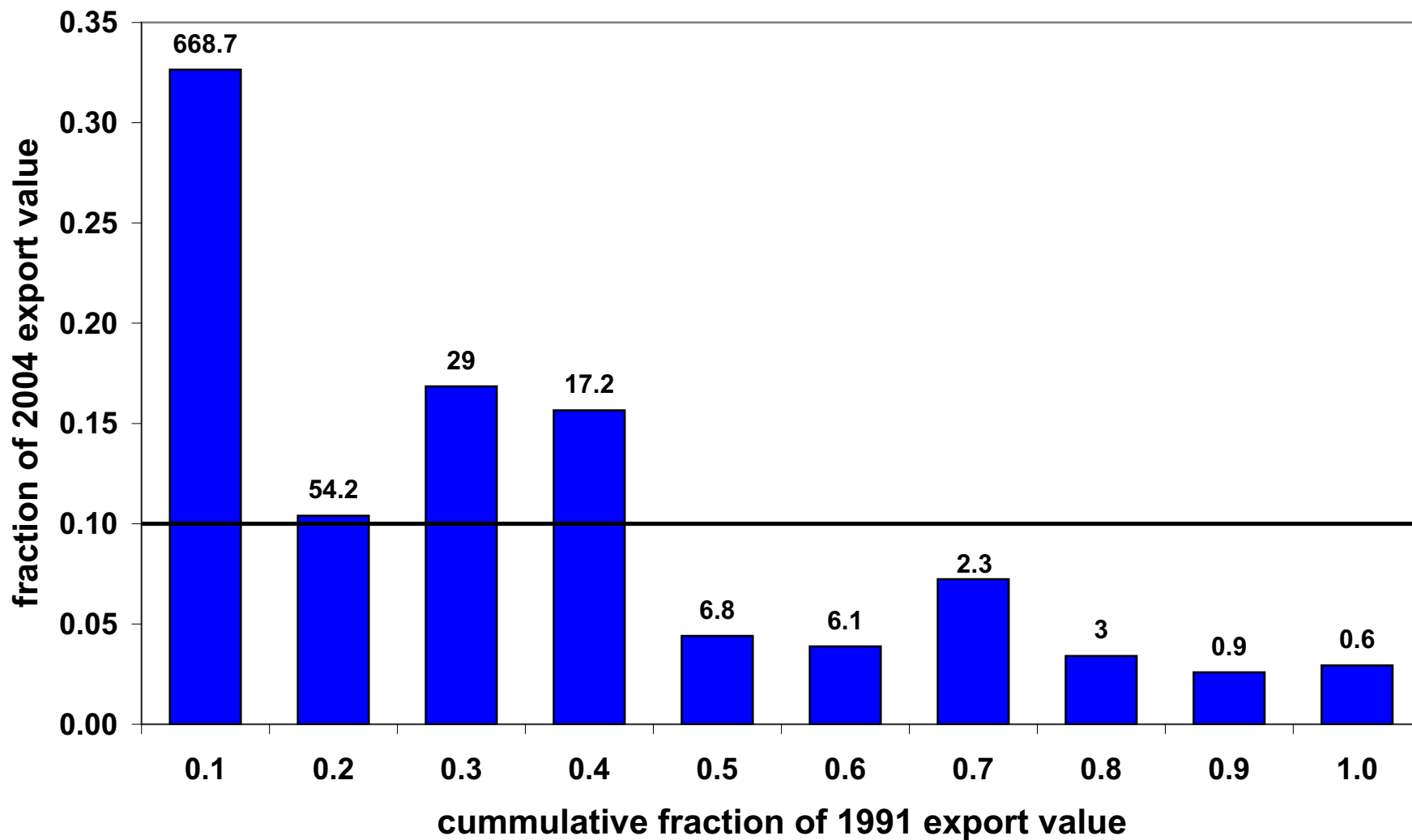
## Composition of Exports: China to the United States



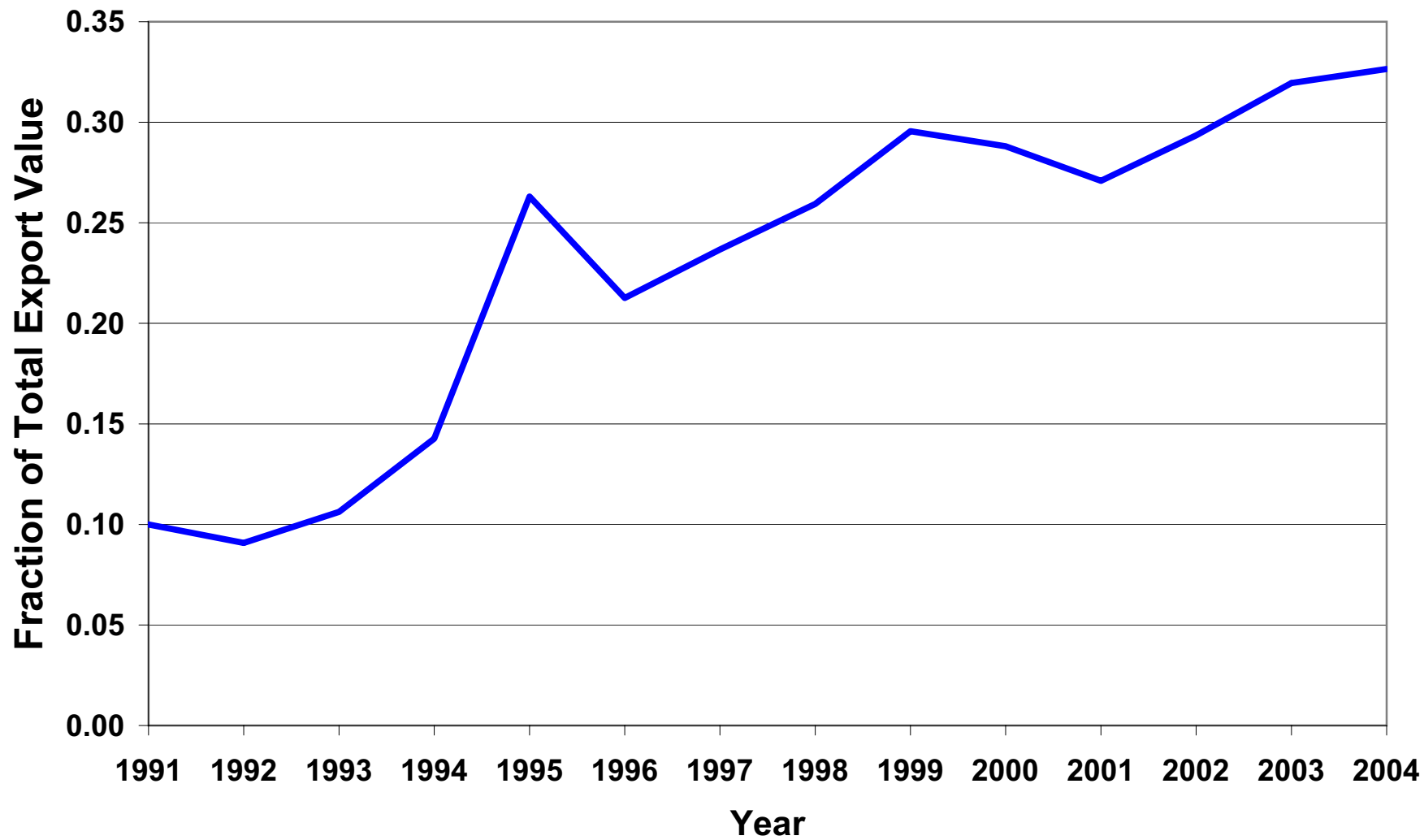
## Exports: China to the United States



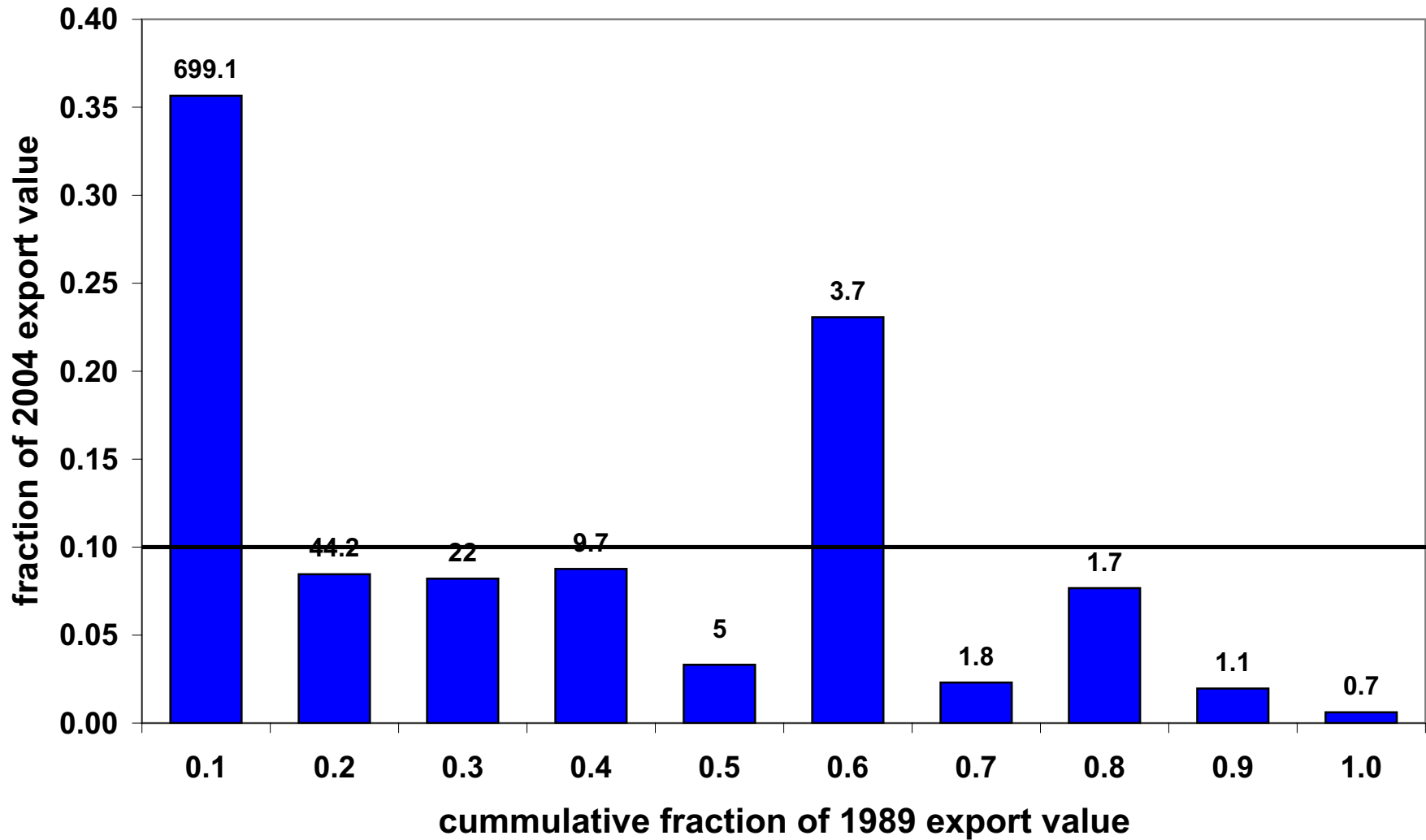
## Composition of Exports: United States to China



## Exports: United States to China

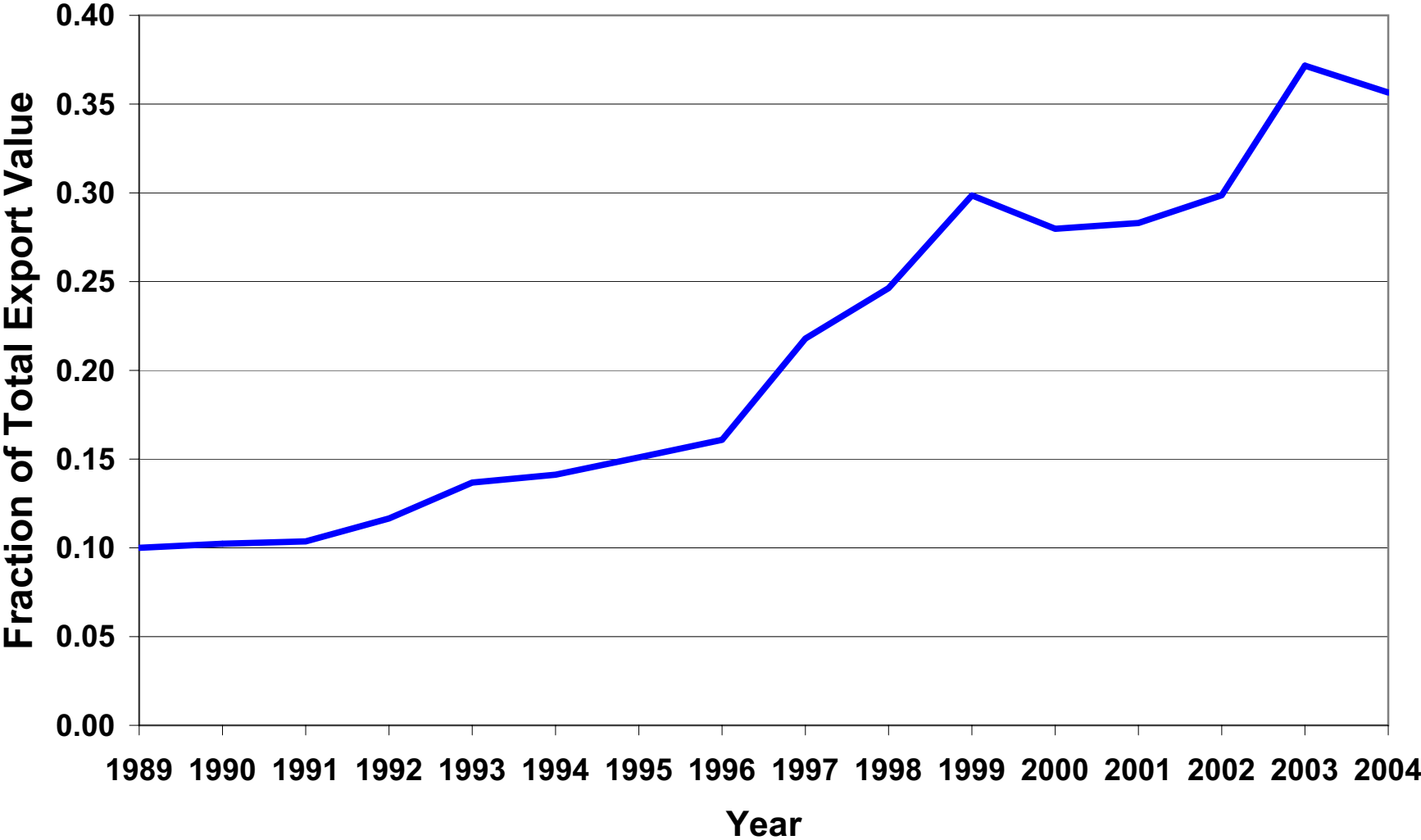


## Composition of Exports: Canada to the United Kingdom

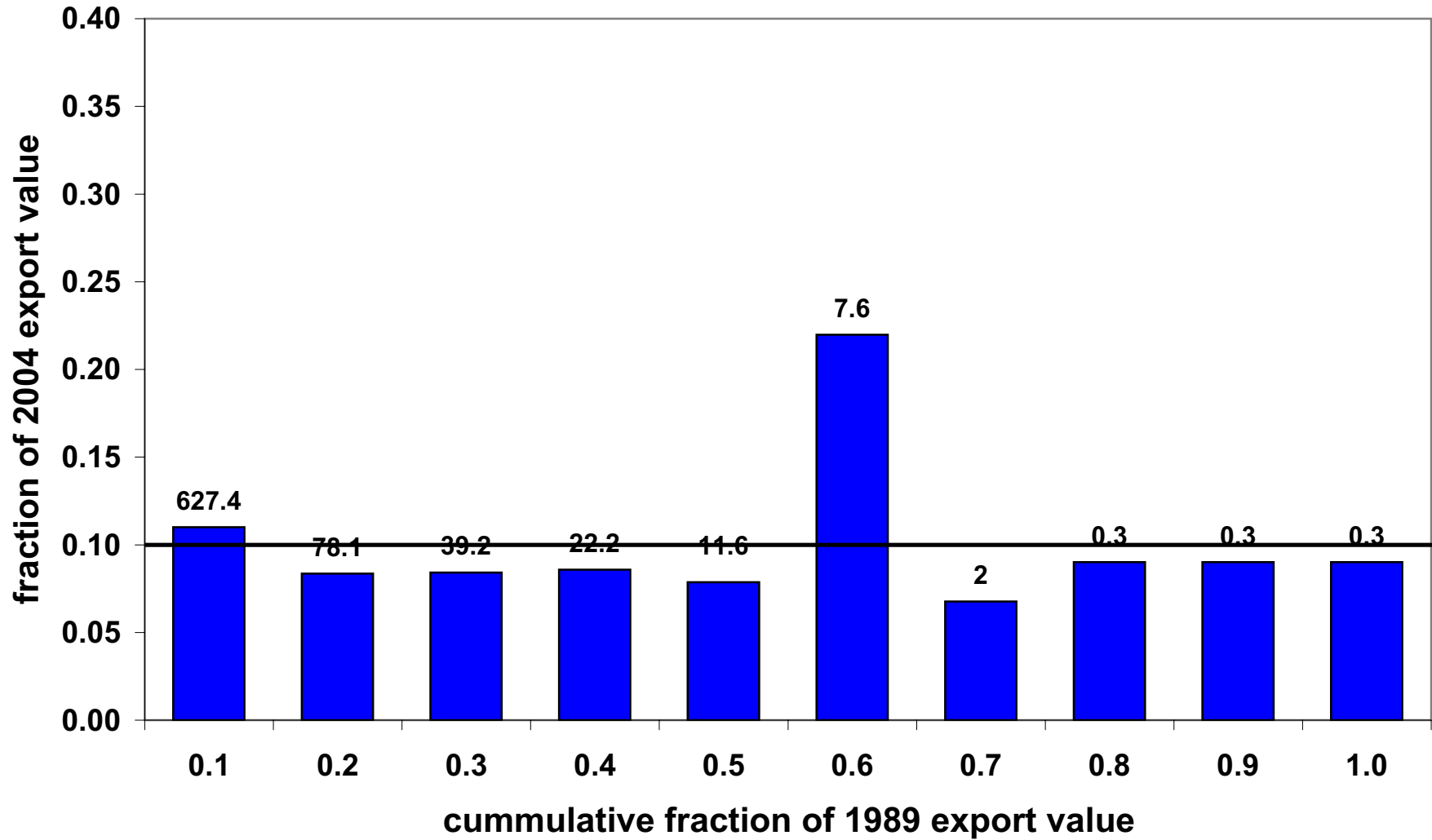




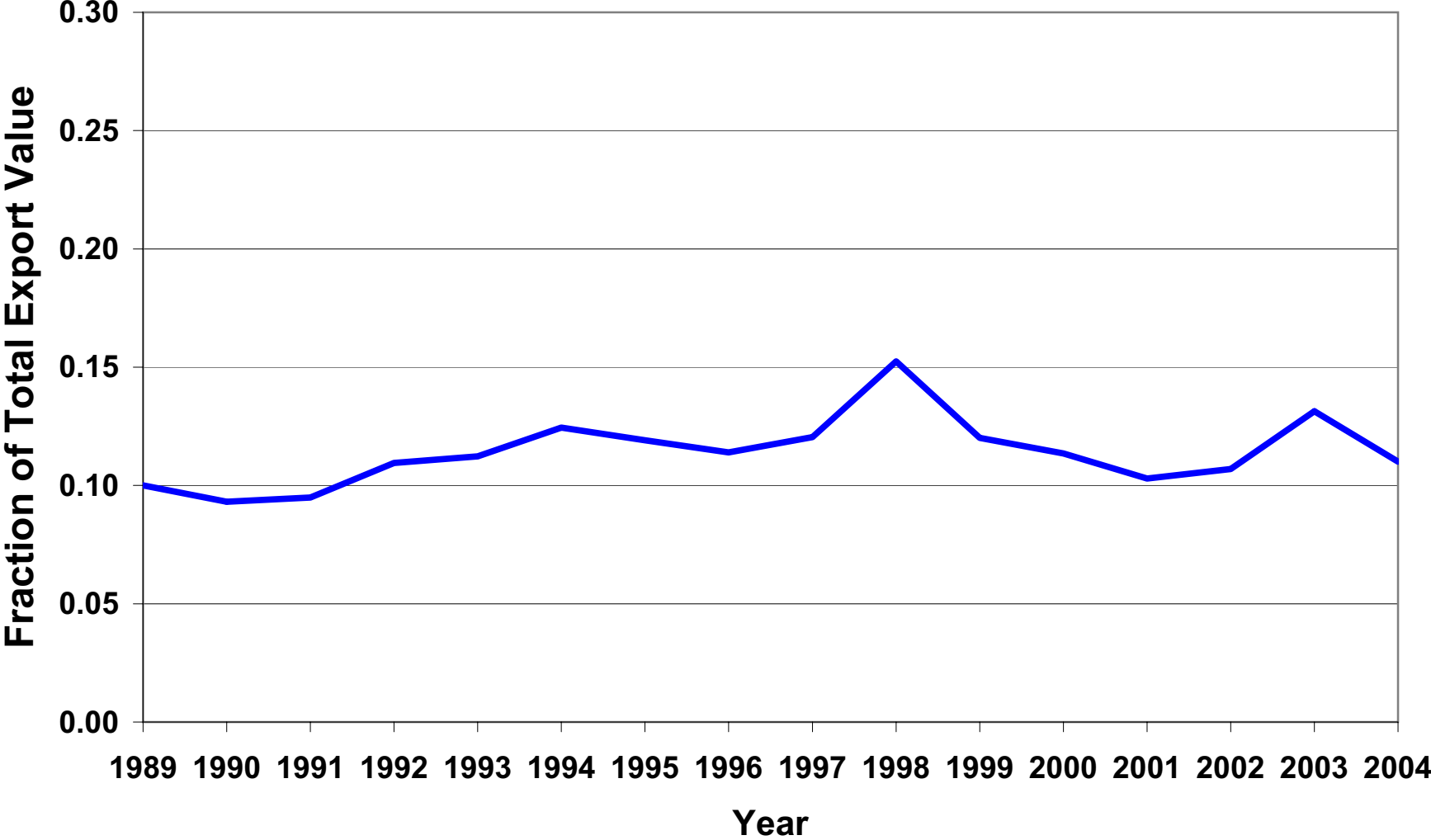
# Exports: Canada to the United Kingdom



## Composition of Exports: United Kingdom to Canada



# Exports: United Kingdom to Canada



## Ricardian model with a continuum of goods $x \in [0,1]$

production technologies  $y(x) = \ell(x)/a(x)$ ,  $y^*(x) = \ell^*(x)/a^*(x)$

*ad valorem* tariffs  $\tau$ ,  $\tau^*$

$$(1 + \tau^*)wa(x) < w^*a^*(x) \Leftrightarrow \frac{a(x)}{a^*(x)} < \frac{w^*}{(1 + \tau^*)w}$$

$\Rightarrow$  home country produces good and exports it to the foreign country.

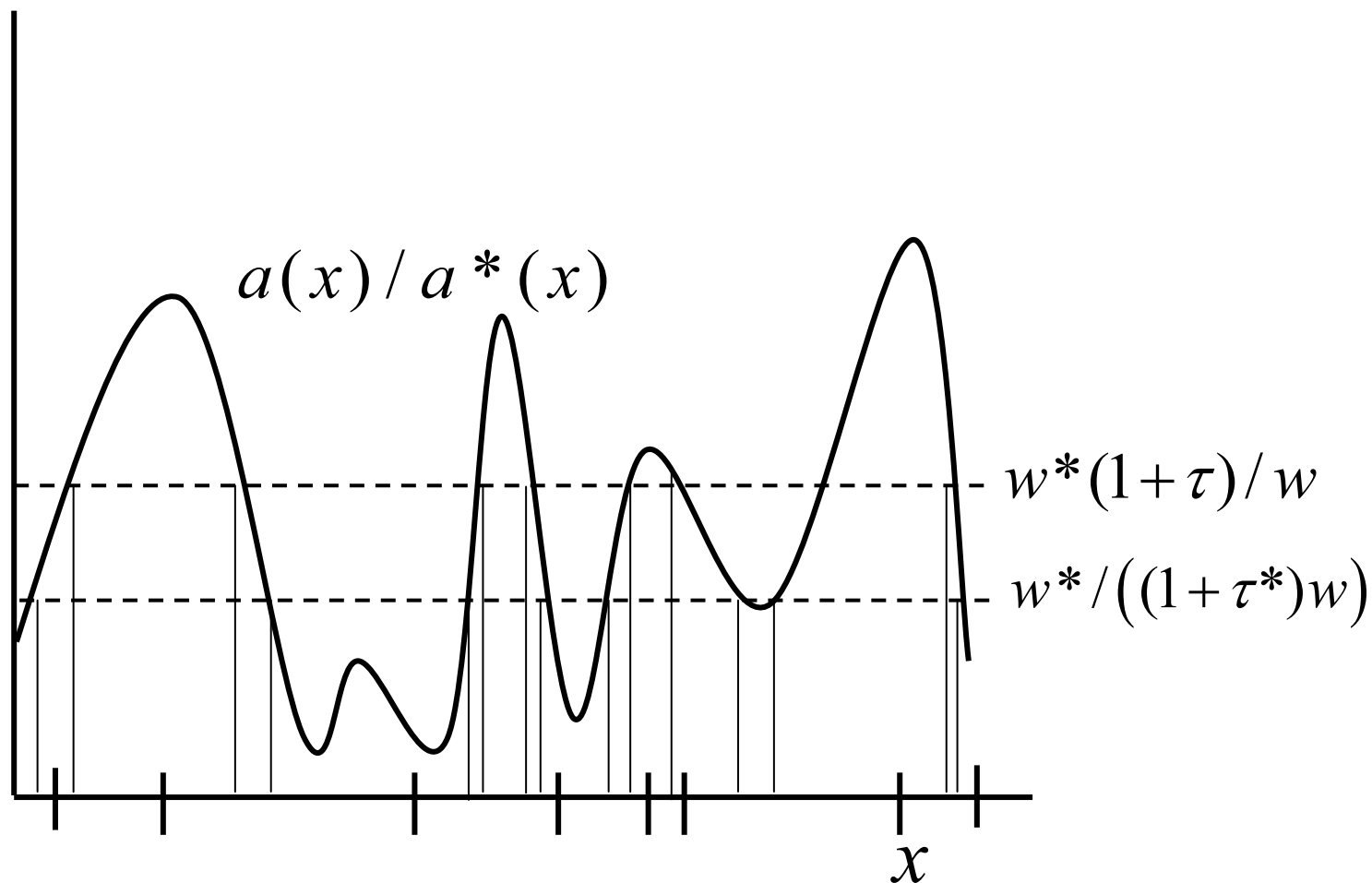
$$\frac{a(x)}{a^*(x)} > \frac{(1 + \tau)w^*}{w}$$

$\Rightarrow$  foreign country produces good and exports it to the home country.

$$\frac{(1 + \tau)w^*}{w} > \frac{a(x)}{a^*(x)} > \frac{w^*}{(1 + \tau^*)w}$$

$\Rightarrow$  good is not traded.

**Lowering tariffs generates trade in previously nontraded goods.**



**4. Fixed costs seem better than Ricardian corner solutions for reconciling time series data on real exchange rate fluctuations with data on trade growth after liberalization experiences.**

K. J. Ruhl, “Solving the Elasticity Puzzle in International Economics,” University of Texas at Austin, 2005.

## The “Armington” Elasticity

- Elasticity of substitution between domestic and foreign goods
- Crucial elasticity in international economic models
- International Real Business Cycle (IRBC) models:
  - Terms of trade volatility
  - Net exports and terms of trade co-movements
- Applied General Equilibrium (AGE) Trade models:
  - Trade response to tariff changes

# The Elasticity Puzzle

- Time series (Business Cycles):
  - Estimates are low
  - Relative prices volatile
  - Quantities less volatile
  
- Panel studies (Trade agreement):
  - Estimates are high
  - Small change in tariffs (prices)
  - Large change in quantities



## **Time Series Estimates: Low Elasticity (1.5)**

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Study	Range
Reinert and Roland Holst (1992)	[0.1, 3.5]
Reinert and Shiells (1993)	[0.1, 1.5]
Gallaway et al. (2003)	[0.2, 4.9]

---

## **Trade Liberalization Estimates: High Elasticity (9.0)**

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Study	Range
Clausing (2001)	[8.9, 11.0]
Head and Reis (2001)	[7.9, 11.4]
Romalis (2002)	[4.0, 13.0]

---

## Why do the Estimates Differ?

- Time series – no liberalization:
  - Change in trade volume from goods already traded
  - Change mostly on the *intensive margin*
- Trade liberalization:
  - Change in intensive margin *plus*
  - New types of goods being traded
  - Change on the *extensive margin*

## **Modeling the Extensive Margin**

- Model: extensive margin from export entry costs
- Empirical evidence of entry costs
  - Roberts and Tybout (1997)
  - Bernard and Wagner (2001)
  - Bernard and Jensen (2003)
  - Bernard, Jensen and Schott (2003)

## The Effects of Entry Costs

- Business cycle shocks:
  - Small extensive margin effect
- Trade liberalization:
  - Big extensive margin effect
- Asymmetry creates different empirical elasticities

## Model Overview

- Two countries:  $\{h, f\}$ , with labor  $L$
- Infinitely lived consumers
- No international borrowing/lending
- Continuum of traded goods plants in each country
  - Differentiated goods
  - Monopolistic competitors
  - Heterogeneous productivity
- Export entry costs
  - Differs across plants: second source of heterogeneity
- Non-traded good, competitive market:  $A$
- Tariff on traded goods (iceberg):  $\tau$

## Uncertainty

- At date  $t$ ,  $H$  possible events,  $\eta_t = 1, \dots, H$
- Each event is associated with a vector of productivity shocks:

$$z_t = [z_h(\eta_t), z_f(\eta_t)]$$

- First-order Markov process with transition matrix  $\Lambda$

$$\lambda_{\eta\eta'} = \text{pr}(\eta_{t+1} = \eta' | \eta_t = \eta)$$

## Traded Good Plants

- Traded good technology:

$$y(\phi, \kappa) = z\phi l$$

- Plant heterogeneity  $(\phi, \kappa)$

- constant, idiosyncratic productivity:  $\phi$
- export entry cost:  $\kappa$
- plant of type  $(\phi, \kappa)$

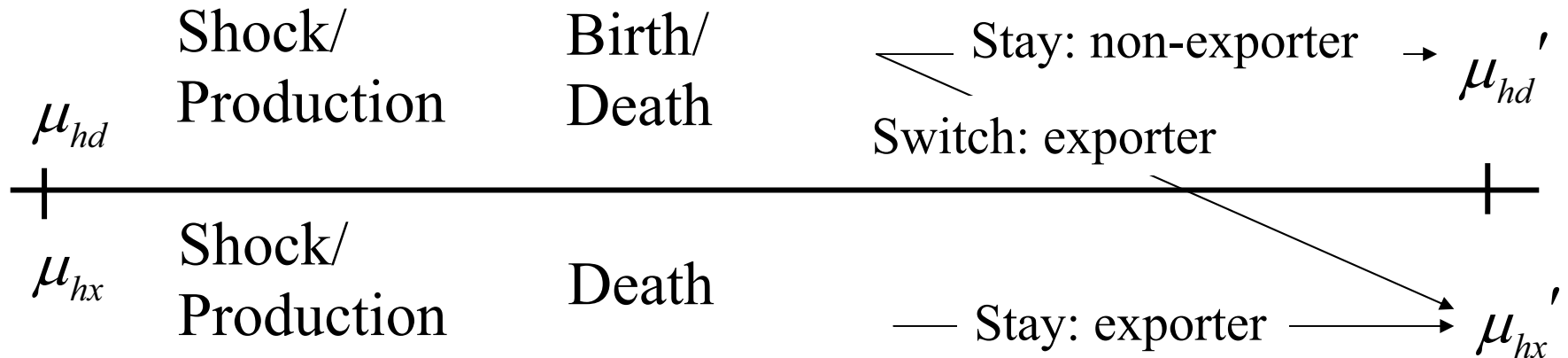
- $\nu$  plants born each period with distribution  $F(\phi, \kappa)$
- Fraction  $\delta$  of plants exogenously die each period

## Timing

$\mu_{hx}(\phi, \kappa)$ : plants of type  $(\phi, \kappa)$  who paid entry cost

$\mu_{hd}(\phi, \kappa)$ : plants of type  $(\phi, \kappa)$  who have not paid entry cost

$$\mu = (\mu_{hd}, \mu_{hx}, \mu_{fd}, \mu_{fx})$$





## Consumers

$$\max_{q, c_h^h(\iota), c_f^h(\iota)} \gamma \log(C) + (1 - \gamma) \log(A)$$

s.t.

$$C = \left[ \int_{\iota \in I_h^h(\mu)} c_h^h(\iota)^\rho d\iota + \int_{\iota \in I_f^h(\mu)} c_f^h(\iota)^\rho d\iota \right]^{\frac{1}{\rho}}$$

$$\int_{\iota \in I_h^h(\mu)} p_h^h(\iota) c_h^h(\iota) d\iota + \int_{\iota \in I_f^h(\mu)} (1 + \tau) p_f^h(\iota) c_f^h(\iota) d\iota + p_{hA} A = L + \Pi_h$$

## Non-traded Good

$$\begin{aligned} \max \quad & p_{hA}(\eta, \mu) A - l \\ \text{s.t.} \quad & A = z_h(\eta) l \end{aligned}$$

Normalize  $w_h = 1$ , implying  $p_{hA}(\eta, \mu) = z_h(\eta)$

## Traded Goods: Static Profit Maximization

$$\pi_d(p_h^h, l; \phi, \kappa, \eta, \mu) = \max_{p_h^h, l} p_h^h z(\eta) \phi l - l$$

$$\text{s.t.} \quad z(\eta) \phi l = \tilde{c}_h^h(p_h^h; \eta, \mu)$$

$$\pi_x(p_h^f, l; \phi, \kappa, \eta, \mu) = \max_{p_h^f, l} p_h^f z(\eta) \phi l - l$$

$$\text{s.t.} \quad z(\eta) \phi l = \tilde{c}_h^f(p_h^f; \eta, \mu)$$

Pricing rules:

$$p_h^h(\phi, \kappa, \eta, \mu) = p_h^f(\phi, \kappa, \eta, \mu) = \frac{1}{\rho \phi z(\eta)}$$

## Dynamic Choice: Export or Sell Domestically

- Exporter's Value Function:

$$V_x(\phi, \kappa, \eta, \mu) = d(\eta, \mu) \left( \pi_d(\phi, \kappa, \eta, \mu) + \pi_x(\phi, \kappa, \eta, \mu) \right) \\ + (1 - \delta) \beta \sum_{\eta'} V_x(\phi, \kappa, \eta', \mu') \lambda_{\eta\eta'}$$

$$\text{s.t. } \mu' = M(\eta, \mu)$$

- $d(\eta, \mu)$  = multiplier on budget constraint

- Non-exporter's Value Function:

$$V_d(\phi, \kappa, \eta, \mu) = \max \left\{ \begin{aligned} &\pi_d(\phi, \kappa, \eta, \mu) d(\eta, \mu) + \beta(1 - \delta) \sum_{\eta'} V_d(\phi, \kappa, \eta', \mu') \lambda_{\eta\eta'}, \\ &[\pi_d(\phi, \kappa, \eta, \mu) - \kappa] d(\eta, \mu) + \beta(1 - \delta) \sum_{\eta'} V_x(\phi, \kappa, \eta', \mu') \lambda_{\eta\eta'} \end{aligned} \right\}$$

$$\text{s.t. } \mu' = M(\eta, \mu)$$

## Equilibrium

- Cutoff level of productivity for each value of the entry cost

- For a plant of type  $(\phi, \kappa)$

If  $\phi \geq \hat{\phi}_{\kappa}(\eta, \mu)$  export and sell domestically

If  $\phi < \hat{\phi}_{\kappa}(\eta, \mu)$  only sell domestically

- In Equilibrium

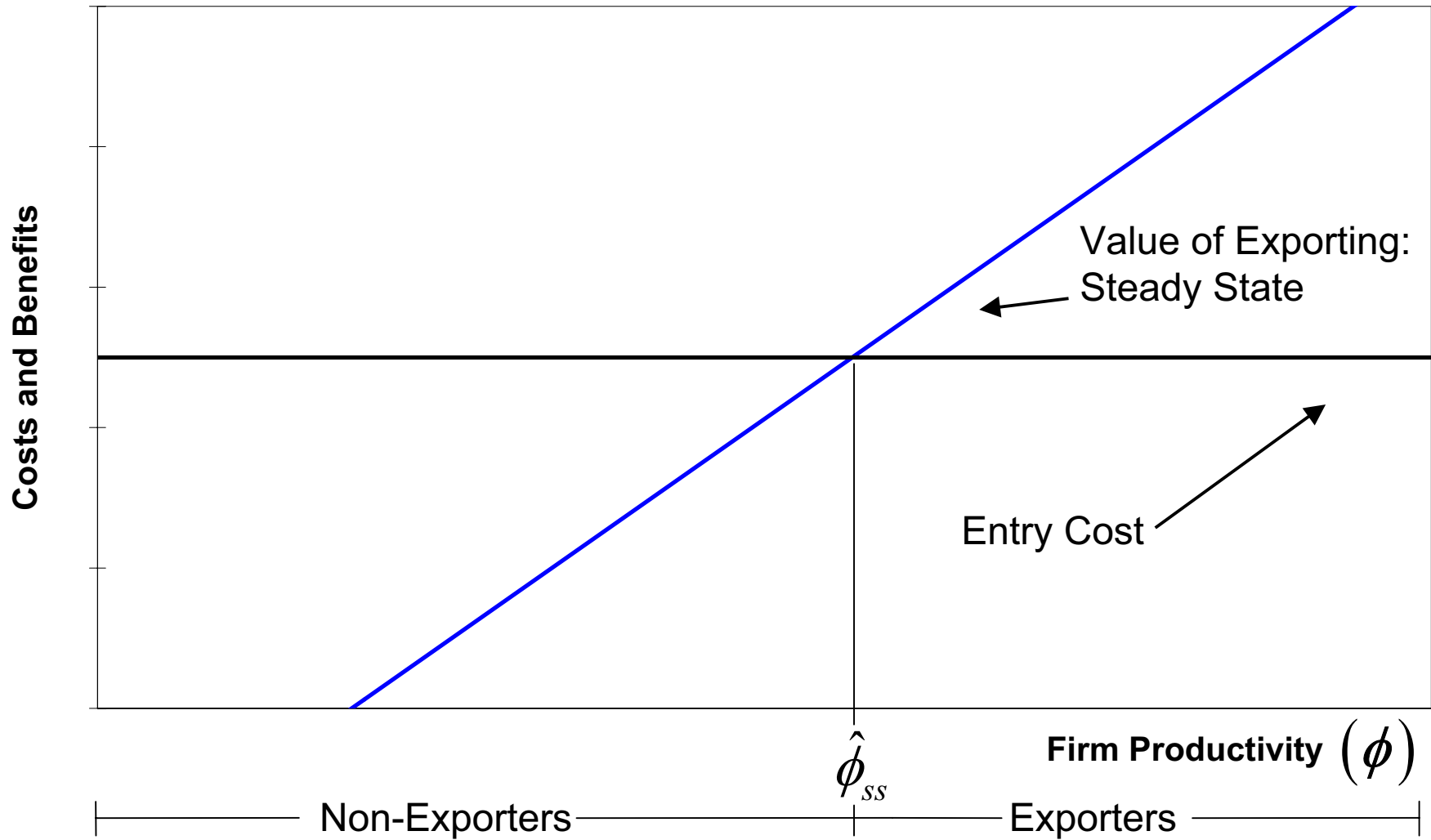
- “Low” productivity/“high” entry cost plants sell domestic
- “High” productivity/“low” entry cost plants also export
- Similar to Melitz (2003)

## Determining Cutoffs

- For the cutoff plant:
  - entry cost = discounted, expected value of exporting
- $\hat{\phi}_\kappa(\eta, \mu)$  is the level of productivity,  $\phi$ , that solves:

$$\underbrace{d(\eta, \mu)\kappa}_{\text{entry cost}} = (1 - \delta)\beta \underbrace{\left[ \sum_{\eta'} V_x(\phi, \kappa, \eta', \mu') \lambda_{\eta\eta'} - \sum_{\eta'} V_d(\phi, \kappa, \eta', \mu') \lambda_{\eta\eta'} \right]}_{\text{expected value of exporting}}$$

## Finding the Cutoff Producer





## Choosing Parameters

- Set  $\sigma = \frac{1}{1-\rho} = 2$  and  $\tau = 0.15$
- Calibrate to the United States (1987) and a symmetric partner.

---

### Parameters

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$\beta$	Annual real interest rate (4%)
$\gamma$	Share of manufactures in GDP (18%)
$\delta$	Annual loss of jobs from plant deaths as percentage of employment (Davis et. al., 1996) (6%)

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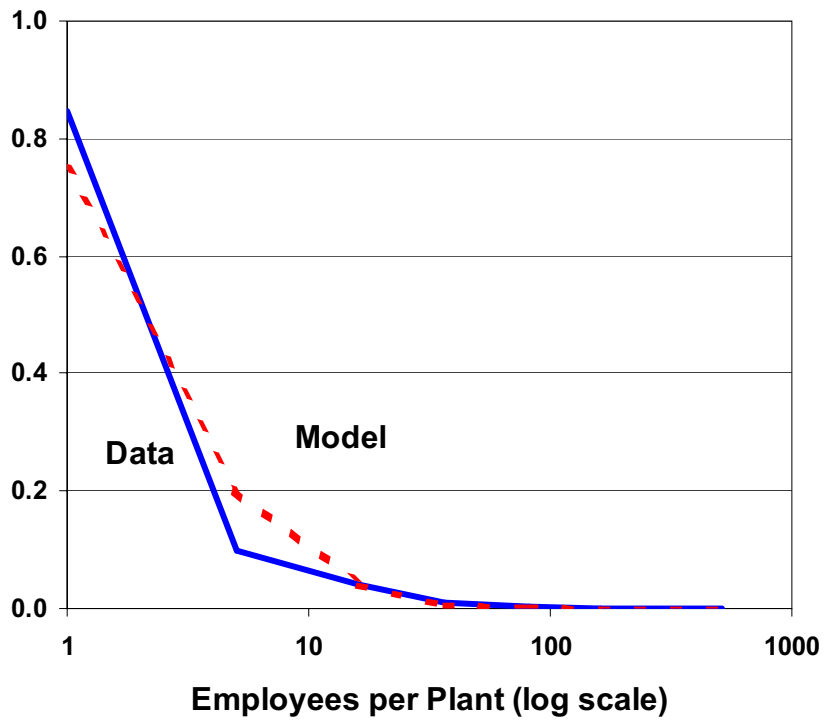
## Other Parameters

- Distribution over new plants:

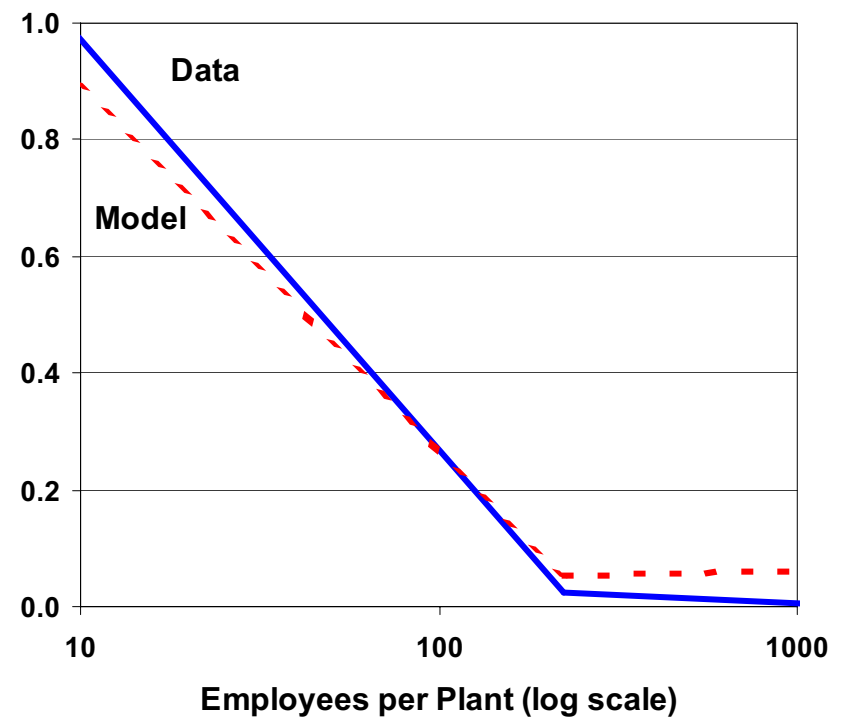
$$F_{\kappa}(\phi) = \frac{1}{\phi^{\theta_{\phi}}} \quad F_{\phi}(\kappa) = \frac{1}{(\bar{\kappa} - \kappa)^{\theta_{\kappa}}}$$

- $\bar{\kappa}, \bar{\phi}, \nu, \theta_{\phi}, \theta_{\kappa}$  jointly determine:
  - Average plant size (12 employees)
  - Standard deviation of plant sizes (892)
  - Average exporting plant size (15 employees)
  - Standard deviation of exporting plant sizes (912)
  - Fraction of production that is exported (9%)

**Plant Size Distribution:  
All Plants**



**Plant Size Distribution:  
Exporting Plants**



## Productivity Process

- Two shocks, low and high:

$$z_i = 1 - \varepsilon$$

$$z_i = 1 + \varepsilon$$

- Countries have symmetric processes with Markov Matrix

$$\Lambda_i = \begin{bmatrix} \bar{\lambda} & 1 - \bar{\lambda} \\ 1 - \bar{\lambda} & \bar{\lambda} \end{bmatrix}$$

- $\varepsilon$ : standard deviation of the U.S. Solow Residuals (1.0%)
- $\bar{\lambda}$ : autocorrelation of the U.S. Solow Residuals (0.90)

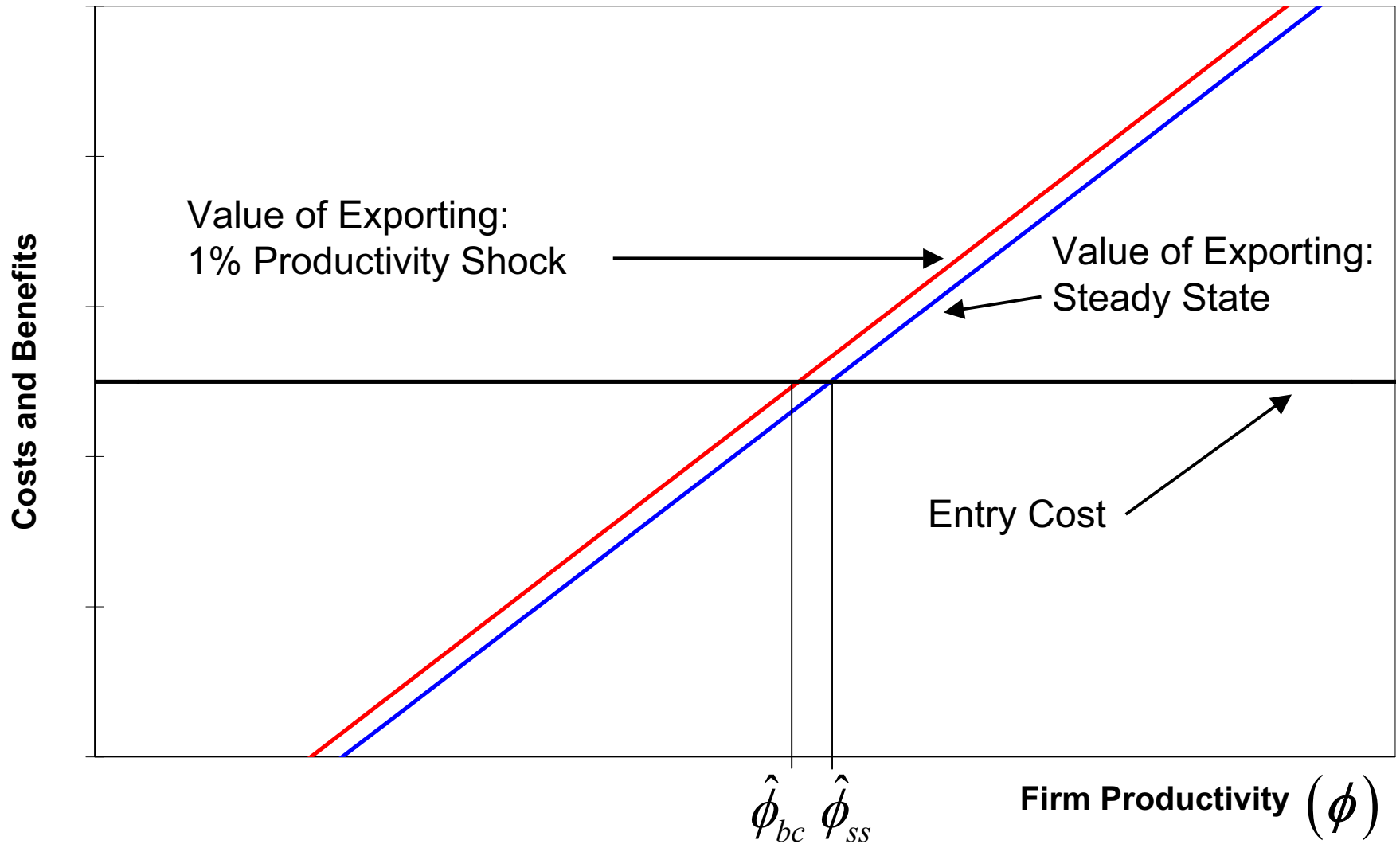
## **How does Trade Liberalization Differ from Business Cycles?**

- Trade liberalization
  - Permanent changes
  - Large magnitudes
- Business cycles
  - Persistent, but not permanent changes
  - Small magnitudes

## **Developing Intuition: Persistent vs. Permanent Shocks**

- 1% positive productivity shock in foreign country
  - Shock is persistent – autocorrelation of 0.90
- 1% decrease in tariffs
  - Change in tariffs is permanent

**Response to 1% Productivity Shock  
Autocorrelation = 0.90**



## Response to a 1% Foreign Productivity Shock

Increase in imports on intensive margin = 1.89%

Increase in imports on extensive margin = 0.16%

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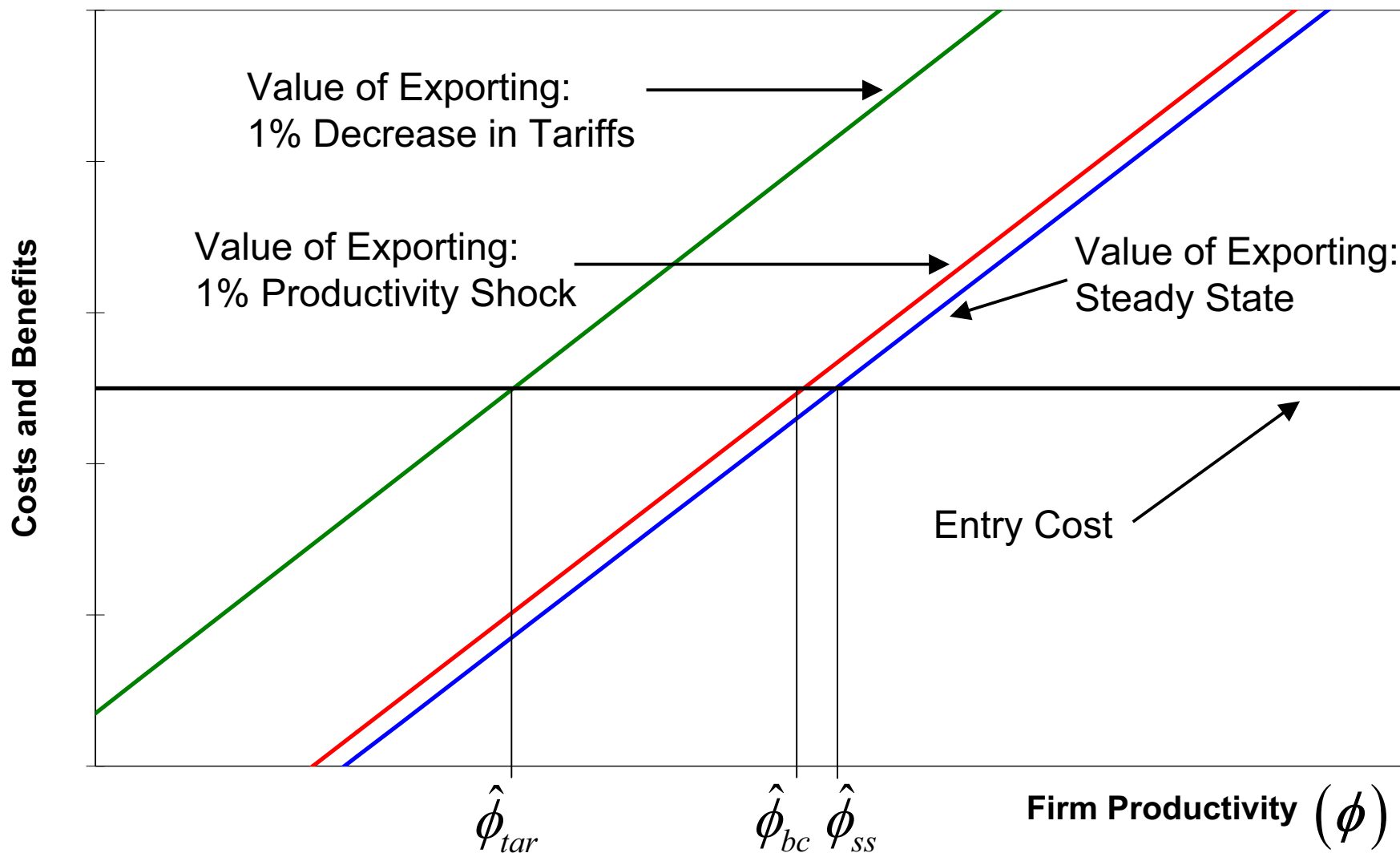
Total increase in imports = 2.05%

Change in consumption of home goods = -0.10%

$$\frac{\% \text{ Change Imports/Dom. Cons.}}{\% \text{ Change Price}} = \frac{2.17}{0.99} = 2.19$$



## Response to 1% Permanent Decrease in Tariffs



## Response to a 1% Tariff Reduction

Increase in imports on intensive margin = 1.42%

Increase in imports on extensive margin = 3.04%

---

Total increase in imports = 4.46%

Change in consumption of home goods = -0.33%

$$\frac{\% \text{ Change Imports/Dom. Cons.}}{\% \text{ Change Tariff}} = \frac{4.81}{1.00} = 4.81$$

## Quantitative Results

- Two experiments
- Trade liberalization
  - Eliminate 15% tariff
  - Compute elasticity across tariff regimes
- Time series regressions
  - Use model to generate simulated data
  - Estimate elasticity as in the literature

## Trade Liberalization Elasticity

<b>Variable</b>	<b>Entry Costs</b> (% change)	<b>No Entry Costs</b> (% change)
Exports	87.1	30.5
Imports/Dom. Cons.	93.0	32.2
Exporting Plants	37.7	0.0
Implied Elasticity	6.2	2.1

## Elasticity in the Time Series

- Simulate: produce price/quantity time series
- Regress:

$$\log\left(C_{f,t} / C_{h,t}\right) = \alpha + \sigma \log\left(p_{h,t} / p_{f,t}\right) + \varepsilon_t$$

---

Parameter	Estimate
$\alpha$ (standard error)	-0.015 (6.36e-04)
$\sigma$ (standard error)	1.39 (0.06)
R- squared	0.30

---

## Conclusion

- Gap between dynamic macro models and trade models
  - Partially closes the gap
  - Modeling firm behavior as motivated by the data
  - Step towards better modeling of trade policy
  
- Single model can account for the elasticity puzzle
  - Time series elasticity of 1.4
  - Trade liberalization elasticity of 6.2

**5. Models of trade with heterogeneous firms imposed fixed costs on firms that decide to export. The focus is on the decision to export. The theory and the data indicate that there is a lot of room for focusing on the decision to import.**

A. Ramanarayanan, “International Trade Dynamics with Intermediate Inputs,” University of Minnesota, 2006.  
<http://www.econ.umn.edu/~tkehoe/papers/Ramanarayan.pdf>.

## **Motivation**

Dynamics of international trade flows

Long-run: Large, gradual changes  
(tariff reform)

Short-run: Small changes  
(fluctuations in relative prices)

Standard Theory: does not capture difference

Constant elasticity of substitution between imports and domestic goods



## **Question**

What accounts for slow-moving dynamics of international trade flows?

## **This Paper's Answer**

Trade in intermediate inputs

Costly, irreversible importing decision at producer-level

## **Previous Literature's Answers**

Lags or costs of adjustment: contracting / distribution

Parameterize to generate slow-moving dynamics

This paper's contribution:

Model mechanism based on micro-level evidence

Quantitative test of theory:

Endogenous aggregate dynamics in line with data

## **Significance of Results**

Effects of trade reform

1. Timing and magnitude of trade growth
2. Welfare gains

## **Data: Aggregate Dynamics**

Armington (1969) elasticity: elasticity of substitution between aggregate imported and domestic goods

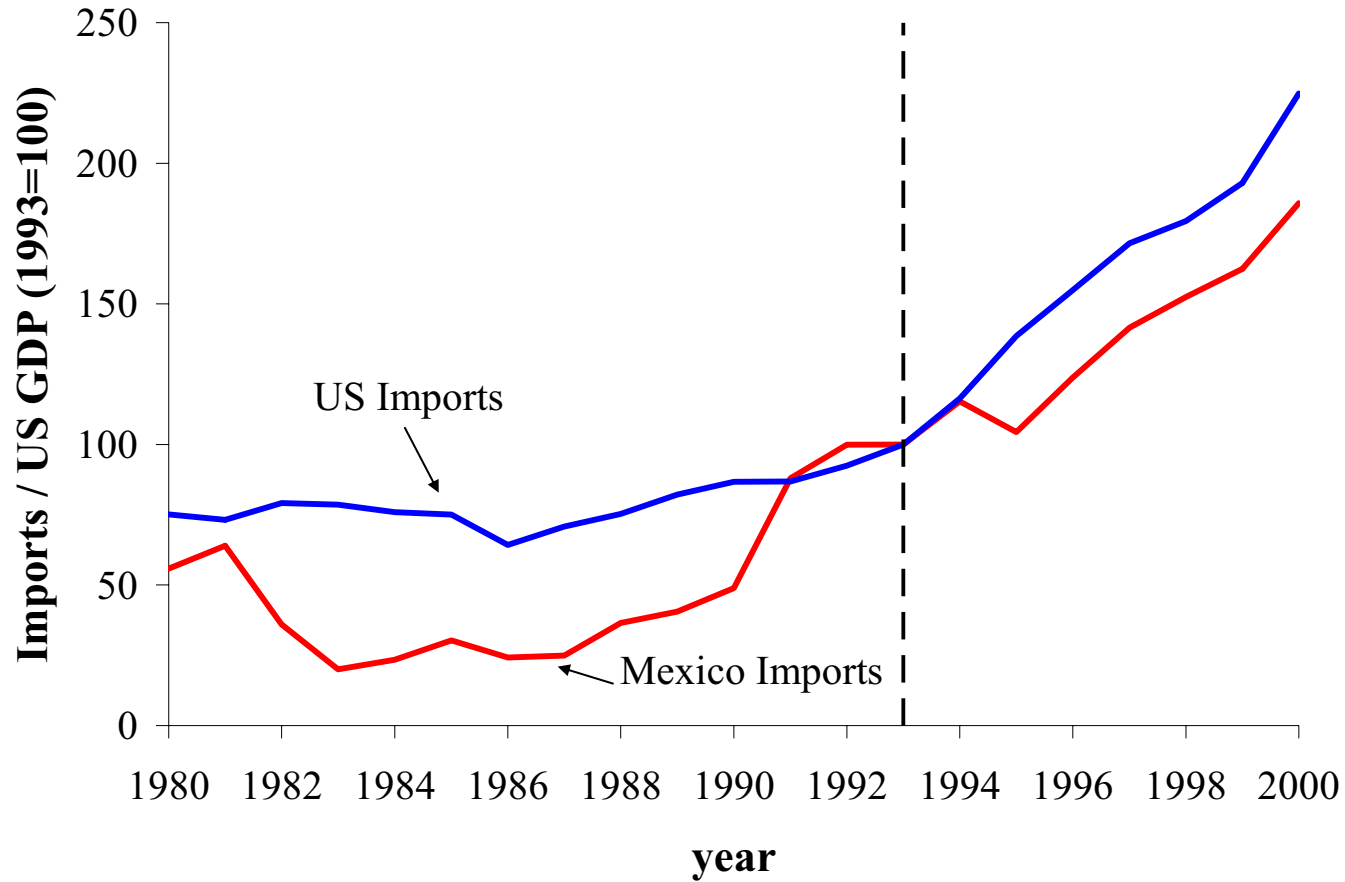
Low estimates from time-series data ( $< 2$ )

High estimates from trade liberalization ( $> 6$ )

## Data: Aggregate Dynamics

Gradual increase in trade after liberalization

NAFTA (Jan 1, 1994)



## **Data: Plant-level**

### *Cross-section*

Not all plants use imported intermediate inputs

Importing plants larger than non-importing plants

### *Panel*

Reallocation between importers / non-importers is significant

## Data: Plant-level Cross-section

		% use imports	Avg. size ratio to non-importers
Chile	average 1979-86	24.1	3.4
US (Kurz, 2006)	1992	23.8	2.3

## Data: Plant-level Dynamics

Decompose changes in aggregate trade volumes

e.g., increase in aggregate imported/total inputs due to:

1. Importers increase ratio (*Within*) +
2. Importers expand, non-importers shrink (*Between*) +
3. Interaction between the two (*Cross*) +
4. Non-importers switch to importing (*Switch*) +
5. Higher proportion of new entrants are importers (*Entry*)

Baily, Hulten, Campbell (1992): productivity growth

## Data: Plant-level Dynamics

Imported / Total Intermediate Inputs: Chile, 1979-1986

	TOTAL	Fraction of Total (%)				
		Within	Between	Cross	Switch	Entry
Avg of 1-year changes	-18%	<b>79</b>	<b>26</b>	<b>-10</b>	<b>3</b>	<b>2</b>
7-year change	-77%	<b>74</b>	<b>42</b>	<b>-30</b>	<b>5</b>	<b>10</b>



# Model

## Heterogeneous Plants

Produce using intermediate inputs

Importing costly, irreversible

Trade growth through *Between* and *Entry* margins

2-country, 2-good real business cycle model

Technology shocks: short-run changes

Tariff reduction: long-run changes

## Time and Uncertainty

Dates  $t = 0, 1, 2, \dots$

Event at date  $t$ :  $s_t$ . State at date  $t$ :  $s^t = (s_0, s_1, \dots, s_t)$ .

$$\Pr(s_t | s^{t-1}) = \phi(s_t | s_{t-1})$$

$$\tilde{\phi}(s^t) = \phi(s_t | s_{t-1})\phi(s_{t-1} | s_{t-2}) \cdots \phi(s_1 | s_0)$$

Commodities and prices are functions  $x(s^t) \rightarrow x_t$

Technology shocks  $A(s^t), A^*(s^t)$

# Representative Consumer

Preferences:

$$E \sum_{t=0}^{\infty} \beta^t U(C_t, 1 - N_t) = \sum_{t=0}^{\infty} \sum_{s^t} \beta^t \tilde{\phi}(s^t) U(C(s^t), 1 - N(s^t))$$

Budget constraint:

$$C_t + \sum_{s_{t+1}} Q(s^t, s_{t+1}) B(s^t, s_{t+1}) \leq w_t N_t + B(s^t) + \Pi_t + T_t$$

Consumer owns plants

## Plants

Heterogeneous in inherent efficiency  $z$ .

Aggregate technology shocks  $A_t$

Within each country, produce homogeneous output

Perfectly competitive, decreasing returns to scale technologies

Two types of decisions

1. Existing plants: static profit maximization
2. New plants: technology choice (import or not)

## Plant technologies

Non-importing

$$f_d(n, d; z) = z^{1-\alpha-\theta} d^\alpha n^\theta$$

Importing

$$f_m(n, d, m; z) = z^{1-\alpha-\theta} \left( \gamma \min \left\{ \frac{d}{\omega}, \frac{m}{1-\omega} \right\} \right)^\alpha n^\theta$$

$$\alpha + \theta < 1, \quad \omega < 1,$$

$\gamma$ : efficiency gain from importing

## Static profit maximization

Non-importing plant with efficiency  $z$  operating at date  $t$

$$\pi_{dt}(z) = \max_{n,d} A_t f_d(n, d; z) - w_t n - d$$

Importing plant

$$\pi_{mt}(z) = \max_{n,d,m} A_t f_m(n, d, m; z) - w_t n - d - (1 + \tau) p_t m$$

No dependence on date of entry

## Plant technologies, costs

Non-importing

$$f_d(n, d; z) = z^{1-\alpha-\theta} d^\alpha n^\theta$$

Price of intermediate input: 1

Importing

$$f_m(n, d, m; z) = z^{1-\alpha-\theta} \left( \gamma \min \left\{ \frac{d}{\omega}, \frac{m}{1-\omega} \right\} \right)^\alpha n^\theta$$

Price of composite intermediate input:  $\frac{1}{\gamma} (\omega + (1 + \tau) p_t (1 - \omega))$

## Plant technologies, costs

Importing technology is more cost-efficient if

$$\gamma > \omega + (1 + \tau)p_t(1 - \omega)$$

Depends on equilibrium price  $p_t$

Estimate  $\gamma$  from plant data

Check that inequality holds along equilibrium path



## Dynamic problem: Timing

Plant pays cost  $\kappa_e$  to get a draw of  $z$  from distribution  $g$

Decide whether to start producing or exit

Pay sunk investment

$\kappa_c$  to use non-importing technology, or

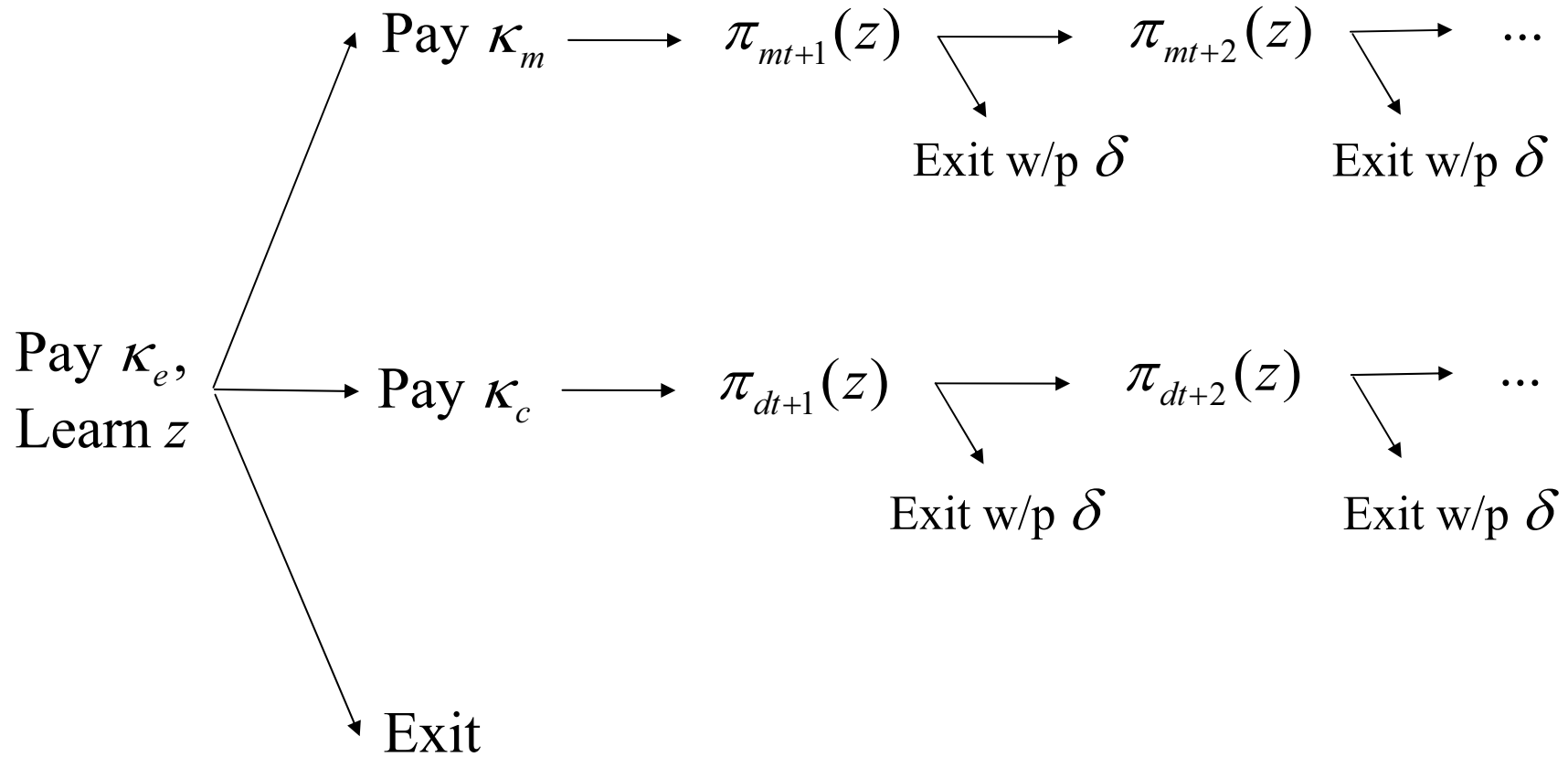
$\kappa_m$  to use importing technology

$$\kappa_m > \kappa_c$$

Face static profit maximization problem each period

Probability  $\delta$  of exit after production each period

# Timing: Plant Entering at date $t$



## Dynamic Problem: Plant entering at date $t$

Present values of static profits:

$$V_{dt}(z) = E_t \sum_{k=1}^{\infty} (1-\delta)^{k-1} P_{t,t+k} \pi_{dt+k}(z)$$

$$V_{mt}(z) = E_t \sum_{k=1}^{\infty} (1-\delta)^{k-1} P_{t,t+k} \pi_{mt+k}(z)$$

with  $P_{t,t+k} = \beta^k \frac{U_{C_{t+k}}}{U_{C_t}}$  (consumer owns plants)

## Technology Choice

$$V_t(z) = \max \{0, -\kappa_c + V_{dt}(z), -\kappa_m + V_{mt}(z)\}$$

Produce using non-importing technology if

$$-\kappa_c + V_{dt}(z) > \max \{0, -\kappa_m + V_{mt}(z)\}$$

Produce using importing technology if

$$-\kappa_m + V_{mt}(z) > \max \{0, -\kappa_c + V_{dt}(z)\}$$

Otherwise exit

## Technology Choice

$V_{dt}(z)$  and  $V_{mt}(z) - V_{dt}(z)$  increasing in  $z$

Cutoffs  $\hat{z}_{dt}$  and  $\hat{z}_{mt}$ ,

$$V_{dt}(\hat{z}_{dt}) = \kappa_c$$

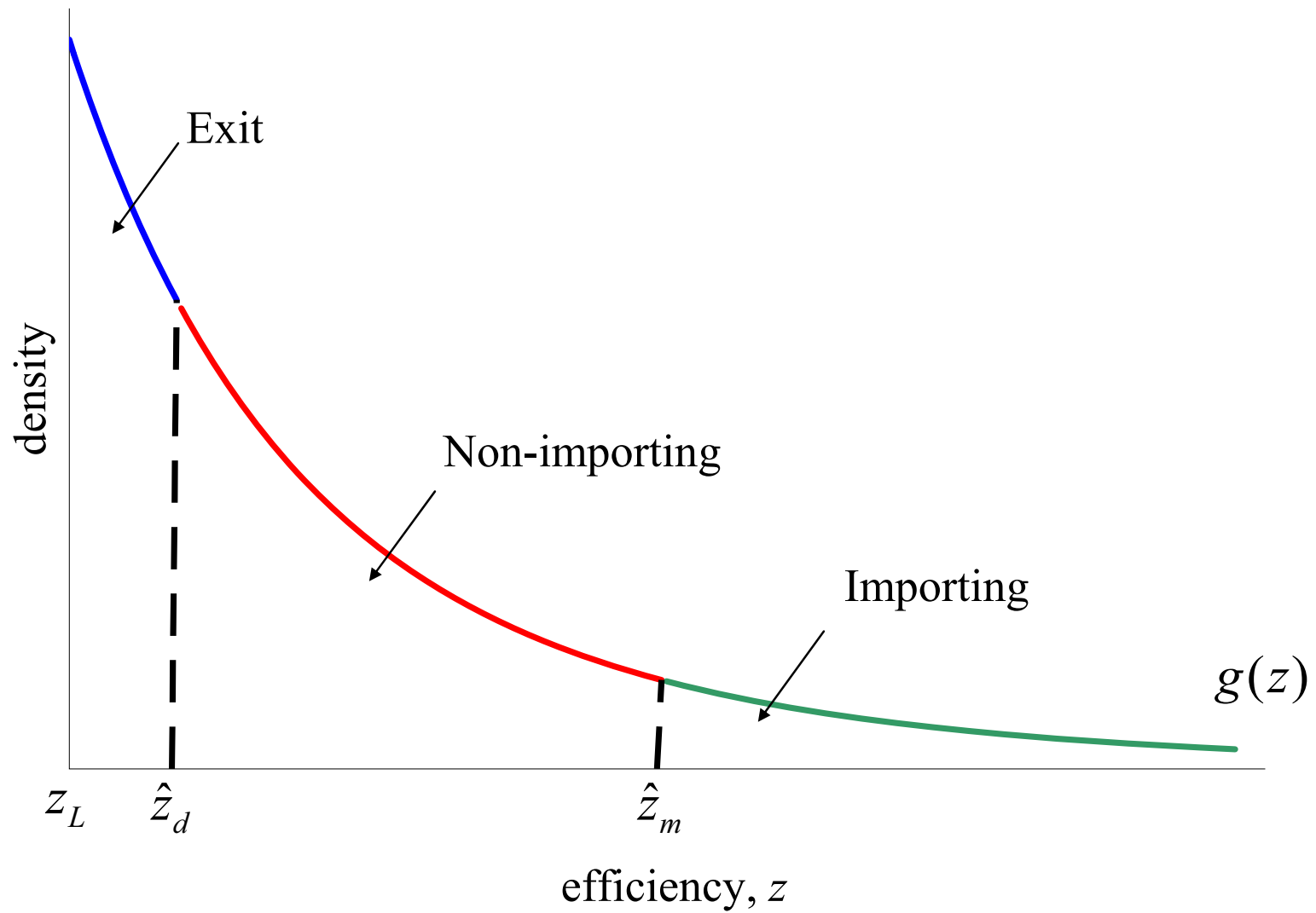
$$V_{mt}(\hat{z}_{mt}) - V_{dt}(\hat{z}_{mt}) = \kappa_m$$

Use importing technology if  $z \in [\hat{z}_{mt}, \infty)$

Use non-importing technology if  $z \in [\hat{z}_{dt}, \hat{z}_{mt})$

Otherwise exit

# Technology Choice: cutoffs



## Equilibrium Conditions: Plant Dynamics

$\mu_{dt}(z)$ : Mass of non-importing plants, efficiency  $z$  at date  $t$ .

$X_t$ : Mass of entrants at date  $t$  (start producing at date  $t + 1$ )

Dynamics of distribution:

$$\mu_{dt+1}(z) = \begin{cases} (1 - \delta)\mu_{dt}(z) + X_t g(z) & \text{if } z \in [\hat{z}_{dt}, \hat{z}_{mt}] \\ (1 - \delta)\mu_{dt}(z) & \text{otherwise} \end{cases}$$

## Equilibrium Conditions: Plant Dynamics

$\mu_{mt}(z)$ : Mass of importing plants, efficiency  $z$  at date  $t$ .

$X_t$ : Mass of entrants at date  $t$  (start producing at date  $t + 1$ )

Dynamics of distribution:

$$\mu_{mt+1}(z) = \begin{cases} (1 - \delta)\mu_{mt}(z) + X_t g(z) & \text{if } z > \hat{z}_{mt} \\ (1 - \delta)\mu_{mt}(z) & \text{otherwise} \end{cases}$$



## Equilibrium Conditions: Feasibility

Goods

$$\begin{aligned} & C_t + X_t \left( \kappa_e + \kappa_c \int_{\hat{z}_{dt}}^{\hat{z}_{mt}} g(z) dz + \kappa_m \int_{\hat{z}_{mt}}^{\infty} g(z) dz \right) \\ & + \int d_{dt}(z) \mu_{dt}(z) dz + \int d_{mt}(z) \mu_{mt}(z) dz + \int m_t^*(z) \mu_{mt}^*(z) dz \\ & = \int y_{dt}(z) \mu_{dt}(z) dz + \int y_{mt}(z) \mu_{mt}(z) dz \end{aligned}$$

Labor

$$\int n_{dt}(z) \mu_{dt}(z) dz + \int n_{mt}(z) \mu_{mt}(z) dz = N_t$$

## Equilibrium Conditions: Free Entry and Asset Market

Expected value of entry is

$$V_{et} = -\kappa_e + \int_{z_L}^{\infty} V_t(z)g(z)dz$$

Free Entry:

$$V_{et} \leq 0, \quad = \text{ if } X_t > 0$$

Asset Market Clearing:

$$B(s^t) + B^*(s^t) = 0$$

## Aggregation

To solve equilibrium conditions, need  $\mu_{dt}(\bullet)$ ,  $\mu_{mt}(\bullet)$

For example:  $\int n_{dt}(z)\mu_{dt}(z)dz$

Let  $Z_{dt} = \int z\mu_{dt}(z)dz$

Plants make decisions proportional to efficiency  $z$ :

$$n_{dt}(z) = \tilde{n}_{dt} \times z$$

So,

$$\int n_{dt}(z)\mu_{dt}(z)dz = \tilde{n}_{dt}Z_{dt}$$

## Aggregation

Replace  $\mu_{dt}(\bullet)$  with  $Z_{dt}$  as state variable:

$$\mu_{dt+1}(z) = \begin{cases} (1 - \delta)\mu_{dt}(z) + X_t g(z) & \text{if } z \in [\hat{z}_{dt}, \hat{z}_{mt}] \\ (1 - \delta)\mu_{dt}(z) & \text{otherwise} \end{cases}$$

⇓

$$Z_{dt+1} = (1 - \delta)Z_{dt} + X_t \int_{\hat{z}_{dt}}^{\hat{z}_{mt}} g(z) dz$$

Same with  $\mu_{mt}(\bullet)$ ,  $\mu_{dt}^*(\bullet)$ ,  $\mu_{mt}^*(\bullet)$

## **Analysis of Model**

1. Aggregate imported / domestic intermediate ratio – what determines substitutability?

Static allocation across plants

Investment decisions of new plants

2. Quantitative analysis

Parameterization

Business Cycle simulation – short-run elasticity

Trade Reform – long-run elasticity; speed of trade growth

## **Import / domestic ratio**

Plant level:

Non-importing plant: fixed, *zero*.

Importing plant: fixed,  $\frac{m_t(z)}{d_{mt}(z)} = \frac{1-\omega}{\omega}$

## Import / domestic ratio

Aggregate:

$$\begin{aligned}\frac{M_t}{D_{mt} + D_{dt}} &= \frac{\tilde{m}_t Z_{mt}}{\tilde{d}_{mt} Z_{mt} + \tilde{d}_{dt} Z_{dt}} \\ &= \frac{\frac{1-\omega}{\omega} \tilde{d}_{mt} Z_{mt}}{\tilde{d}_{mt} Z_{mt} + \tilde{d}_{dt} Z_{dt}}\end{aligned}$$

Increasing in:

$\frac{\tilde{d}_{mt}}{\tilde{d}_{dt}}$  : non-importing / importing plant with same  $z$ ;

$\frac{Z_{mt}}{Z_{dt}}$  : mass of importers / non-importers ( $z$ -weighted)

## Effects of increase in relative price $(1 + \tau)p_t$ :

1. At date  $t$ : allocation between plants,

$$\frac{\tilde{d}_{mt}}{\tilde{d}_{dt}} = \left( \frac{\gamma}{\omega + (1 + \tau)p_t(1 - \omega)} \right)^{\alpha/(1 - \alpha - \theta)}$$

Decreasing in  $(1 + \tau)p_t$

Importers less profitable; allocated less inputs in equilibrium



**Effects of increase in relative price  $(1 + \tau)p_t$  if persistent:**

2. At date  $t + 1$ : new plants *entering at date t*,

$$\frac{Z_{mt+1}}{Z_{dt+1}} = \frac{(1 - \delta)Z_{mt} + X_t \int_{\hat{z}_{mt}}^{\infty} g(z)dz}{(1 - \delta)Z_{dt} + X_t \int_{\hat{z}_{dt}}^{\hat{z}_{mt}} g(z)dz}$$

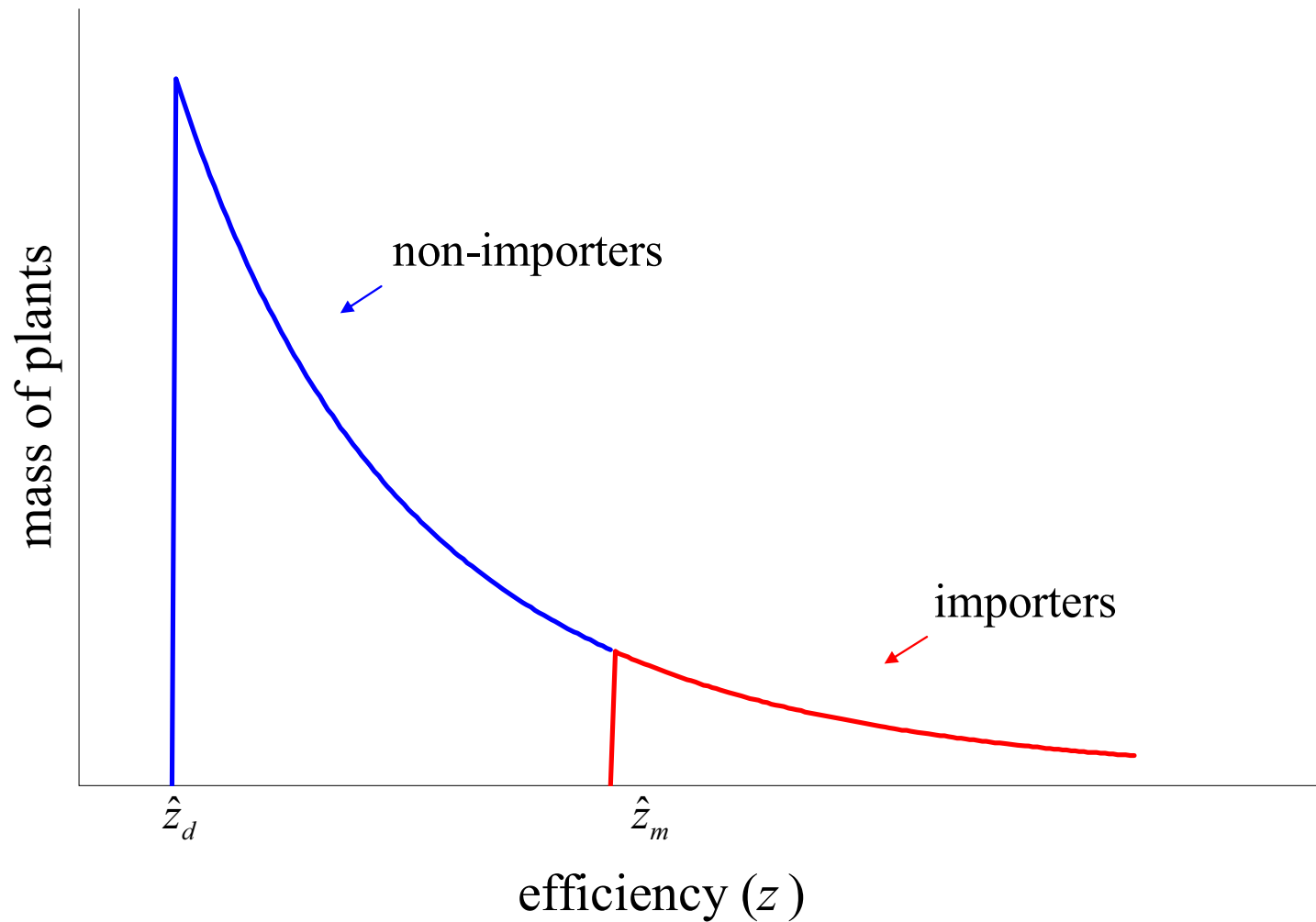
Decreasing in  $(1 + \tau)p_t$

Importing less profitable; fewer new plants choose importing.

$$\hat{z}_{mt} \downarrow, \hat{z}_{dt} \uparrow$$

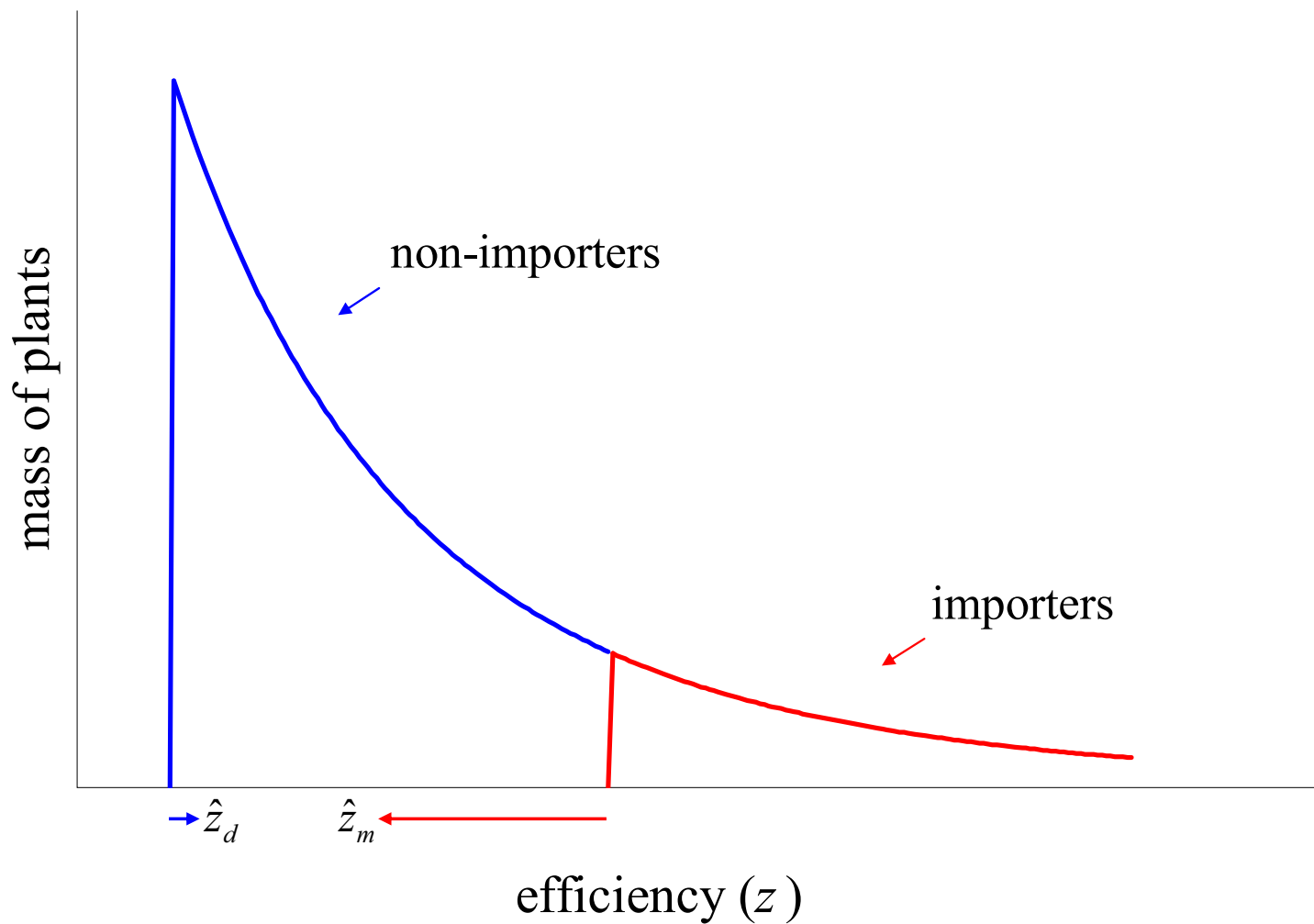
# Dynamic effect of decrease in $(1 + \tau)p_t$

Distribution of Plants,  $t$



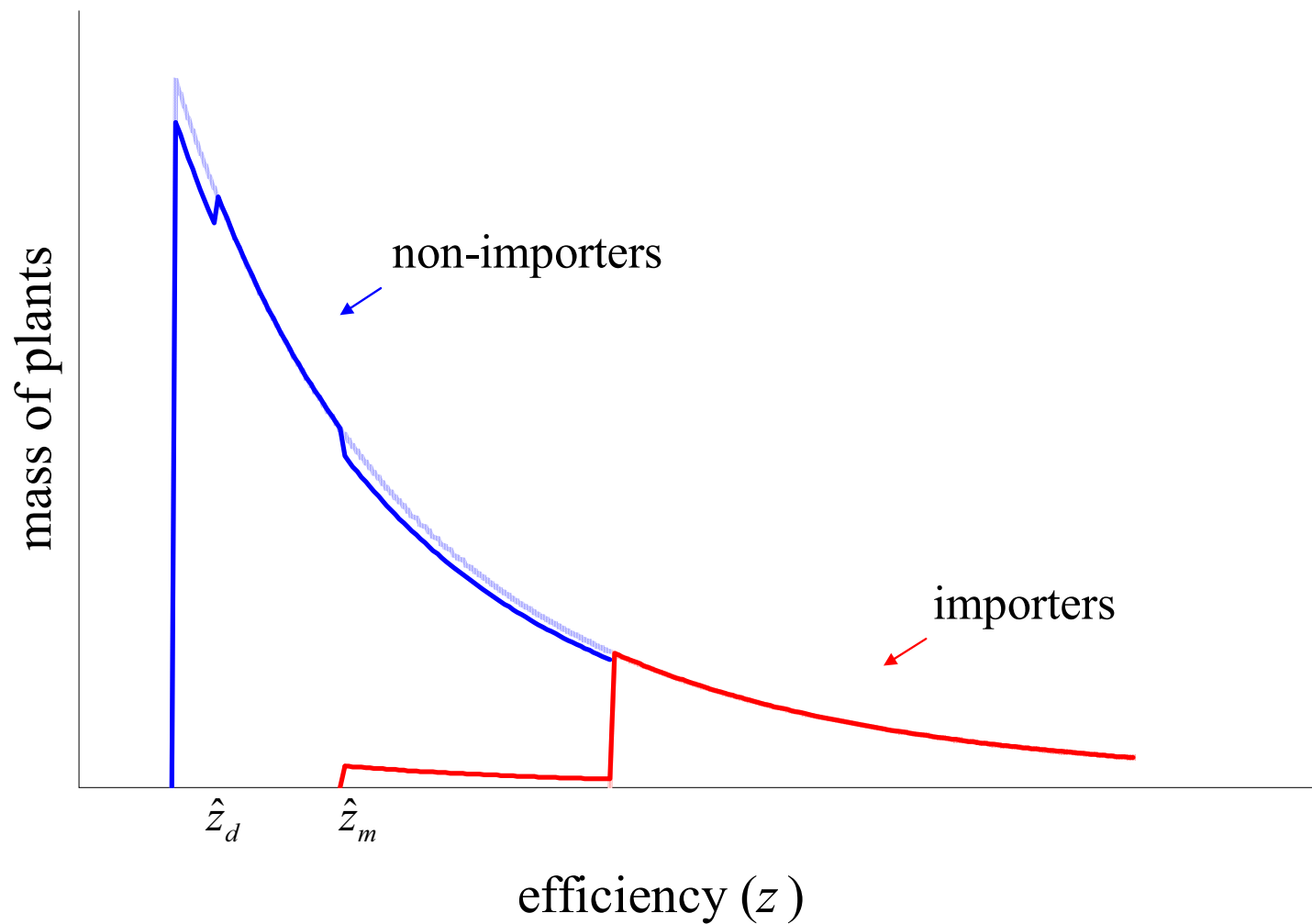
# Dynamic effect of decrease in $(1 + \tau)p_t$

Distribution of Plants,  $t$



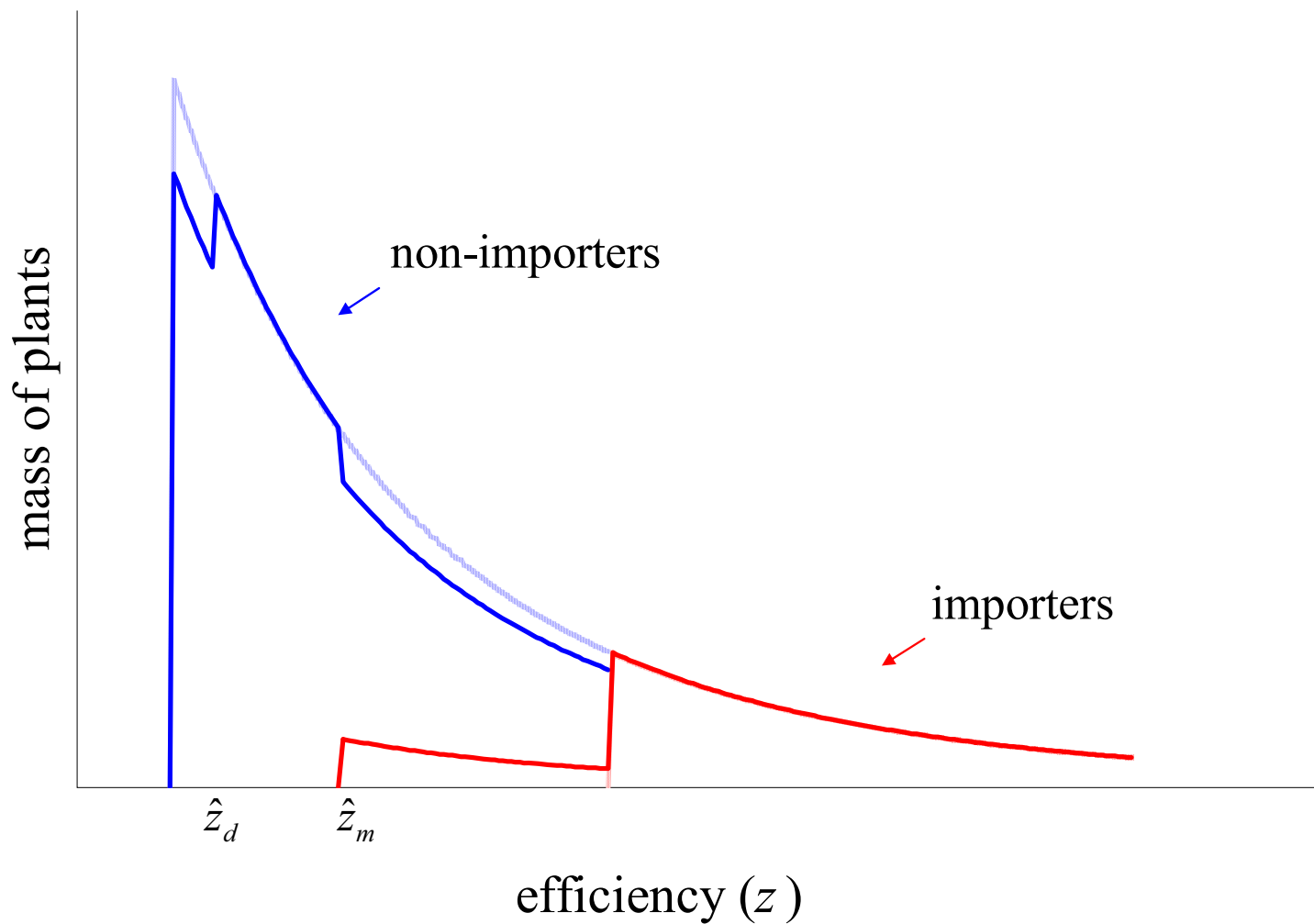
# Dynamic effect of decrease in $(1 + \tau)p_t$

Distribution of Plants,  $t+1$



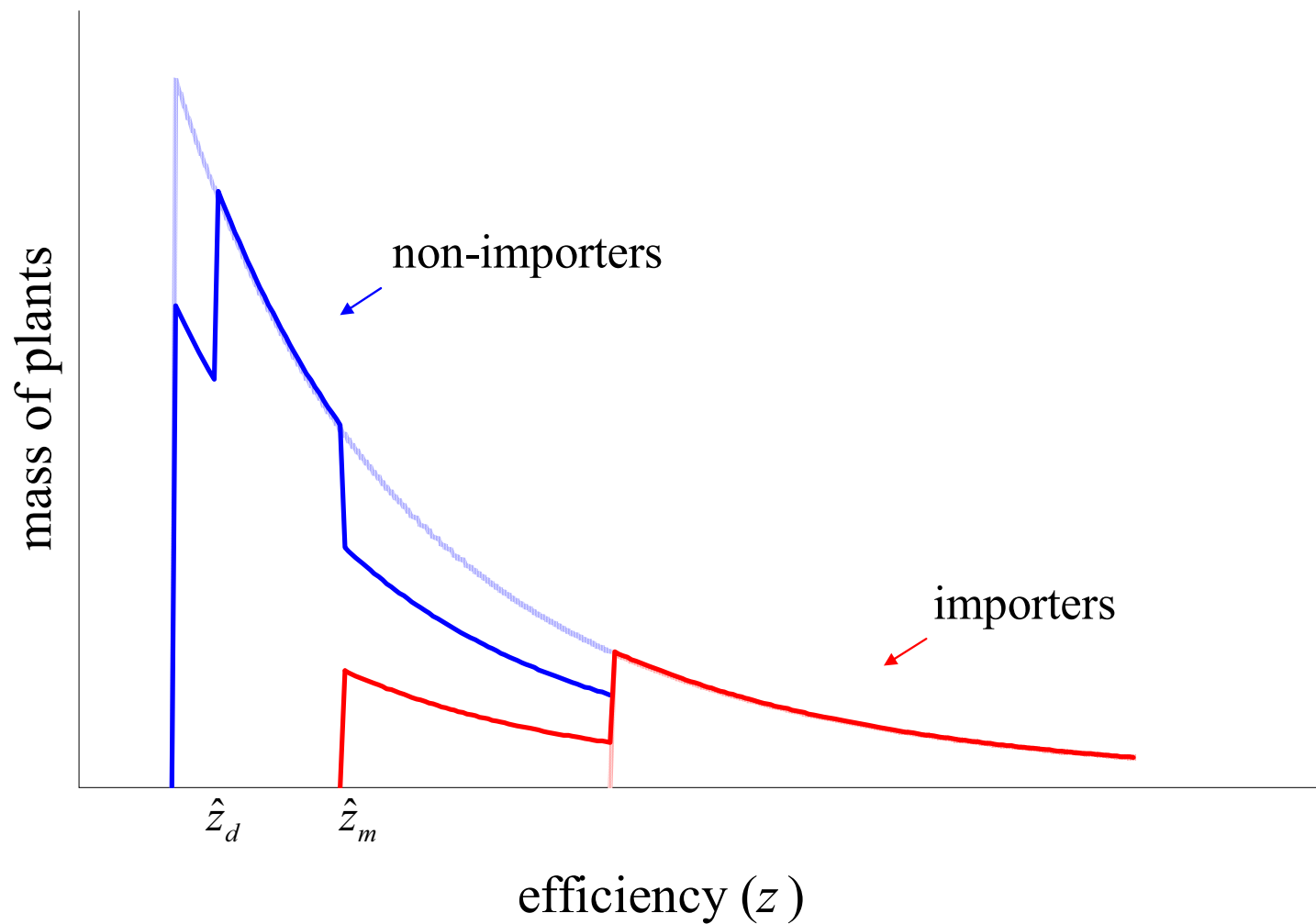
# Dynamic effect of decrease in $(1 + \tau)p_t$

Distribution of Plants,  $t+2$



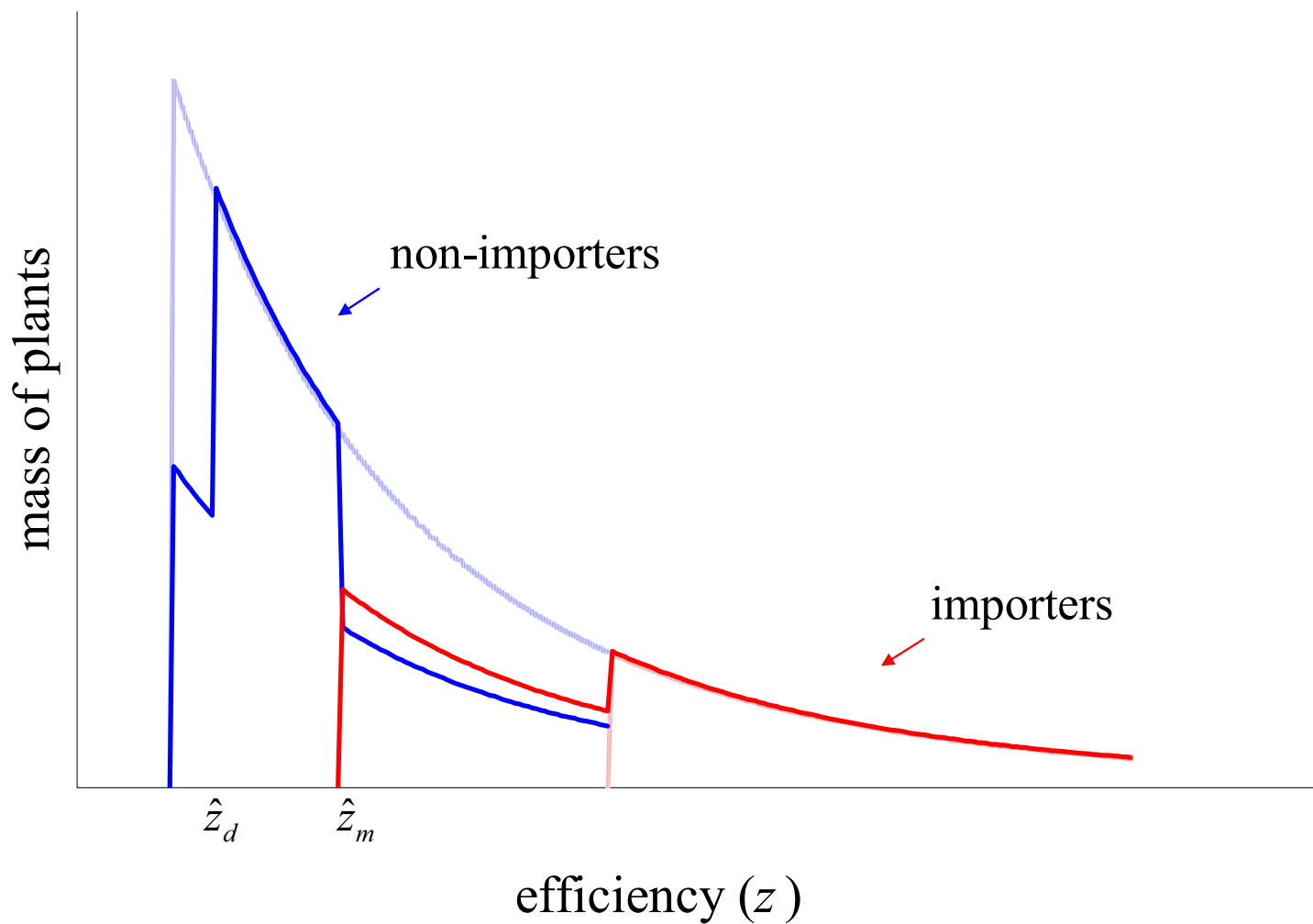
# Dynamic effect of decrease in $(1 + \tau)p_t$

Distribution of Plants,  $t+5$



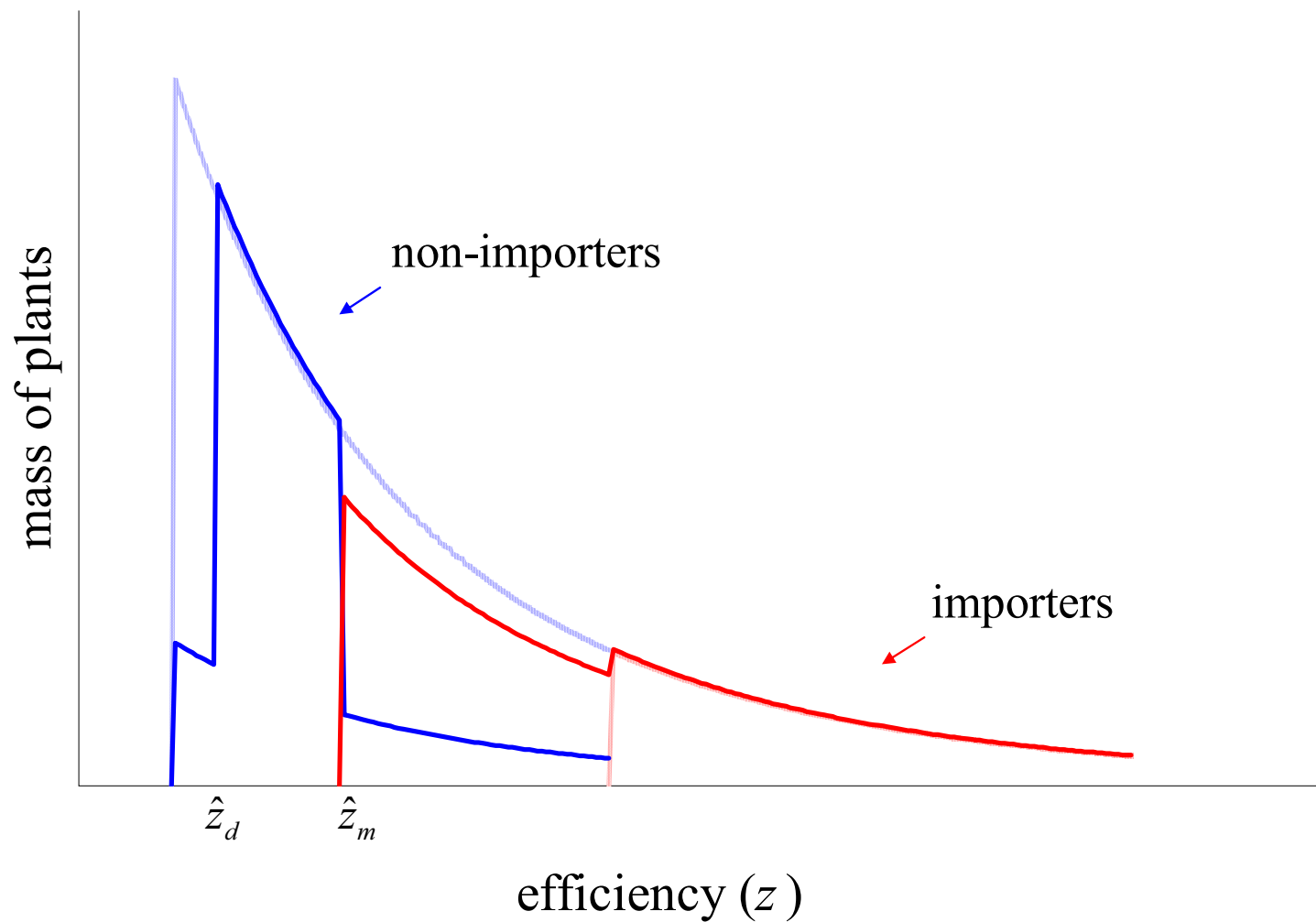
# Dynamic effect of decrease in $(1 + \tau)p_t$

Distribution of Plants,  $t+10$



# Dynamic effect of decrease in $(1 + \tau)p_t$

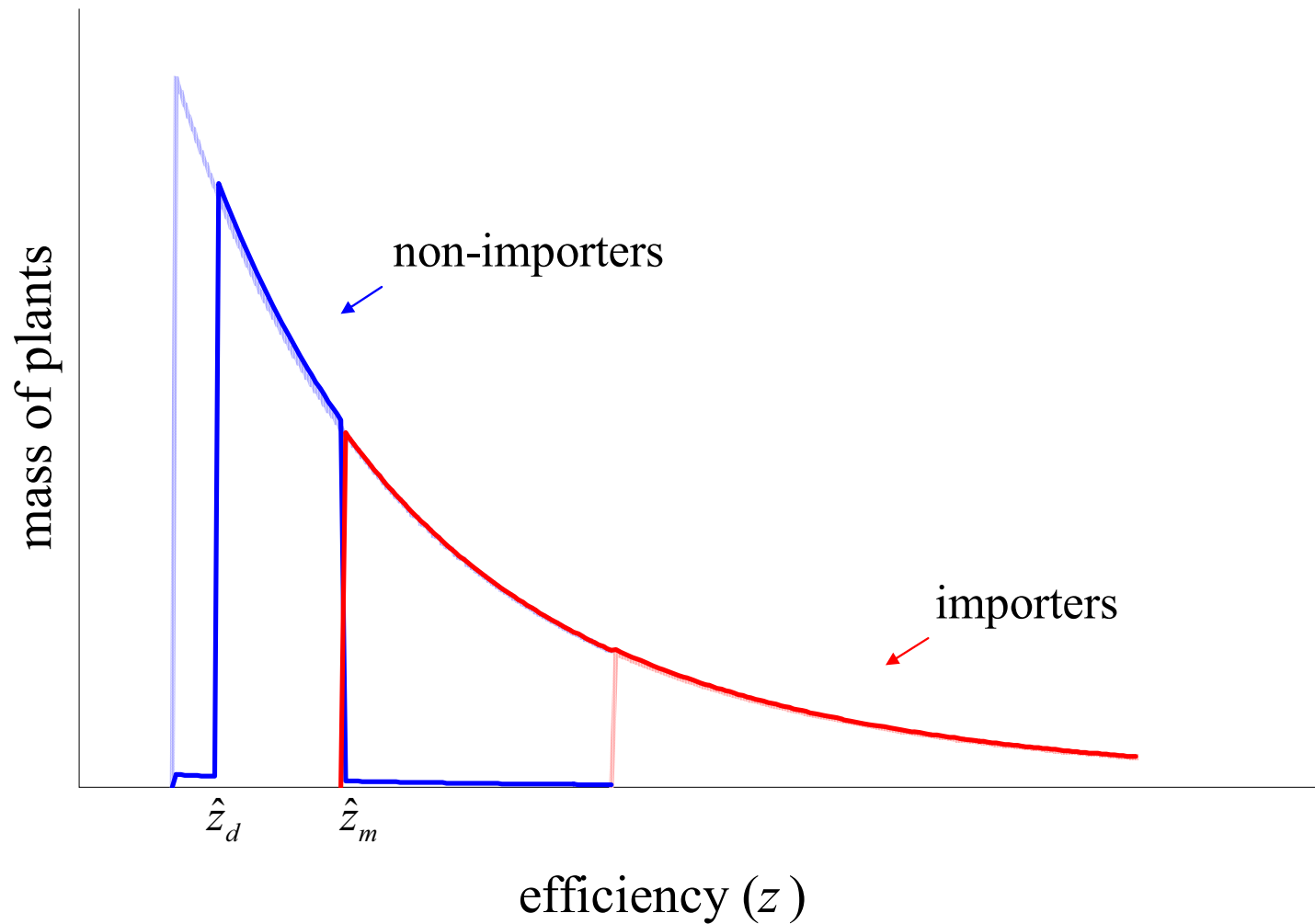
Distribution of Plants,  $t+20$





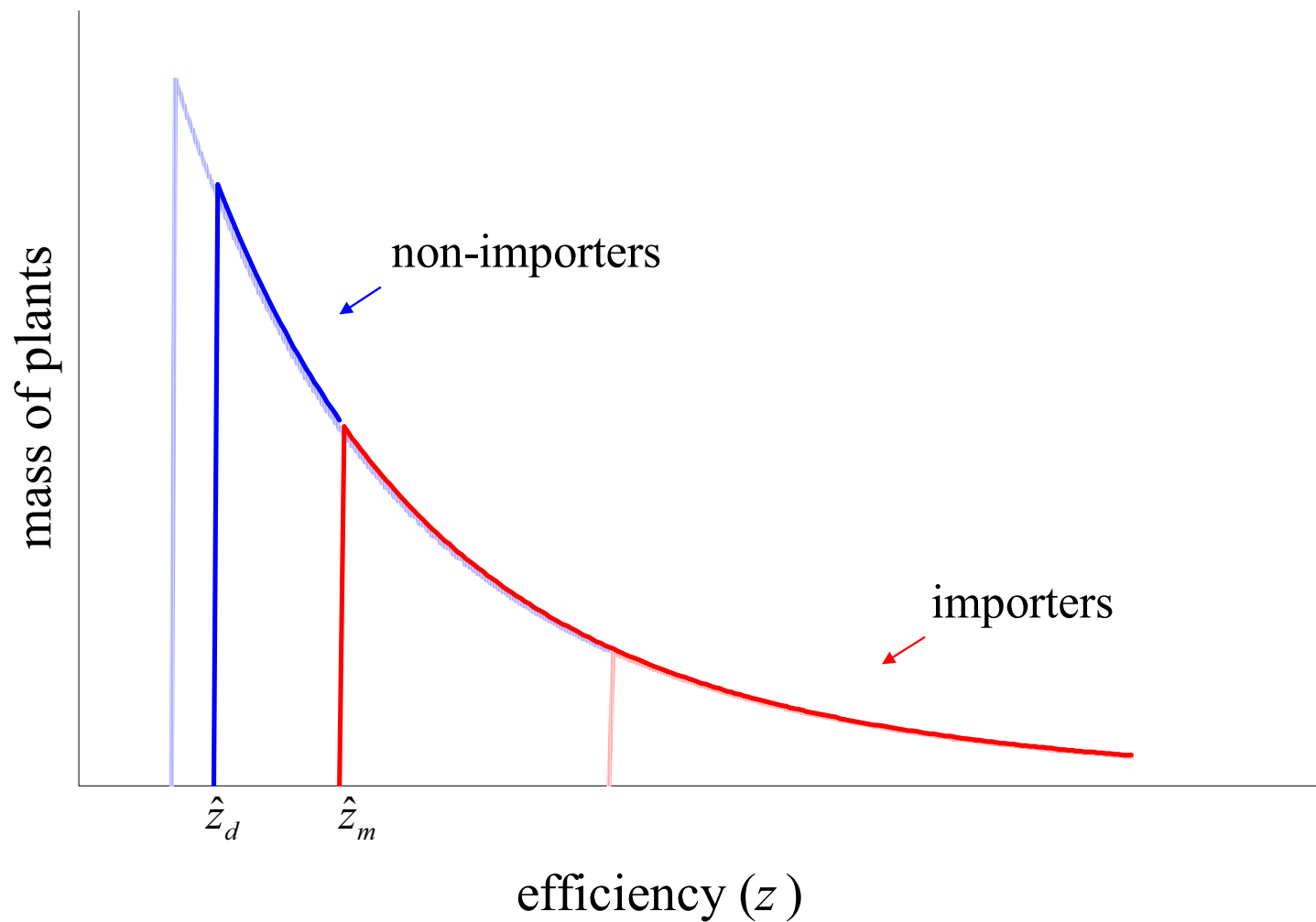
# Dynamic effect of decrease in $(1 + \tau)p_t$

Distribution of Plants,  $t+50$



# Dynamic effect of decrease in $(1 + \tau)p_t$

Distribution of Plants,  $t + \infty$



1. Cyclical fluctuations: static reallocation dominant  
Low aggregate elasticity of substitution ( $\sim 1.3$ )
2. Trade liberalization: gradual change in ratio of plants  
High aggregate elasticity of substitution ( $\sim 7$ )  
Gradual increase in trade

## **Conclusions**

Heterogeneity and irreversibility in importing at producer level

Slow-moving dynamics at aggregate level

Significant implications for welfare gains from trade reform

**6. Models with uniform fixed cost across firms with heterogeneous productivity have implications that are sharply at odds with micro data. A model with increasing costs of accessing a fraction of a market has many of features of models with fixed costs without these undesirable properties.**

C. Arkolakis, “Market Access Costs and the New Consumers Margin in International Trade,” University of Minnesota, 2006.  
<http://www.econ.umn.edu/~tkehoe/papers/Arkolakis.pdf>.

# Two Key Observations in Trade Data

## Key Observation 1: Who exports and how much

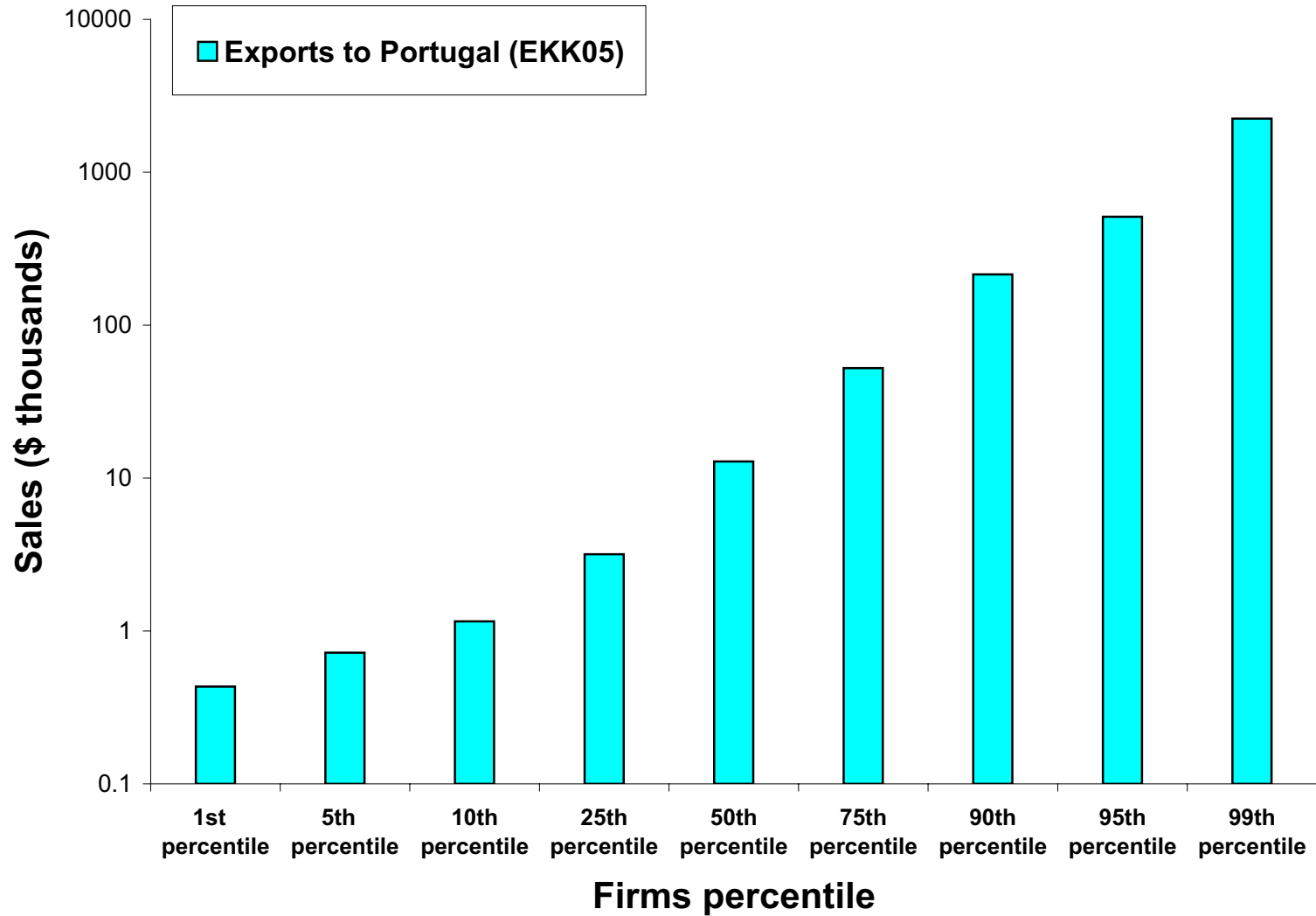
(Eaton Kortum and Kramarz '05)

- Most firms do not export **and**
- Large fraction of firms exporting to each country sell tiny amounts there

## Example

- Only 1.9% of French firms export to Portugal **and**
- More than 25% of French firms exporting to Portugal  $< 10K$  there

**Example:** 1.9% of French firms export to Portugal, mostly tiny amounts



# Two Key Observations in Trade Data

## **Key Observation 1: Who exports and how much**

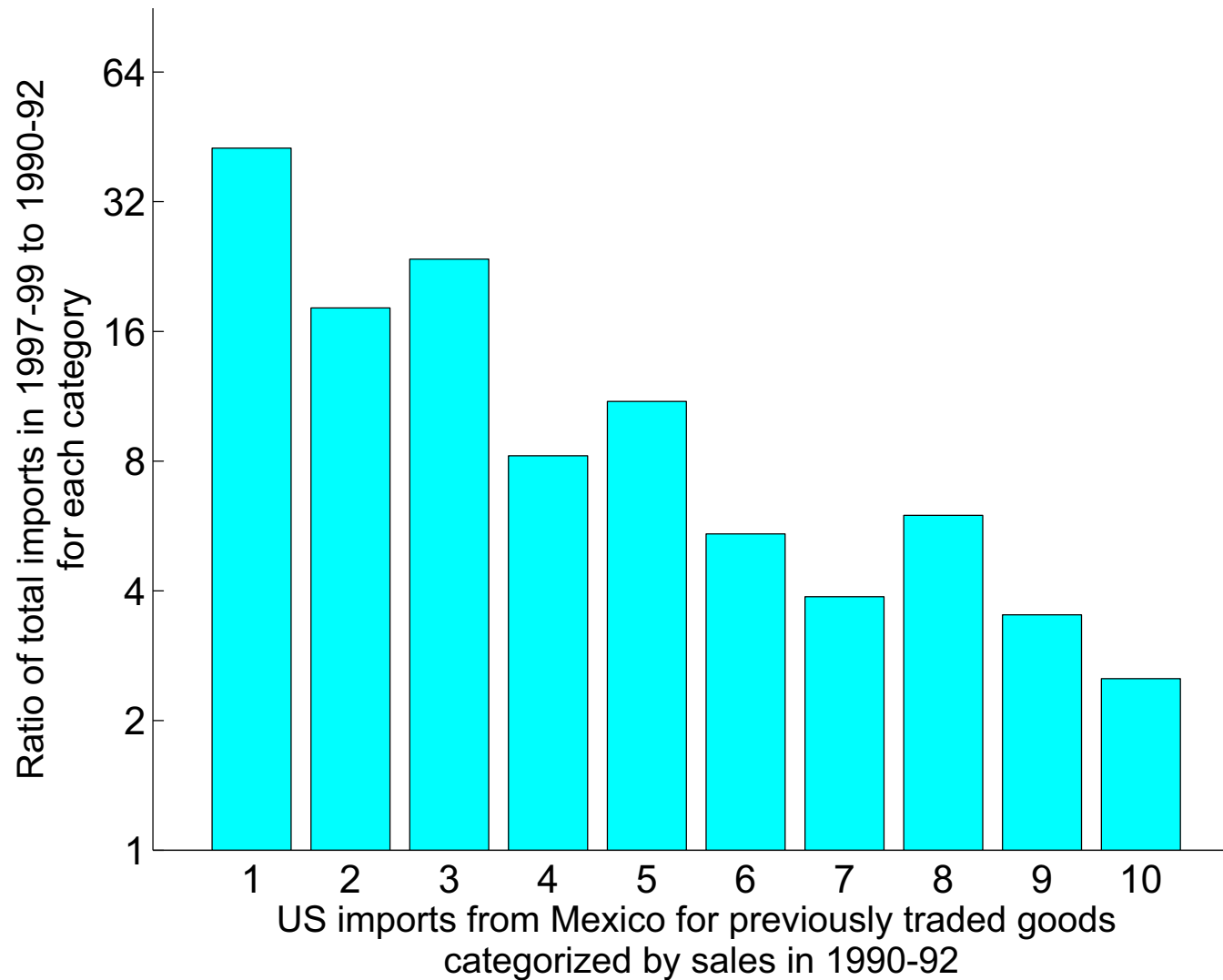
- Most firms do not export **and**
- Large fraction of firms exporting to each country sell tiny amounts there

## **Key Observation 2: Trading decisions after a trade liberalization**

(Kehoe '05, Kehoe & Ruhl '03)

- Large increases in trade for goods with positive but little trade

**Example:** Large increases in goods with positive but little trade prior NAFTA





# Existing Firm-Level Models of Trade

- Models such as those of Melitz '03 and Chaney '06 assume
  - Differentiated products
  - Heterogeneous productivity firms
  - Fixed market access cost of exporting
  
- Yield 2 puzzles related to 2 key observations

# Two Puzzles for Theory with Fixed Costs

- **Puzzle 1: Fixed Cost model needs**
  - Large fixed cost for most firms not to export
  - Small fixed cost for small exporters
- **Puzzle 2: Fixed Cost model relies solely on Dixit-Stiglitz demand**
  - Predicts symmetric changes for all previously positively traded goods
- This paper points out the shortcomings of the Fixed Cost model
  - Proposes a **theory of marketing** that can resolve them

# A Theory of Marketing: The Basic Idea

**Example:** TV channel, each ad randomly reaches 50% of consumers

	<b>1st ad</b>	<b>2nd ad</b>	<b>3rd ad</b>
fraction reached	50%		
cost per consumer	2		

# A Theory of Marketing: The Basic Idea

**Example:** TV channel, each ad randomly reaches 50% of consumers

	<b>1st ad</b>	<b>2nd ad</b>	<b>3rd ad</b>
fraction reached	50%	+25%	
cost per consumer	2	4	

# A Theory of Marketing: The Basic Idea

**Example:** TV channel, each ad randomly reaches 50% of consumers

	<b>1st ad</b>	<b>2nd ad</b>	<b>3rd ad</b>
fraction reached	50%	+25%	+12.5%
cost per consumer	2	4	8

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## **Properties of marketing cost per consumer**

- a) Costly to reach first consumer
- b) Increasing marketing cost per consumer to reach additional consumers

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## **Properties of marketing cost per consumer**

- a) Costly to reach first consumer
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*Model with a)+b) can account for observation 1, namely,*

- Most firms do not export **and**
- Large fraction of firms exporting to each country sell tiny amounts there

# A Theory of Marketing: The Basic Idea

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## **Properties of marketing cost per consumer**

- a) Costly to reach first consumer
- b) Increasing marketing cost per consumer to reach additional consumers
- c) More ads bring fewer new consumers (saturation)



# A Theory of Marketing: The Basic Idea

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## **Properties of marketing cost per consumer**

- Costly to reach first consumer
- Increasing marketing cost per consumer to reach additional consumers
- More ads bring fewer new consumers (saturation)

*Model with c) can account for observation 2, namely,*

- Large increases in trade for goods with positive but little trade

# Model Environment

Builds on Melitz '03 and Chaney '06

- **Countries**

- Index by  $i$  when exporting,  $j$  when importing,  $i, j = 1, \dots, N$
- $L_j$  consumers
- Firms sell locally and/or export

# Model Environment

Builds on Melitz '03 and Chaney '06

- **Representative Consumers**

- Sell unit of labor, own shares of domestic firms
- Symmetric CES Dixit-Stiglitz preferences over continuum of goods
- Buy the goods they have access to

- **Firms**

- Indexed by productivity  $\phi$  (drawn from same distribution), nationality  $i$
- Each sells 1 good
- Determine probability a consumer in a market has access to their good

# Demand Faced by a Type $\phi$ Firm from Country $i$

- $n_{ij}(\phi)$  : probability a type  $\phi$  firm from  $i$  reaches a repres. consumer in  $j$
- Large number of consumers
  - thus firm **reaches** fraction  $n_{ij}(\phi)$  of them
- Effective demand for firm  $\phi$ :

$$\underbrace{n_{ij}(\phi)L_j}_{\text{consumers that firm reaches}} \underbrace{\frac{p_{ij}(\phi)^{-\sigma}}{P_j^{1-\sigma}} y_j}_{\text{D-S demand per consumer}}$$

$p_{ij}(\phi)$  : price that type  $\phi$  firm from  $i$  charges in  $j$ ,  $y_j$ : output (income) per capita

$P_j$  : D-S price aggregator,  $\sigma$  : elasticity of substitution ( $\sigma > 1$ , demand is elastic)

# Firm's Problem

Type  $\phi$  firm from country  $i$  solves for each country  $j = 1, \dots, N$

$$\pi_{ij} = \max_{n_{ij}, p_{ij}, q_{ij}} p_{ij} q_{ij} - w_i \frac{\tau_{ij} q_{ij}}{\phi} - w_i f(n_{ij}, L_j)$$

$$s.t. \quad q_{ij} = n_{ij} L_j \frac{p_{ij}^{-\sigma}}{P_j^{1-\sigma}} y_j, \quad n_{ij} \in [0, 1]$$

- Uses production function  $q_{ij} = \phi l_{ij}$  to produce good
- $\tau_{ij}$ : iceberg cost to ship a unit of good from  $i$  to  $j$  (in terms of labor)
- $f(n_{ij}, L_j)$ : marketing to reach fraction  $n_{ij}$  of a population with size  $L_j$

# Firm's Problem

- **Result:** Price is the usual markup over unit production cost,

$$p_{ij}(\phi) = \tilde{\sigma} \frac{\tau_{ij} w_j}{\phi}, \quad \tilde{\sigma} = \frac{\sigma}{\sigma-1}$$

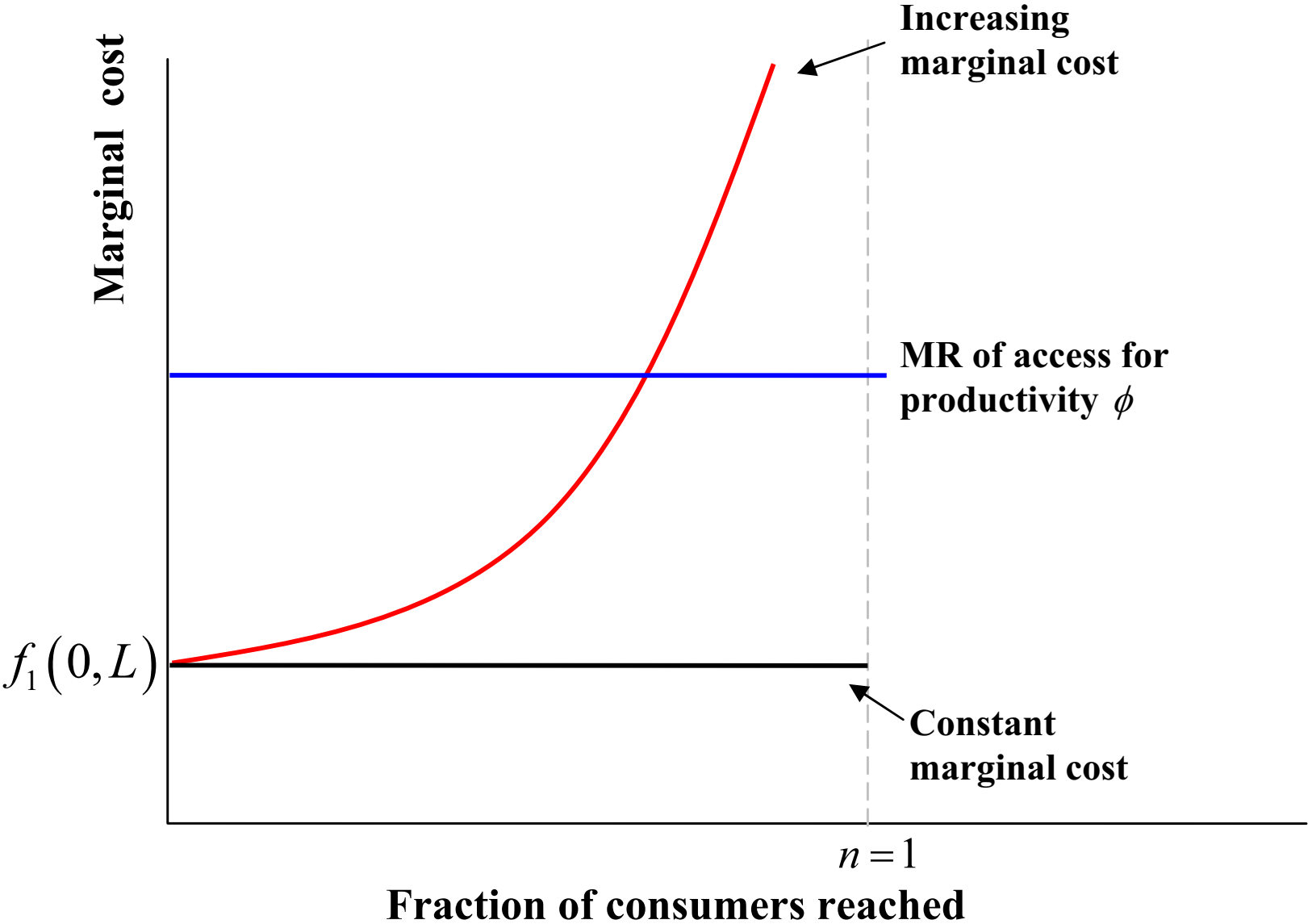
- Given price markup rule firm solves:

$$\pi_{ij} = \max_{n_{ij}} n_{ij} L_j \underbrace{\phi^{\sigma-1} \frac{(\tau_{ij} w_j \tilde{\sigma})^{1-\sigma}}{P_j^{1-\sigma}} \frac{y_j}{\sigma}}_{\substack{\text{Revenue per consumer} \\ \text{(net of labor production cost)}}} - w_j f(n_{ij}, L_j)$$

s.t  $n_{ij} \in [0, 1]$

- Look at marginal decision of reaching additional fractions of consumers

# Marginal Revenue & Cost from Reaching Additional Consumers



# The Market Access Cost Function

- Solve the differential equation

$$n'(S) = [1 - n(S)]^\beta L^{1-\alpha} \frac{1}{L}, \quad \text{s.t. } n(0) = 0$$

- Obtain Market Access Cost function

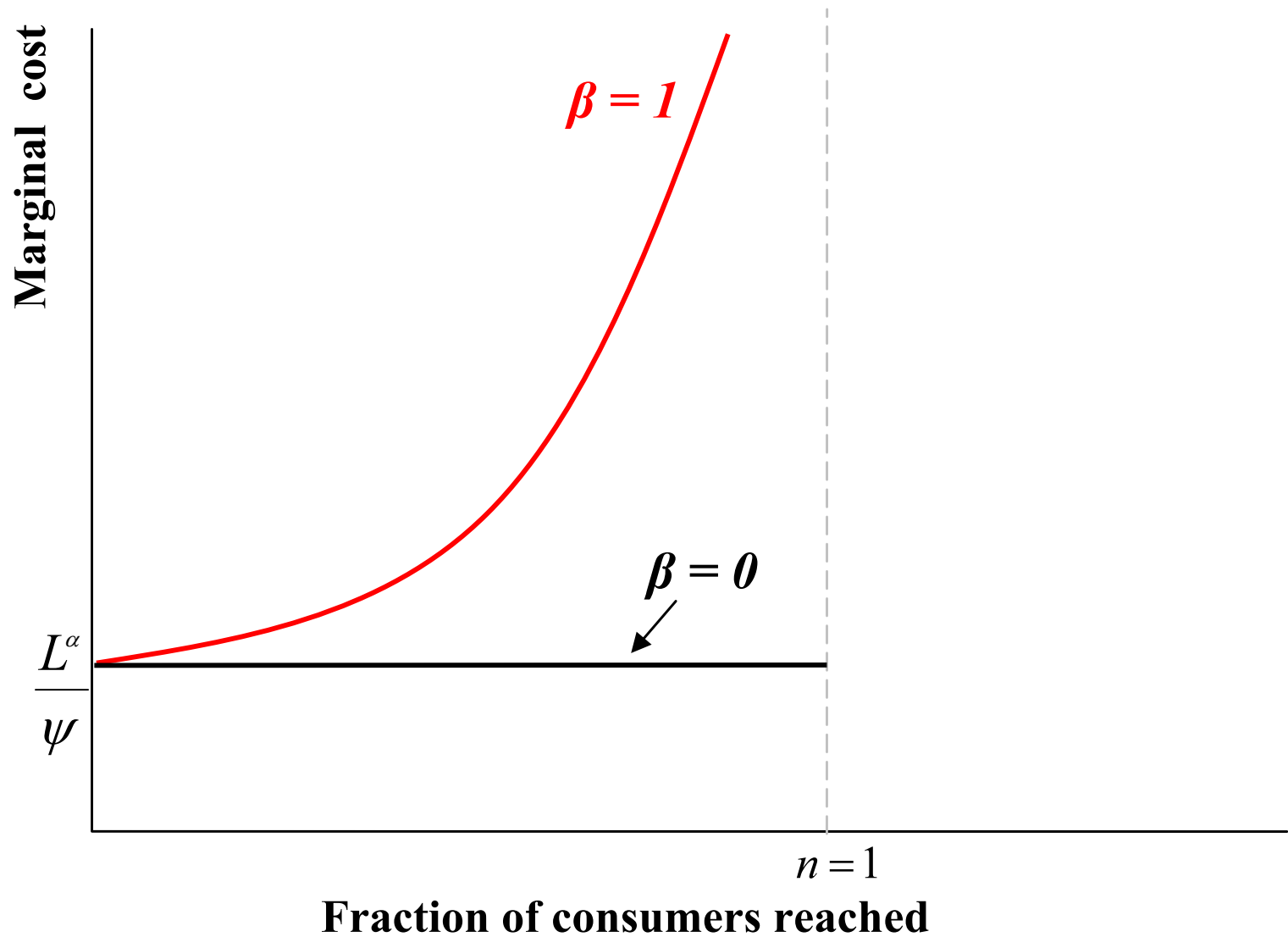
- Assuming that  $\frac{1}{\psi}$  is the labor required for each ad

$$f(n, L) = \begin{cases} \frac{L^\alpha}{\psi} \frac{1 - (1-n)^{-\beta+1}}{-\beta+1} & \text{if } \beta \in [0, 1) \cup (1, +\infty) \\ -\frac{L^\alpha}{\psi} \log(1-n) & \text{if } \beta = 1 \end{cases}$$

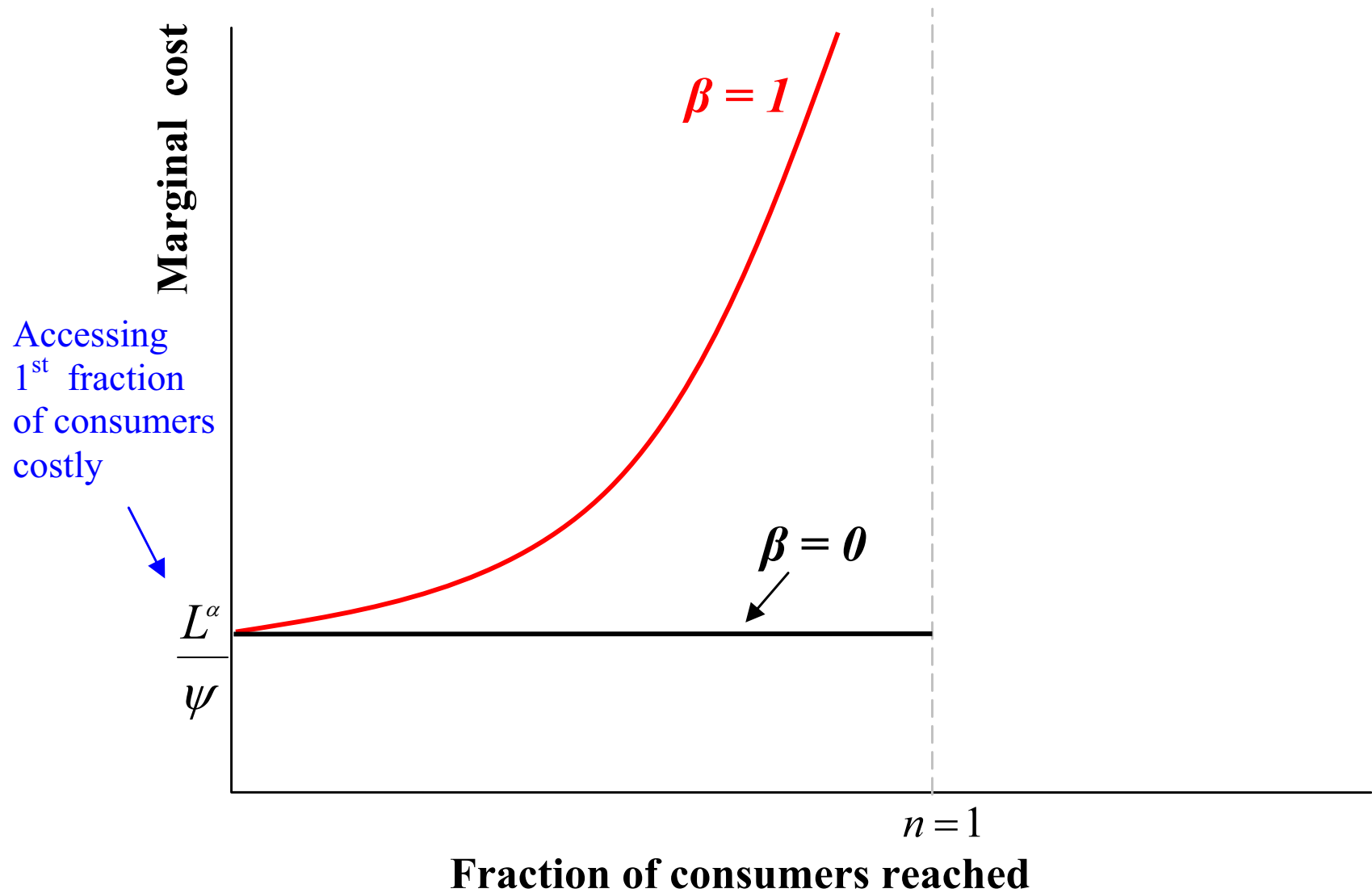
where  $\alpha \in [0, 1]$



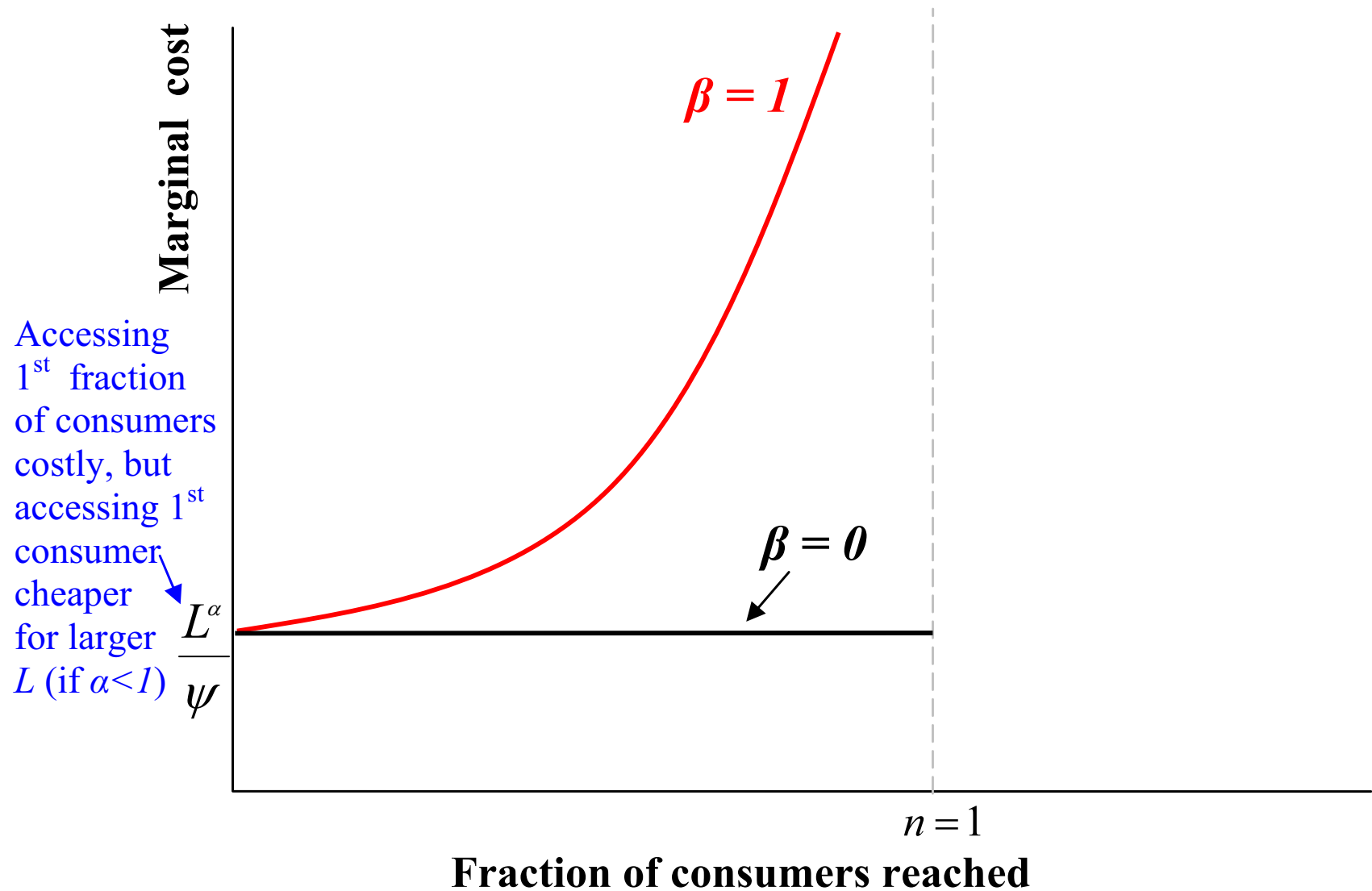
# The properties of the Market Access Cost function



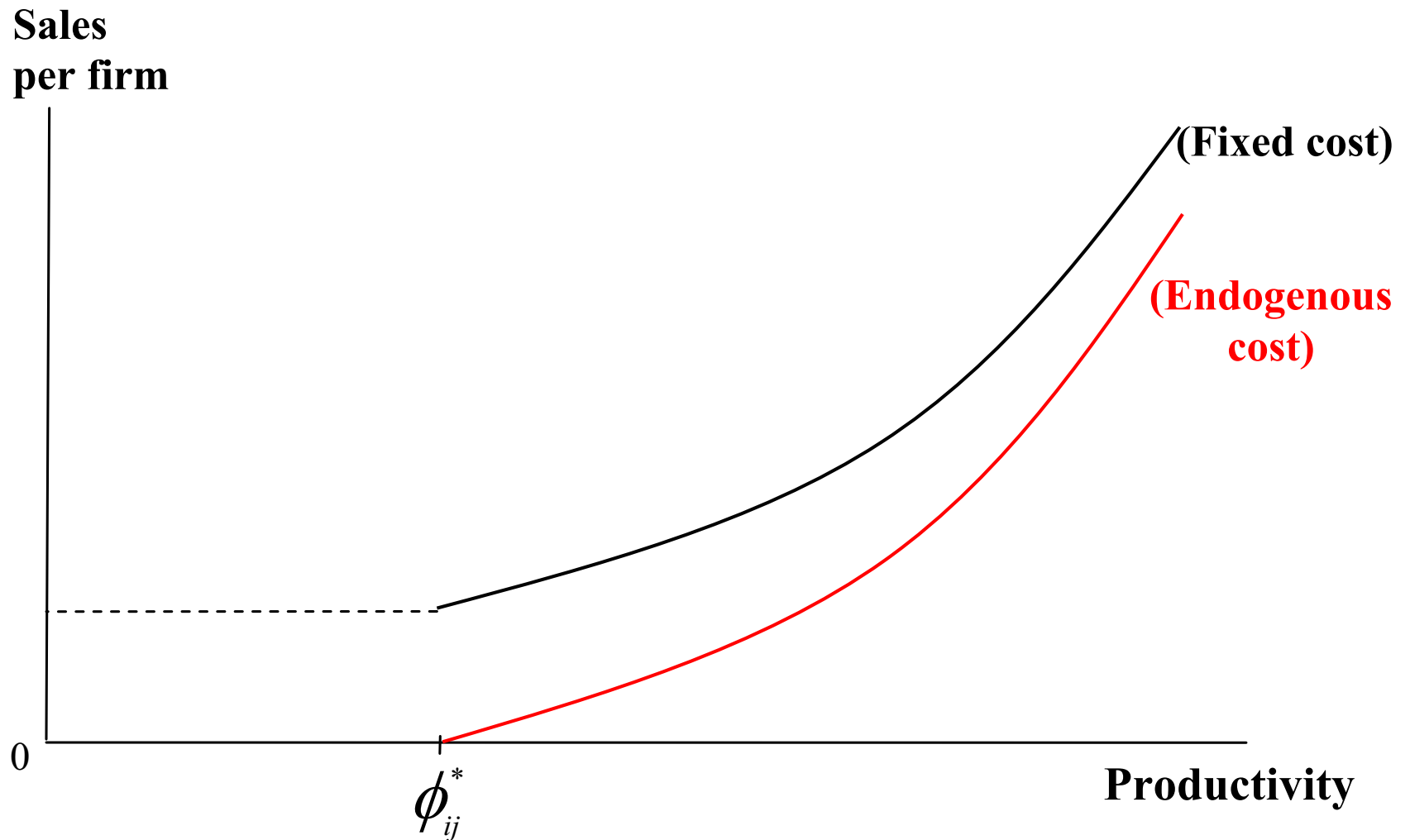
# The properties of the Market Access Cost function



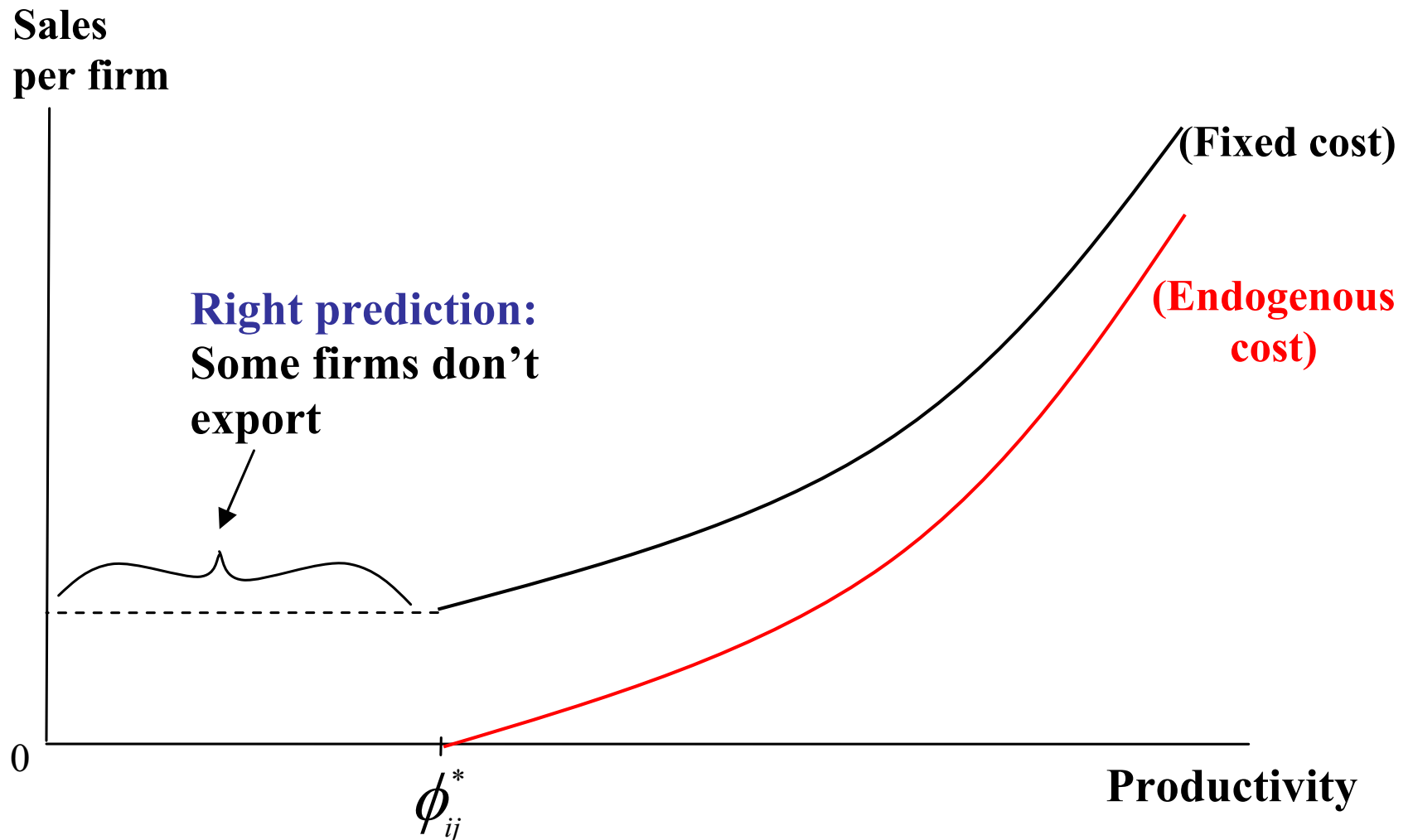
# The properties of the Market Access Cost function



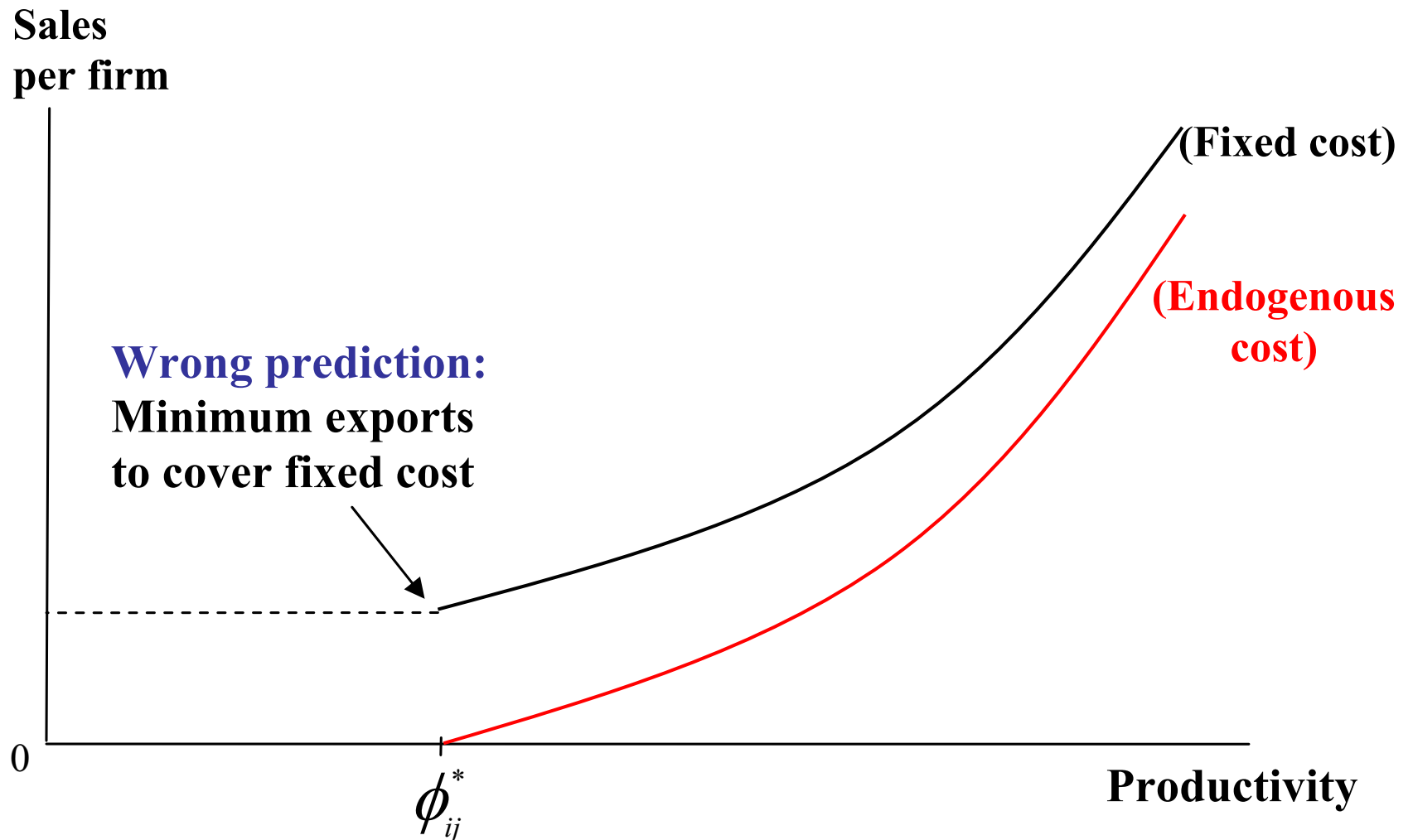
The product of the two margins: total sales per firm



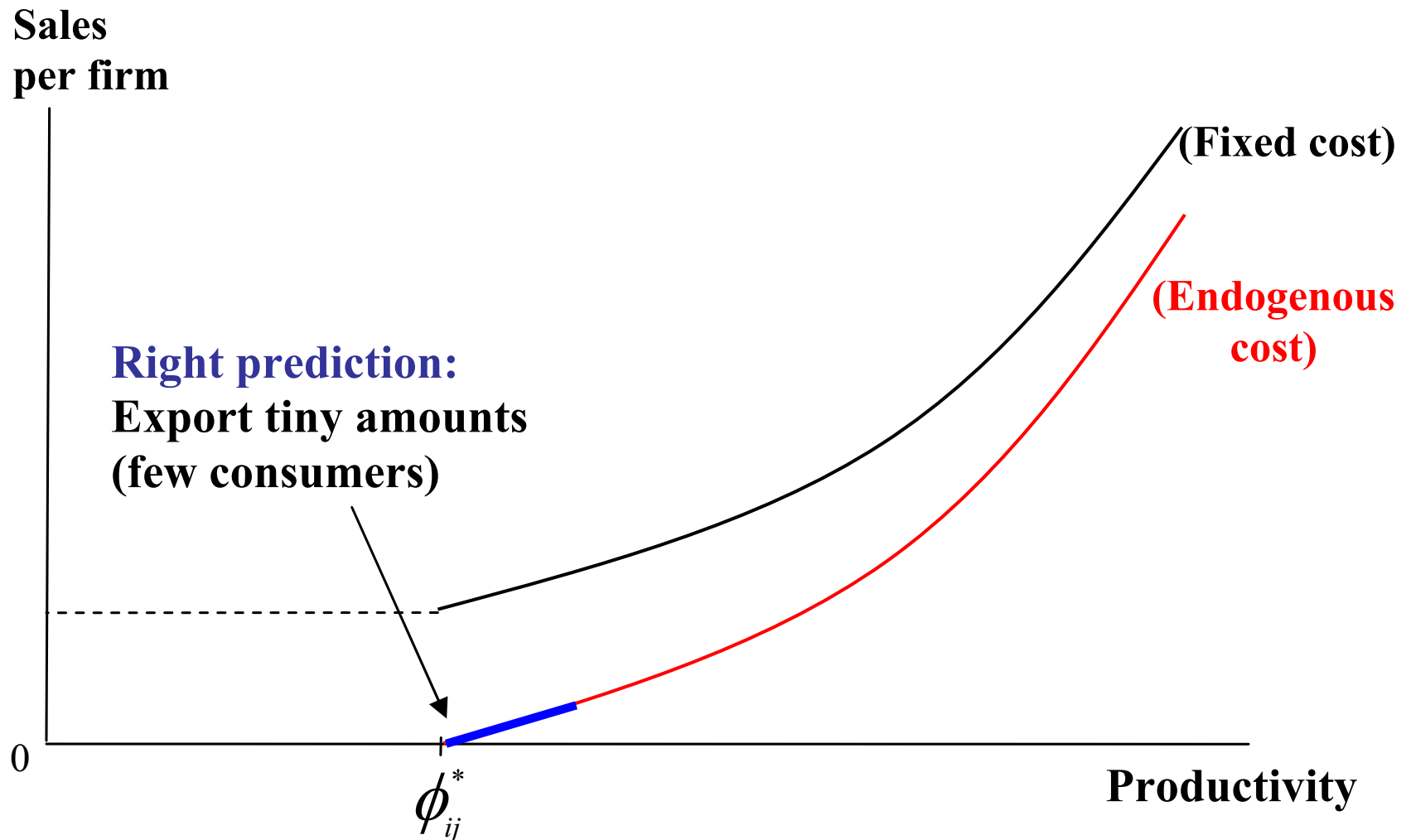
# Models' predictions on which firms export



# Models' predictions on how much firms export



# Models' predictions on how much firms export

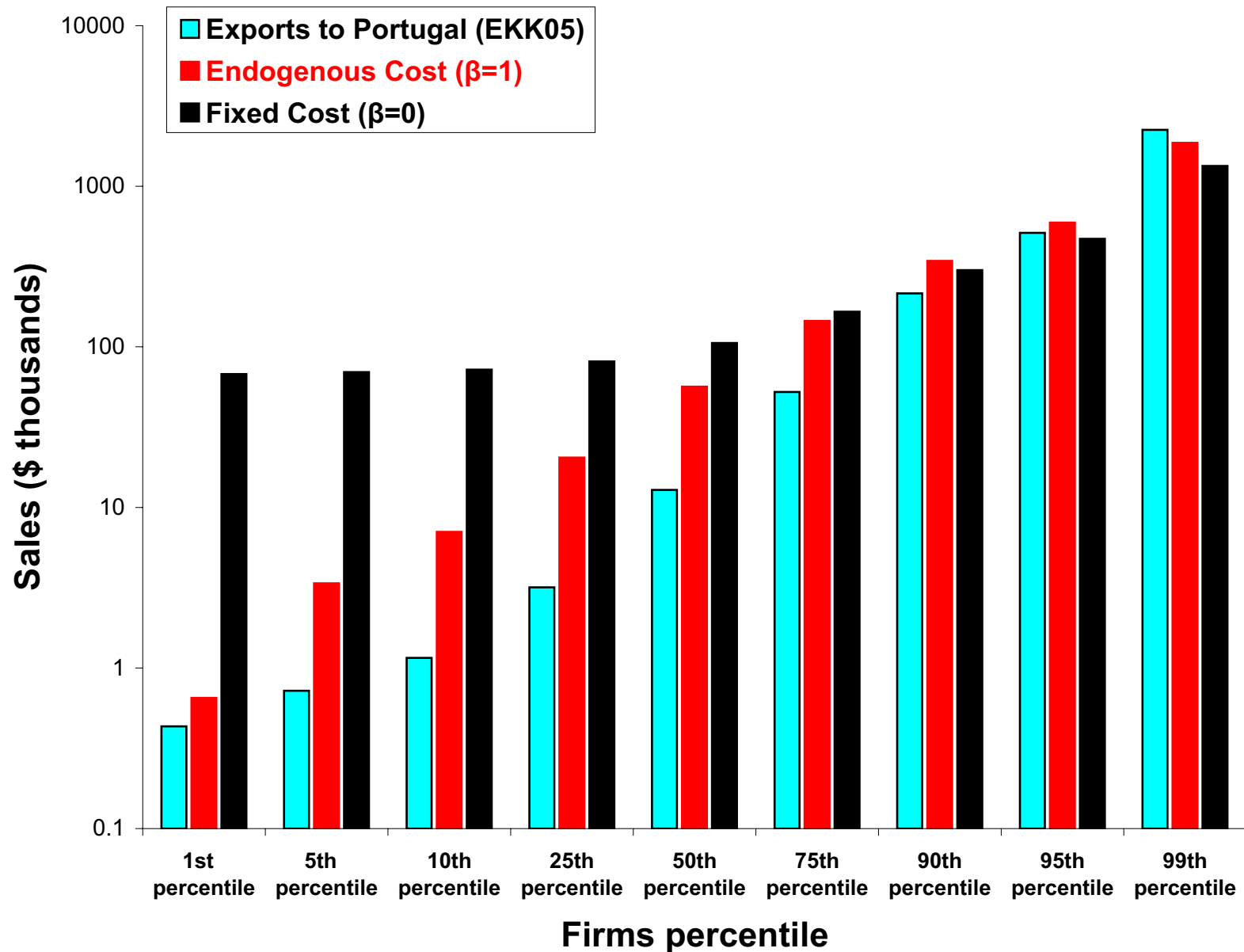


# Comparing the Calibrated Model to French Data

- Look at the sales distribution for the model with  $\beta = 0, 1$
- Remember:  $\beta = 1$  calibrated to match higher sales in France of French firms exporting to more countries
- $\frac{1}{\psi}, \alpha$  calibrated to match number of French exporters to each country



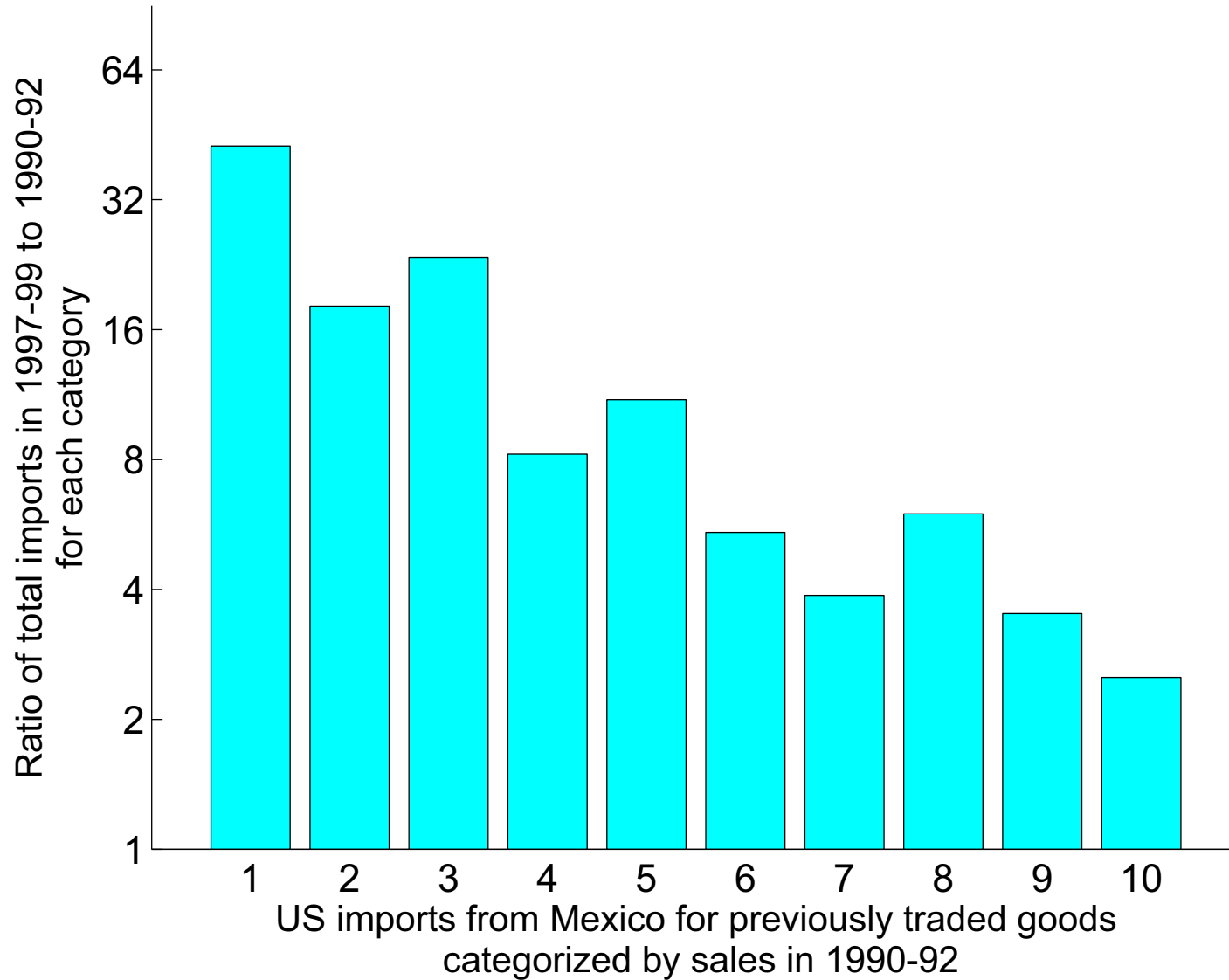
# Calibrated Endogenous Cost model accounts for large fraction of small exporters



# Observation 2: Trading Decisions After Trade Liberalization

- **Data:** Large increases in trade in least traded goods, Kehoe & Ruhl '03
- Look at US-Mexico trade liberalization; extend Kehoe-Ruhl analysis
- **Compute growth of positively traded goods prior to NAFTA**
  1. Data: US imports from Mexico '90-'99, 6-digit HS,  $\approx$  5400 goods
  2. Keep goods traded throughout '90-'92,  $\approx$  2900 goods
  3. Rank goods in terms of sales '90-'92
  4. Categorize **traded** goods in 10 bins

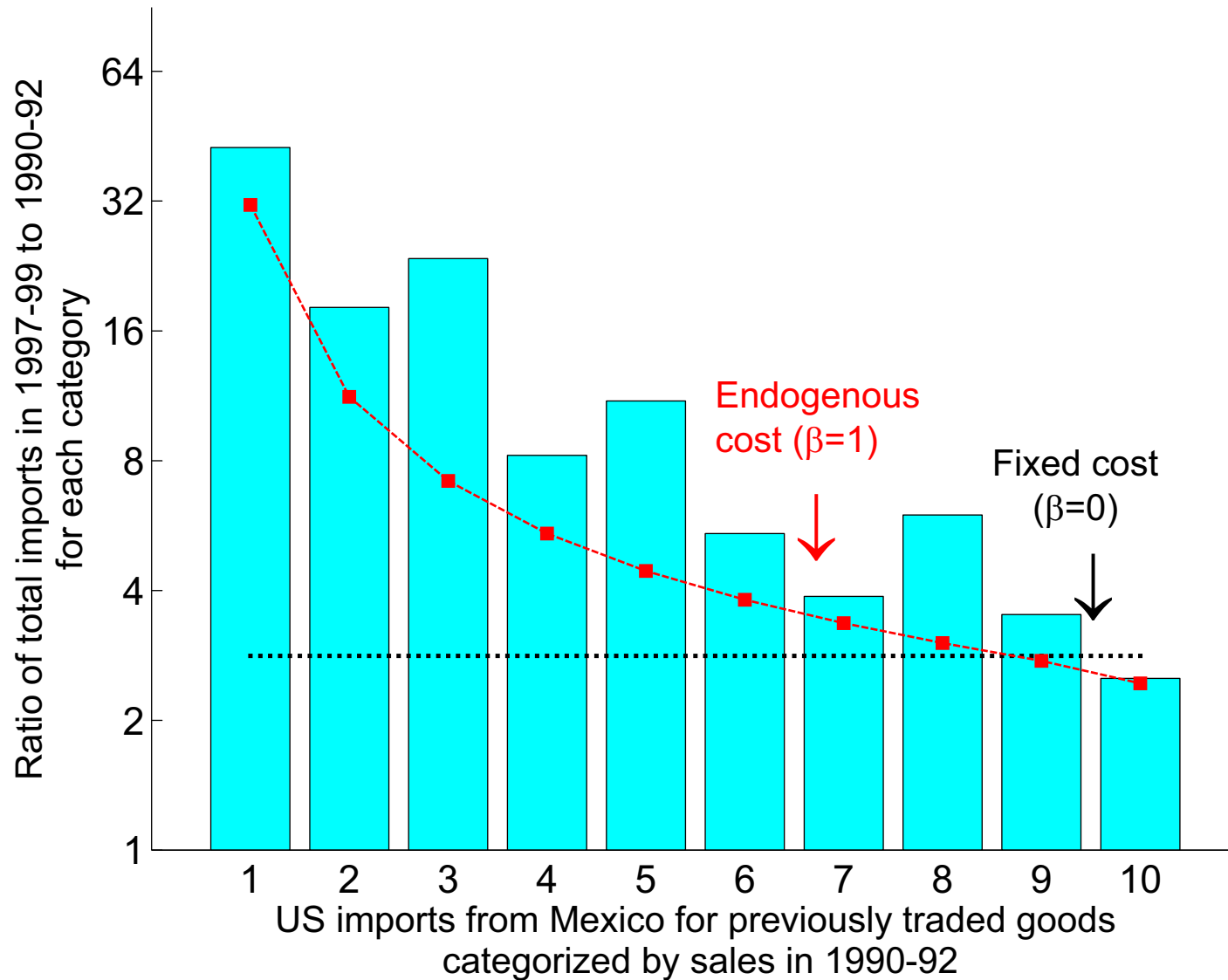
## Large increases in trade for least traded goods



# Comparing Calibrated Model to Data from NAFTA Episode

- Look at growth of trade for previously traded goods for  $\beta = 0, 1$
- Use calibrated parameters, consider a firm as a good
- Change variable trade costs symmetrically across goods
  - Match increase in trade in previously traded goods
    - Fixed Cost model: 12.5% decrease in variable trade costs
    - My model: 9.5% decrease in variable trade costs (e.g.  $\tau'_{ij} = 0.905\tau_{ij}$ )

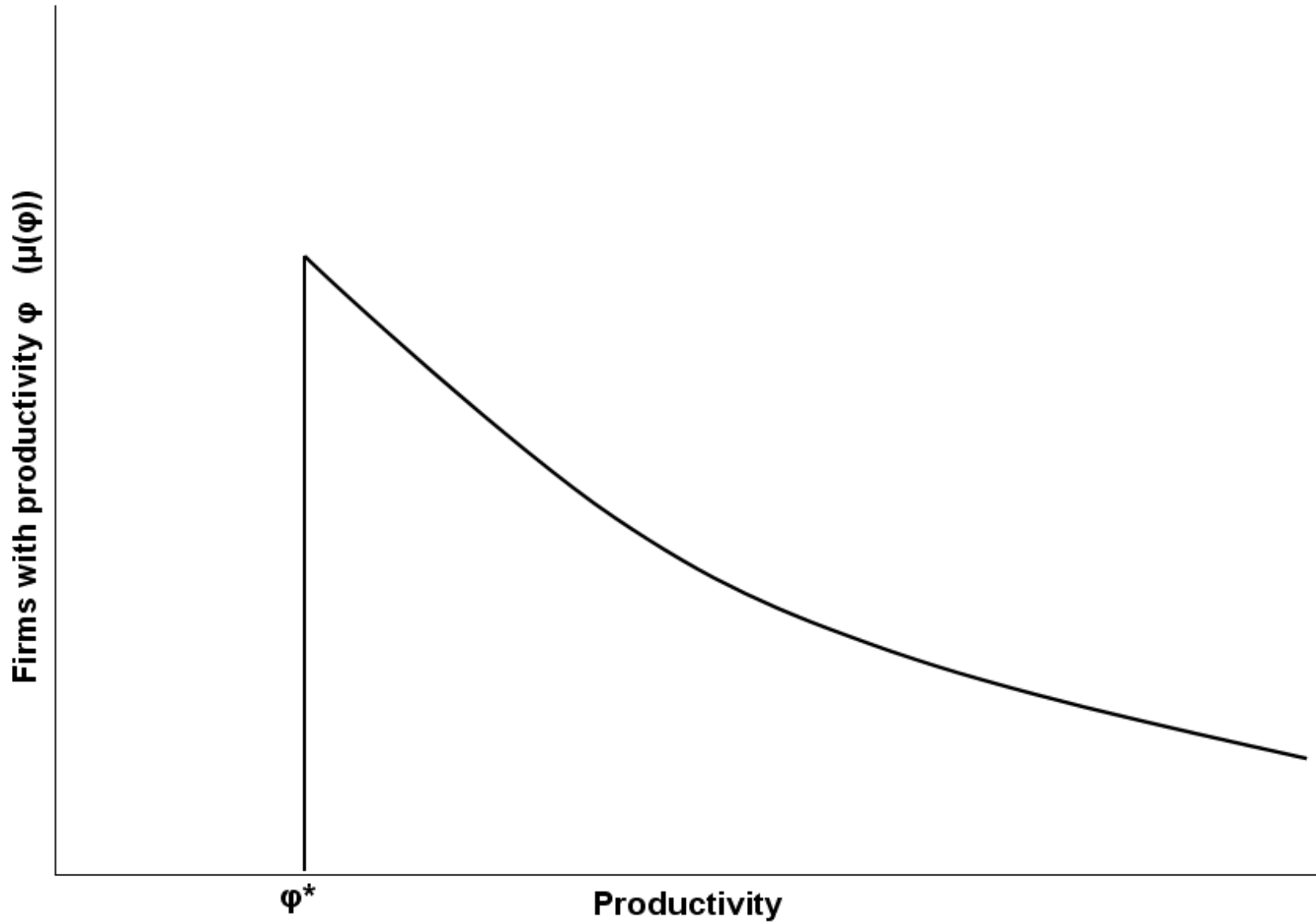
# Calibrated Endogenous Cost model predicts increases in trade for least traded goods



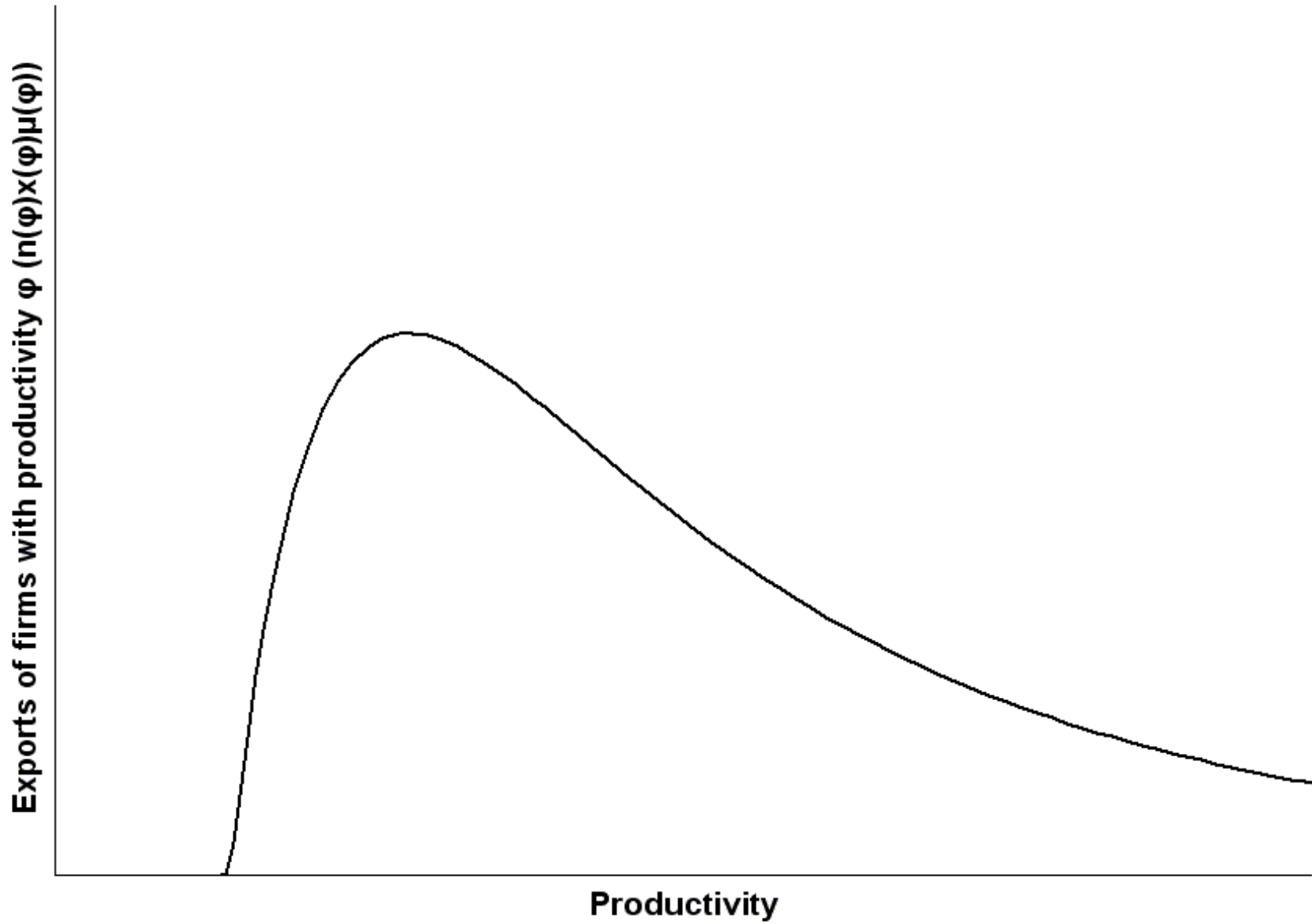
# New Consumers Margin and New Trade

- Recent theory emphasizes increase in trade due to **many** new firms (EK02, Chaney '06 à la Melitz '03)
- **Decompose contribution of the 3 margins to total trade**
  - **Intensive margin growth** (total growth in sales per consumer)
  - **New consumers margin** (total growth in extensive margin of consumers)
  - **New firms margin** (total growth in extensive margin of firms)

# Pareto Density and Number of Firms with Productivity $\phi$

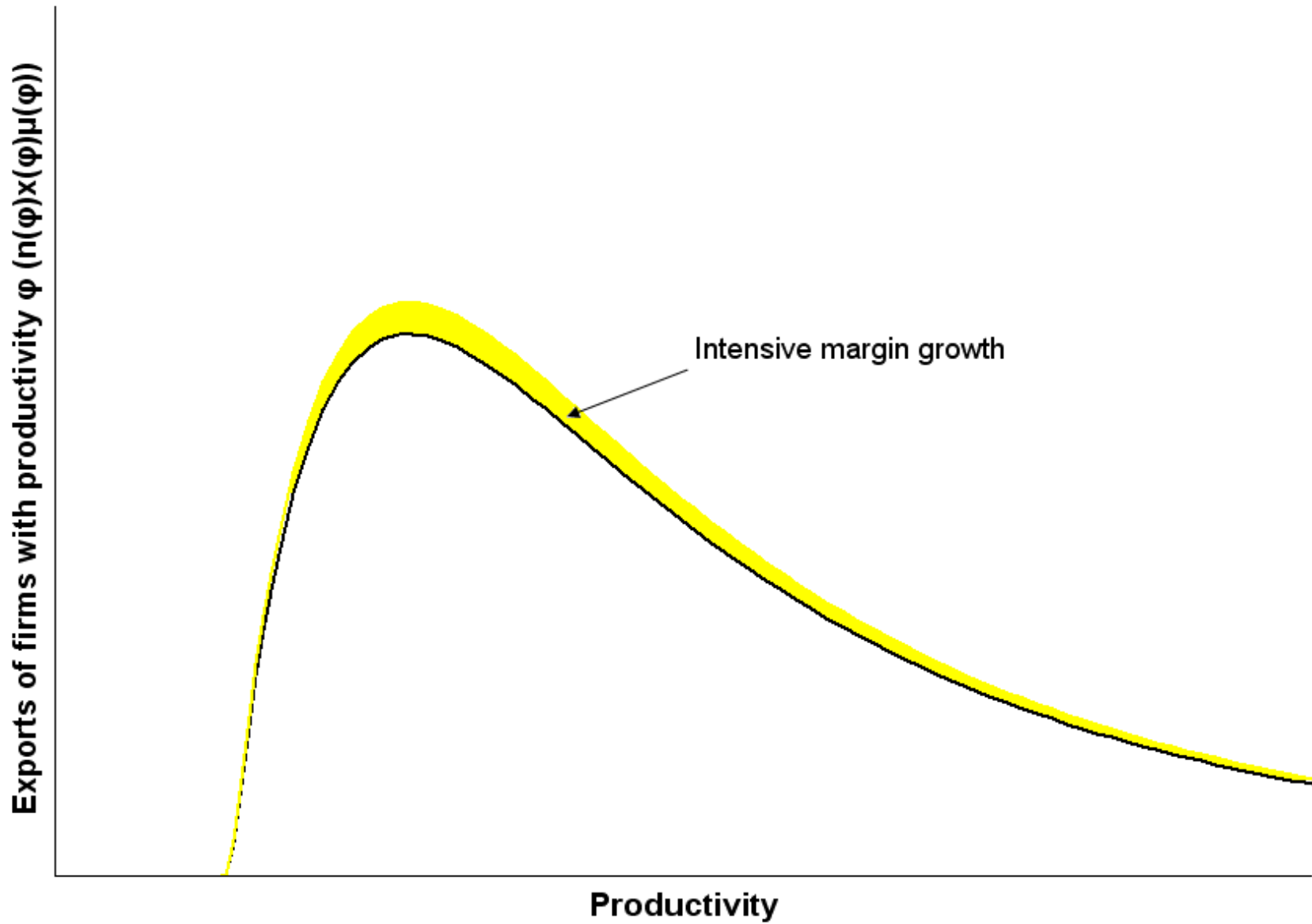


# Density of exports

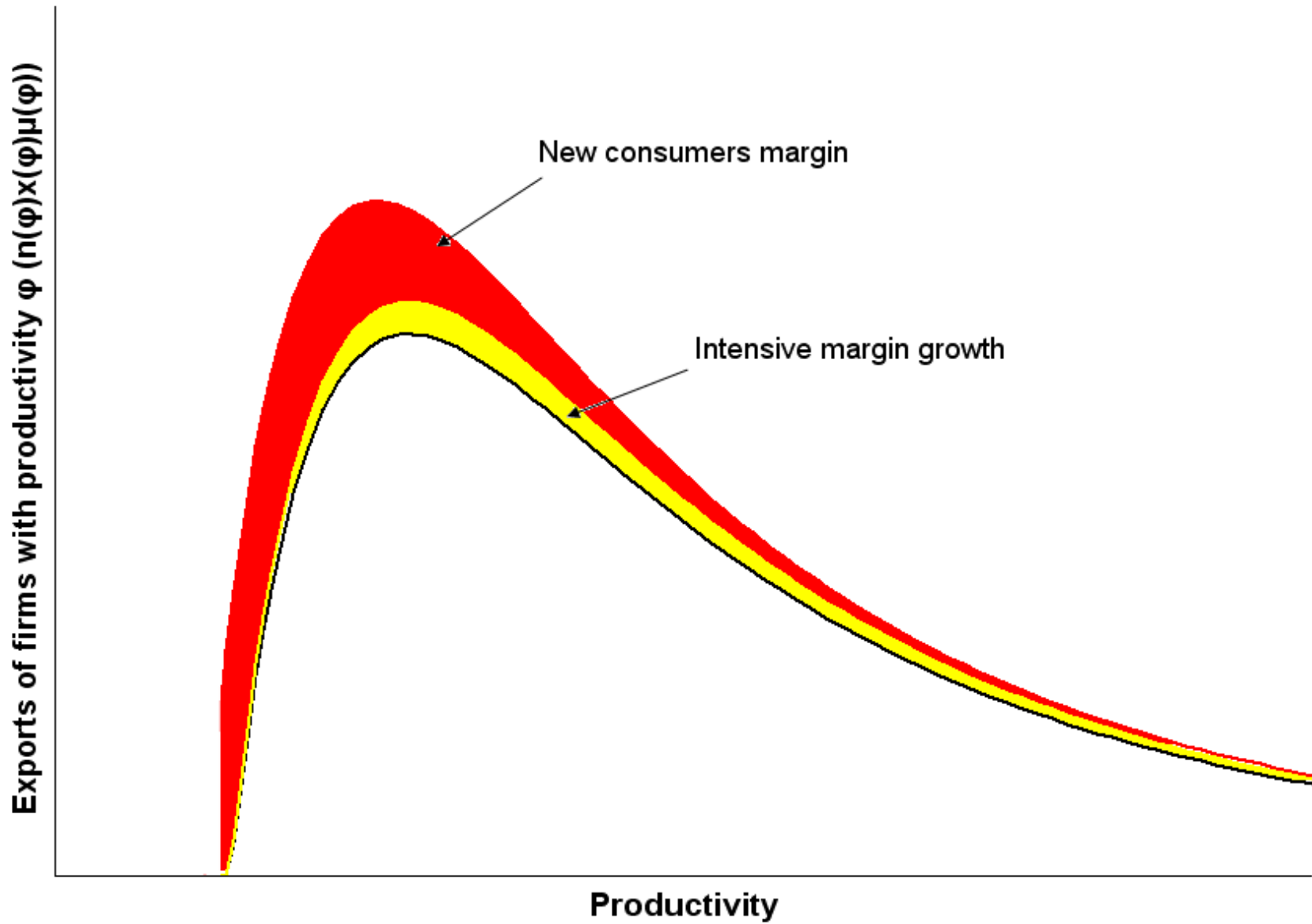




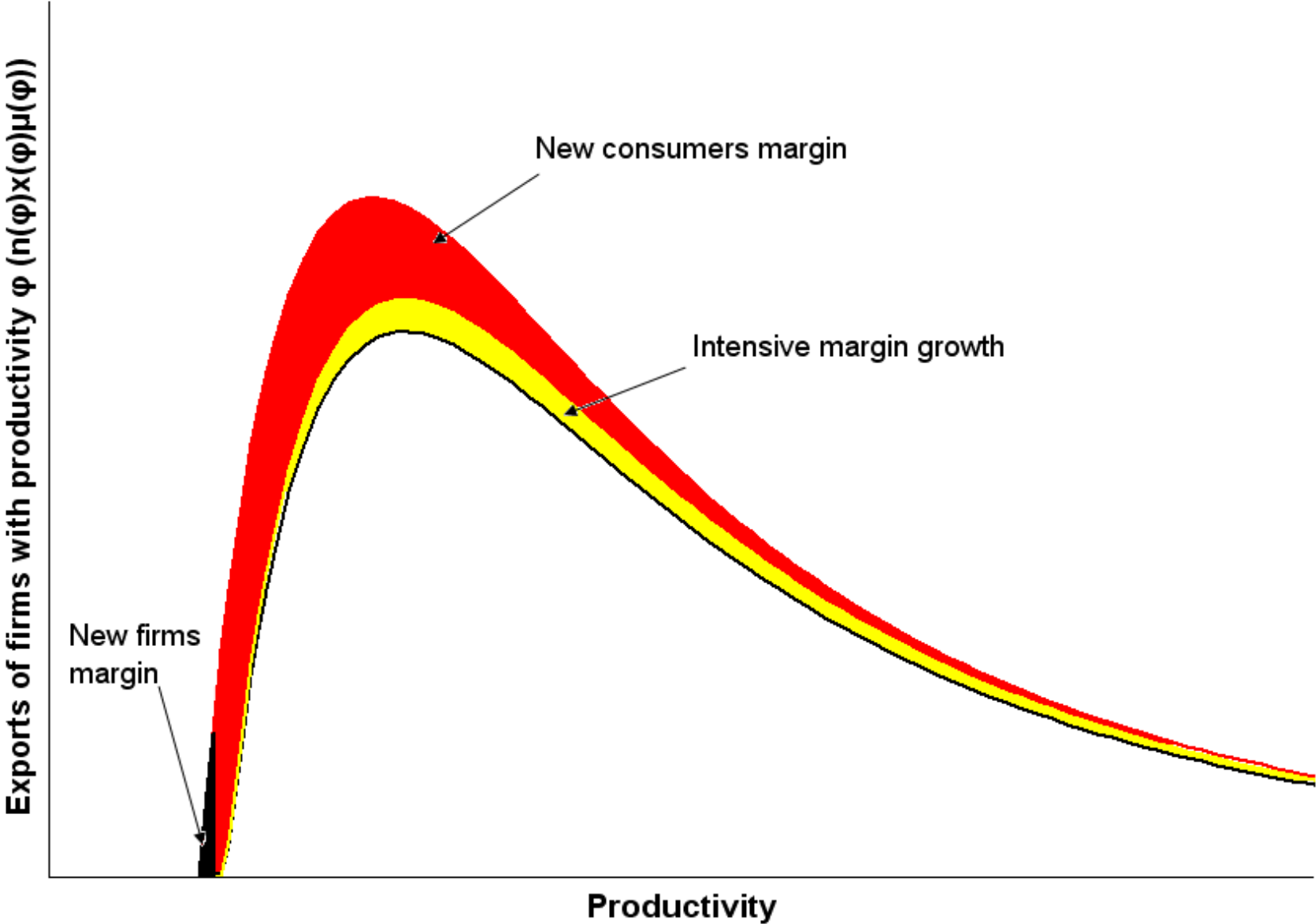
## New Consumers Margin and new trade



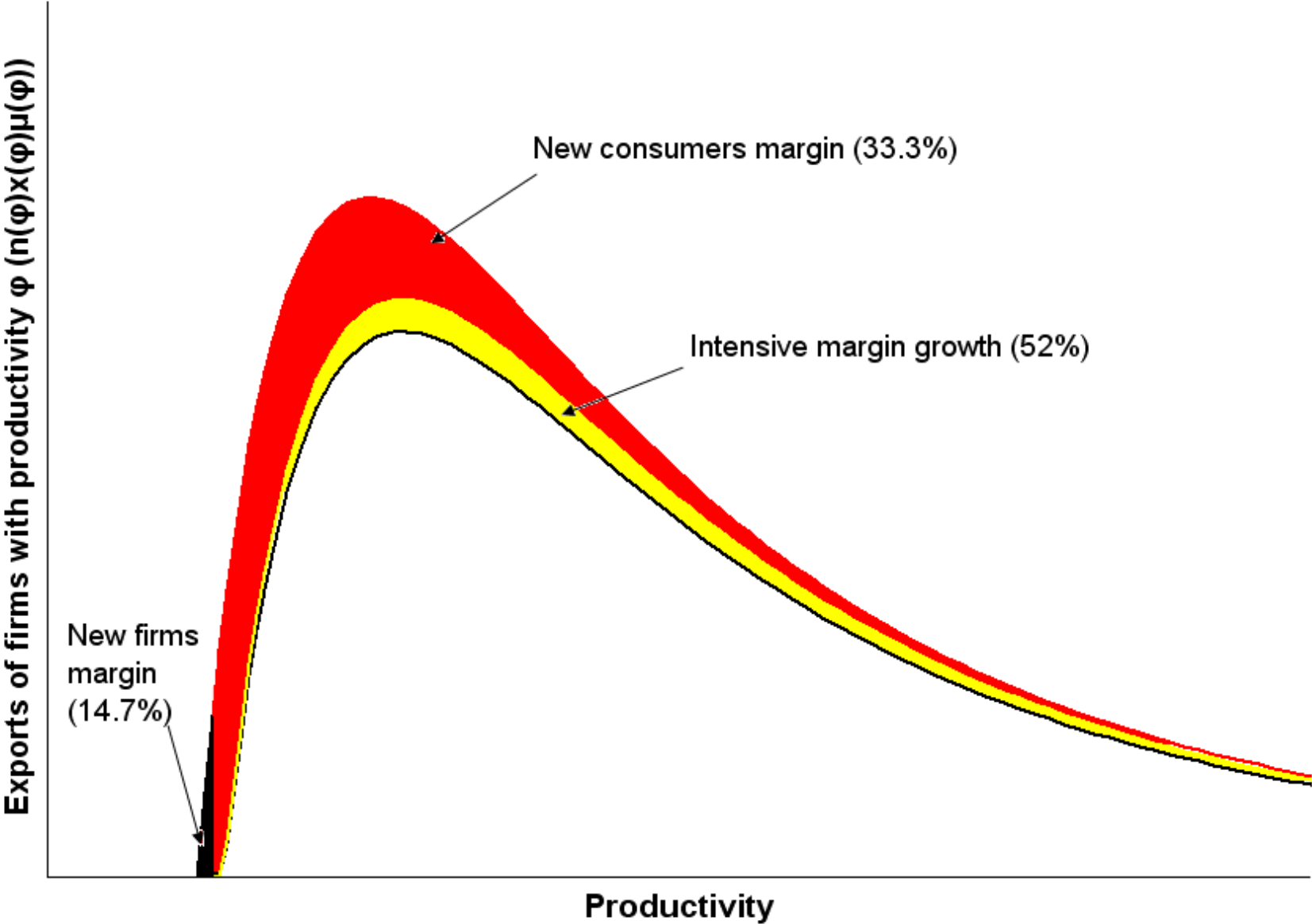
## New Consumers Margin and new trade



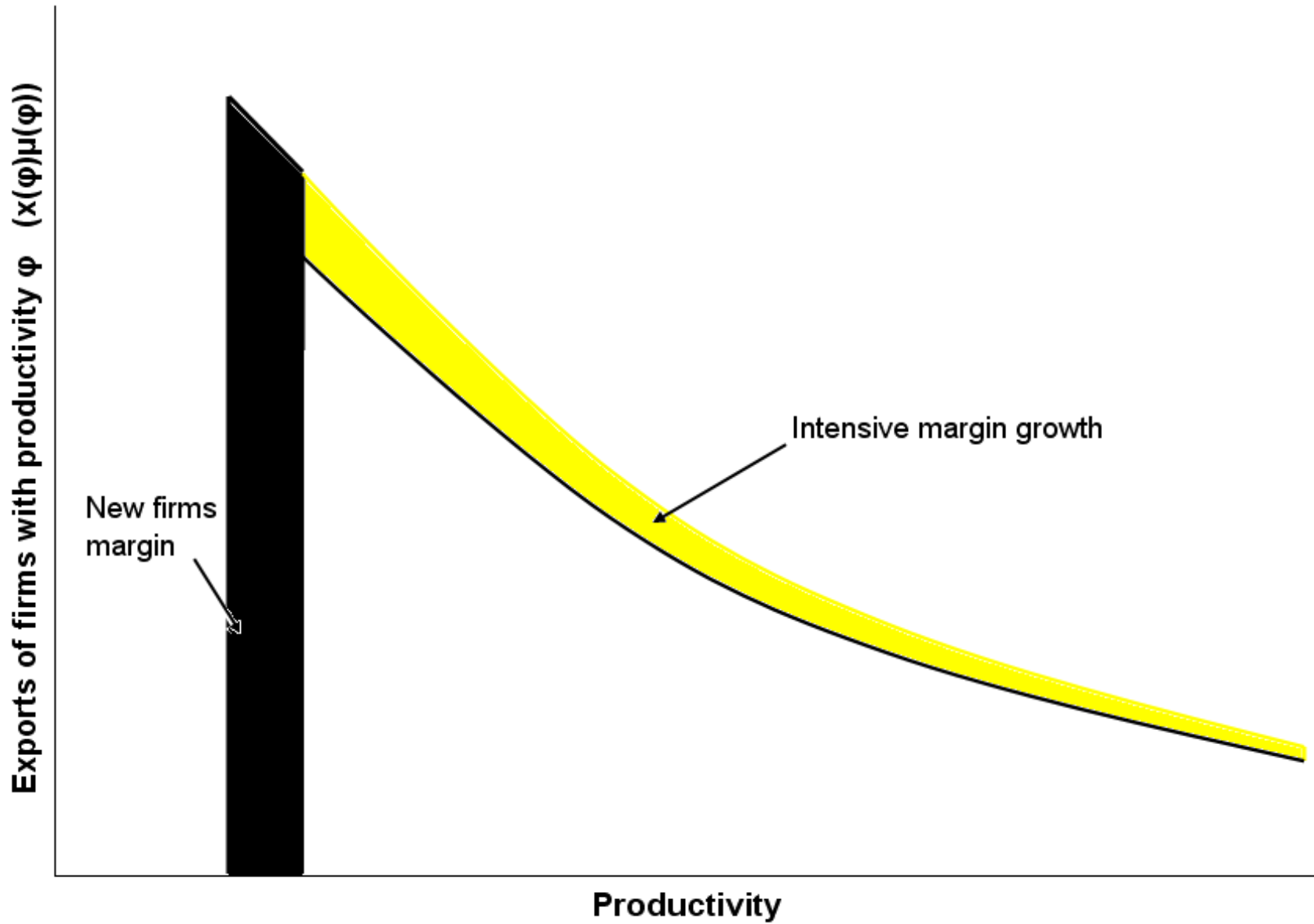
# New Consumers Margin and new trade



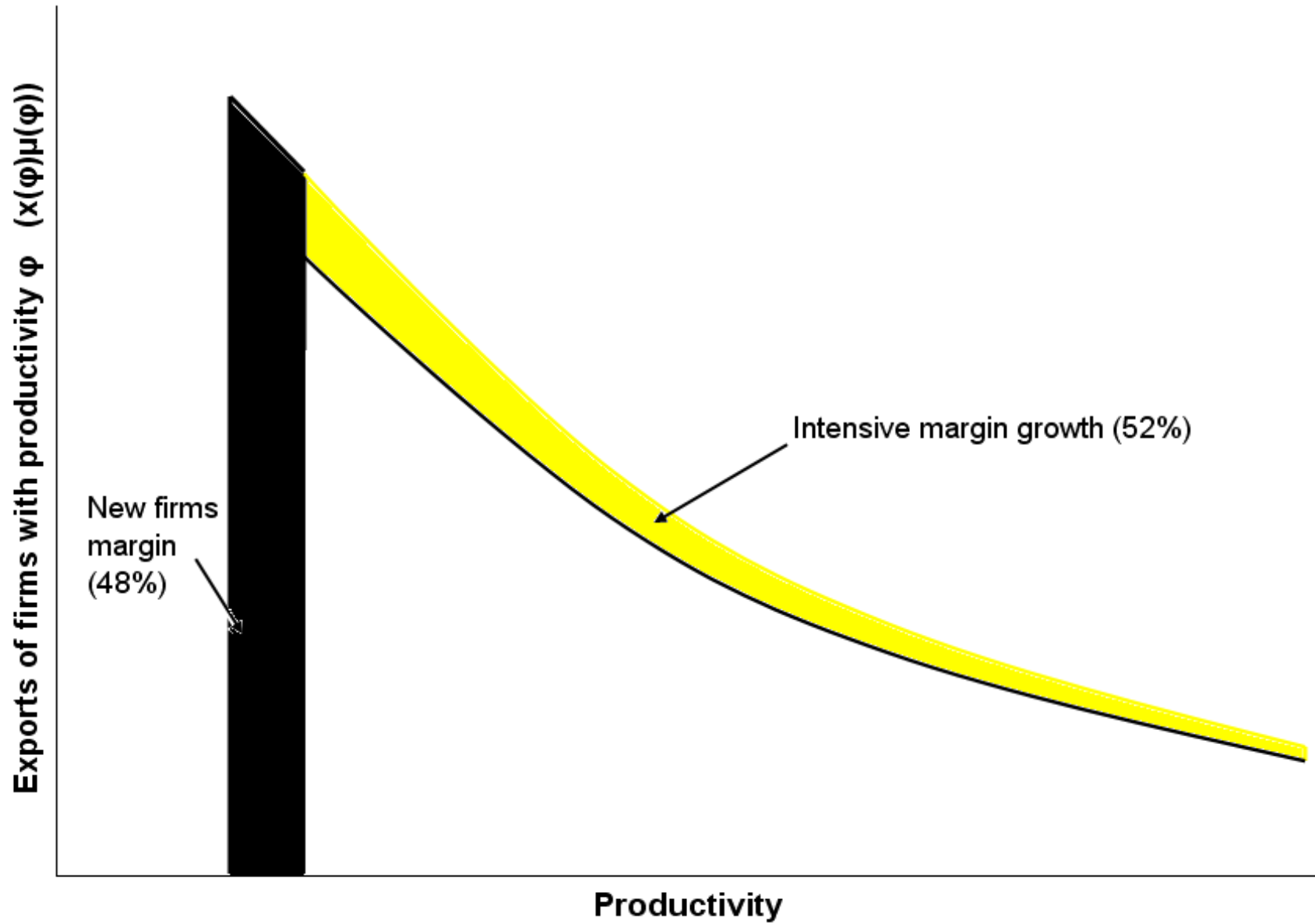
# New Consumers Margin and new trade



# New Firms Margin and the Fixed Cost model ( $\beta = 0$ )



# New Firms Margin and new trade ( $\beta = 0$ )



**7. Growth theory needs to be reconsidered in the light of trade theory. In particular, a growth model that includes trade can have the opposite convergence properties from a model of closed economies.**

C. Bajona and T. J. Kehoe, “Trade, Growth, and Convergence in a Dynamic Heckscher-Ohlin Model,” Federal Reserve Bank of Minneapolis, 2006.

## Trade and Growth

In 2004 Mexico has income per capita of 6500 U.S. dollars. In 1935 the United States had income per capita of about 6600 U.S. dollars (real 2004 U.S. dollars).

To study what will happen in Mexico over the next 70 years, should we study what happened to the United States since 1935?

...or should we take into account that the United States was the country with the highest income in the world in 1935, while Mexico has a very large trade relation with the United States — a country with a level of income per capita approximately 6 times larger in 2004?

We study this question using the Heckscher-Ohlin model of international trade: Countries differ in their initial endowments of capital per worker.



# The General Dynamic Heckscher-Ohlin Model

$n$  countries

countries differ in initial capital-labor ratios  $\bar{k}_0^i$   
and in size of population  $L^i$ .

two traded goods — a capital intensive good and a labor intensive good

$$y_j = \phi_j(k_j, \ell_j)$$

$$\frac{\phi_{1L}(k/\ell, 1)}{\phi_{1K}(k/\ell, 1)} < \frac{\phi_{2L}(k/\ell, 1)}{\phi_{2K}(k/\ell, 1)}$$

nontraded investment good

$$x = f(x_1, x_2)$$

Feasibility:

$$\sum_{i=1}^n L^i (c_{jt}^i + x_{jt}^i) = \sum_{i=1}^n L^i y_{jt}^i = \sum_{i=1}^n L^i \phi_j(k_{jt}^i, \ell_{jt}^i).$$

$$k_{1t}^i + k_{2t}^i = k_t^i$$

$$\ell_{1t}^i + \ell_{2t}^i = 1$$

$$k_{t+1}^i - (1 - \delta)k_t^i = x_t^i = f(x_{1t}^i, x_{2t}^i)$$

## Infinitely-Lived Consumers

consumer in country  $i$ ,  $i = 1, \dots, n$ :

$$\begin{aligned} & \max \sum_{t=0}^{\infty} \beta^t u(c_{1t}^i, c_{2t}^i) \\ \text{s.t. } & p_{1t} c_{1t}^i + p_{2t} c_{2t}^i + q_t^i x_t^i + b_{t+1}^i = w_t^i + (1 + r_t^{bi}) b_t^i + r_t^i k_t^i \\ & k_{t+1}^i - (1 - \delta) k_t^i = x_t^i \\ & c_{jt}^i \geq 0, x_t^i \geq 0, b_t^i \geq -B \\ & k_0^i = \bar{k}_0^i, b_0^i = 0. \end{aligned}$$

Notice that since  $p_{1t}$  and  $p_{2t}$  are equalized across countries by trade, we can set

$$q_t^i = q_t = 1.$$

The factor prices  $w_t^i$  and  $r_t^i$  are potentially different across countries.  
International borrowing and lending:

$$\sum_{i=1}^n L^i b_t^i = 0,$$

No international borrowing and lending:

$$b_t^i = 0.$$

International borrowing and lending implies that  $r_t^{bi} = r_t^b$ ,  $t = 1, 2, \dots$ . No arbitrage implies that  $r_t^i = r_t = r_t^b + \delta$ .

## **Integrated Equilibrium Approach**

Characterization and computation of equilibrium is relatively easy when we can solve for equilibrium of an artificial world economy in which we ignore restrictions on factor mobility and then disaggregate the consumption, production, and investment decisions.

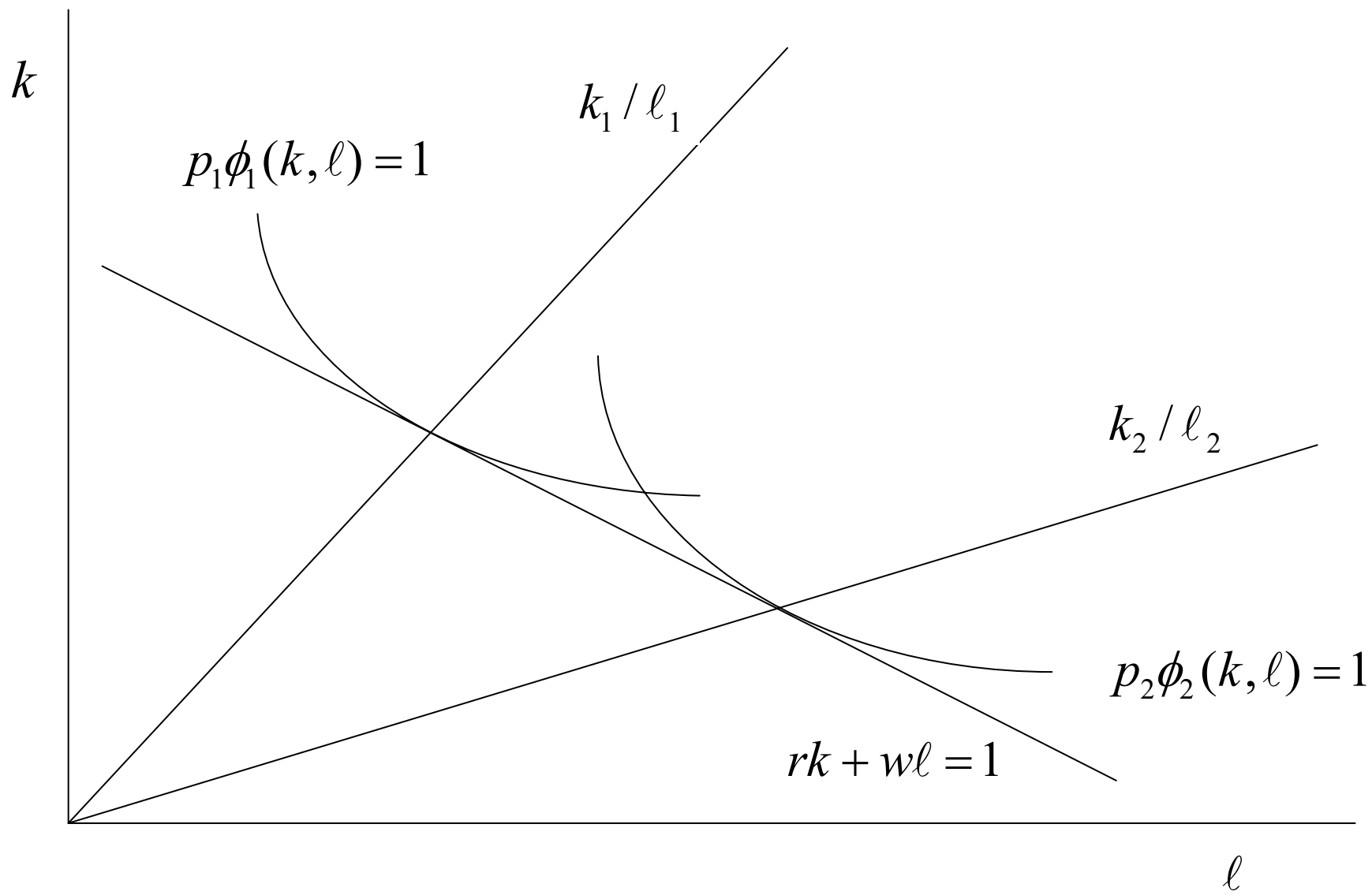
This is a guess-and-verify approach: We first solve for the integrated equilibrium of the world economy and then we see if we can disaggregate the consumption, production, and investment decisions.

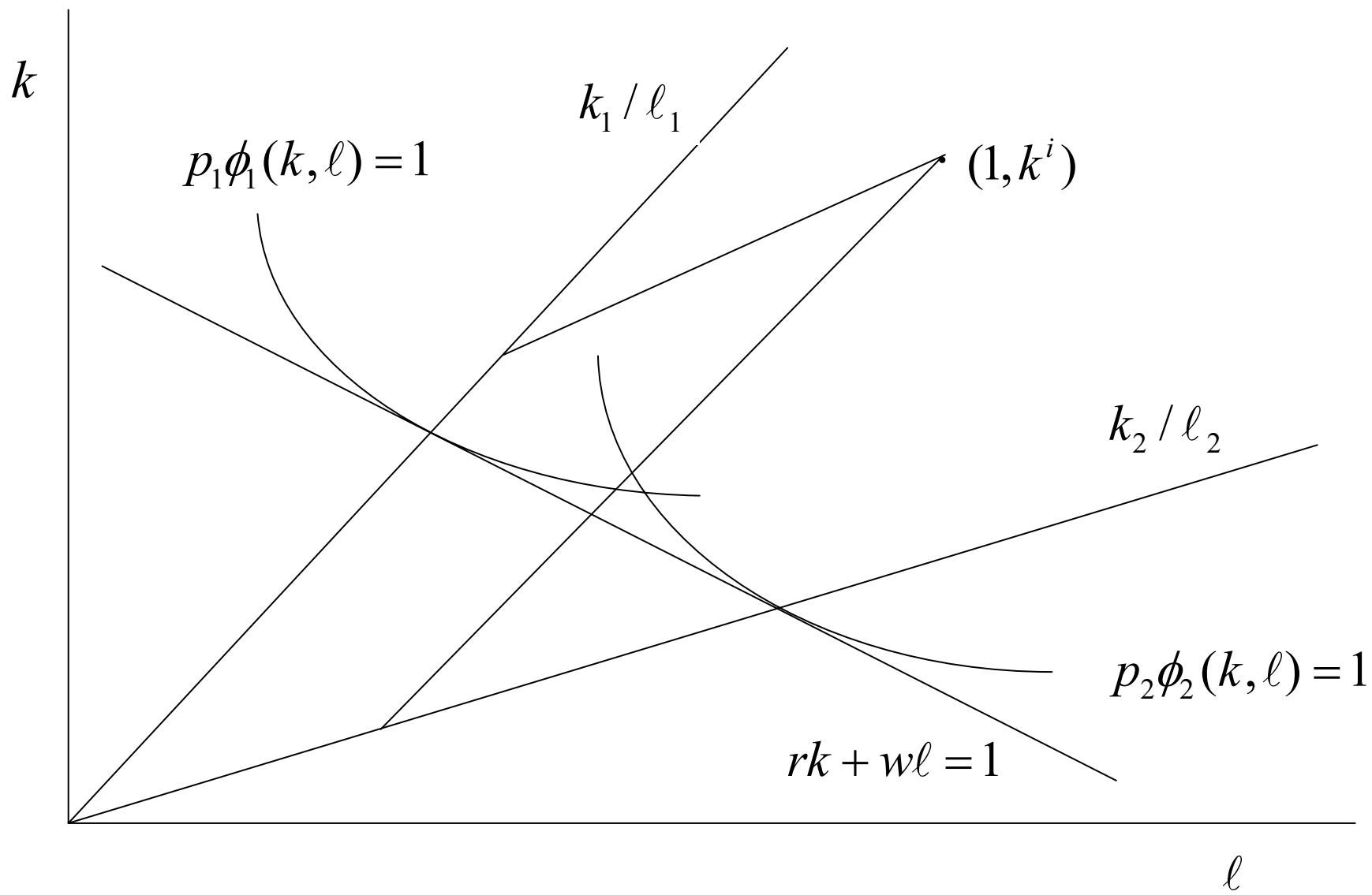
Potential problem: We cannot assign each country nonnegative production plans for each of the two goods while maintaining factor prices equal to those in the world equilibrium.

Another potential problem: We cannot assign each country nonnegative investment.

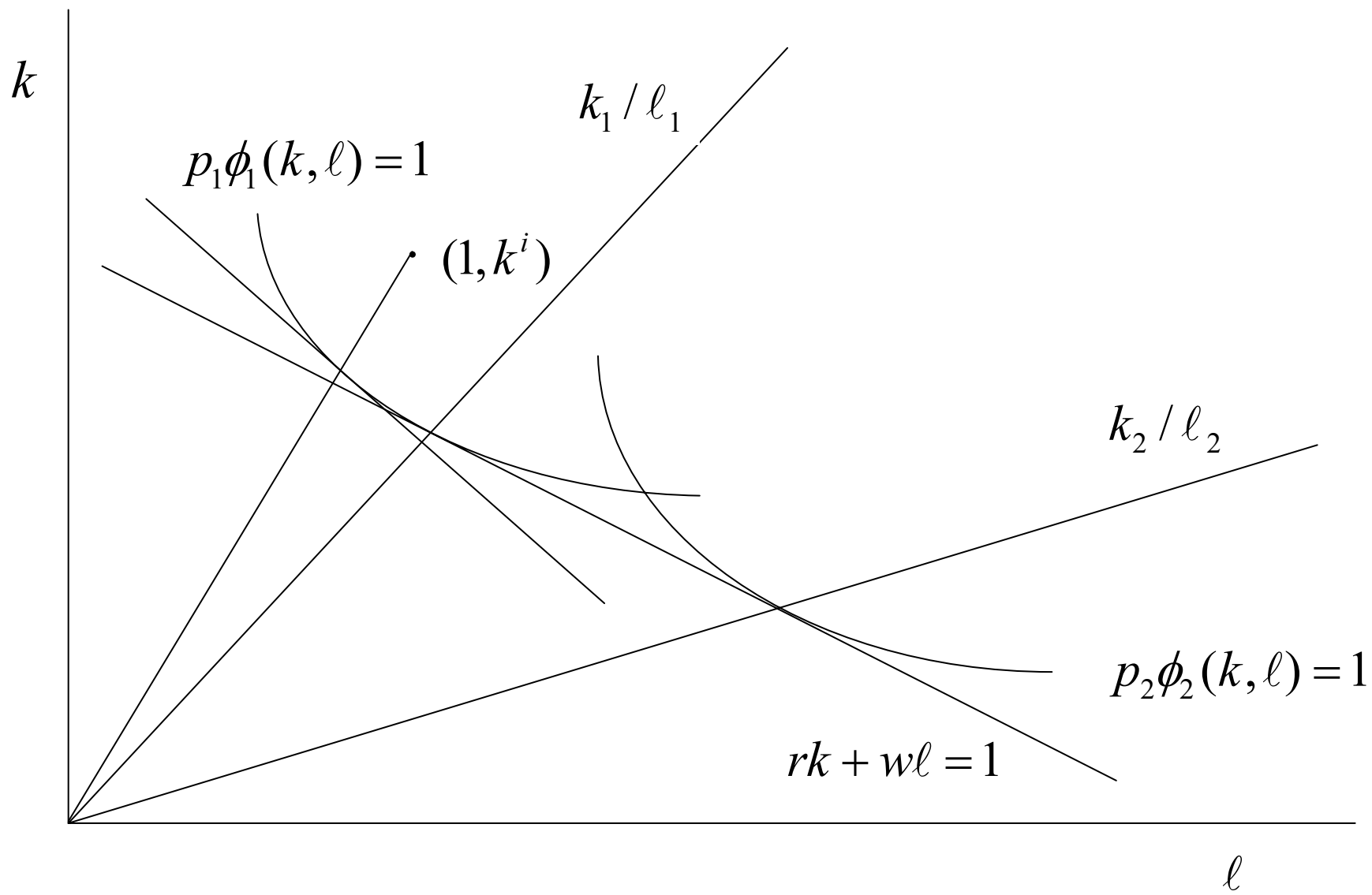
If the integrated equilibrium approach does not work, it could be very difficult to calculate an equilibrium.

We would have to determine the pattern of specialization over an infinite time horizon.









## Results for General Model

International borrowing and lending implies factor price equalization in period  $t = 1, 2, \dots$ . Production plans and international trade patterns are indeterminate.

Any steady state or sustained growth path has factor price equalization.

If there exists a steady state in which the total capital stock is positive or a sustained growth path, then there exists a continuum of such steady states or sustained growth paths, indexed by the distribution of world capital  $\hat{k}^1 / \hat{k}, \dots, \hat{k}^n / \hat{k}$ .

International trade occurs in every steady state or sustained growth path of the model in which  $\hat{k}^i / \hat{k} \neq 1$  for some  $i$ .

**We focus on models with no international borrowing and lending.**

For analysis of general model with infinitely lived consumers and comparison with model with overlapping generations, see

C. Bajona and T. J. Kehoe (2006), “Demographics in Dynamic Heckscher-Ohlin Models: Overlapping Generations versus Infinitely Lived Consumers.”

## Ventura Model

$$u(c_1, c_2) = v(f(c_1, c_2)) = \log(f(c_1, c_2))$$

$$\phi_1(k_1, \ell_1) = k_1$$

$$\phi_2(k_2, \ell_2) = \ell_2$$

$$f(x_1, x_2) = \begin{cases} d(a_1 x_1^b + a_2 x_2^b)^{1/b} & \text{if } b \neq 0 \\ dx_1^{a_1} x_2^{a_2} & \text{if } b = 0 \end{cases}.$$

Ventura (1997) examines the continuous-time version of this model.

In the Ventura model, we can solve for the equilibrium of the world economy by solving a one-sector growth model in which  $c_t = f(c_{1t}, c_{2t})$ :

$$\begin{aligned} & \max \sum_{t=0}^{\infty} \beta^t \log c_t \\ & \text{s.t. } c_t + x_t = f(k_t, 1) \\ & \quad k_{t+1} - (1 - \delta)k_t = x_t \\ & \quad c_t \geq 0, k_t \geq 0 \\ & \quad k_0 = \bar{k}_0. \end{aligned}$$

If  $b < 0$  and  $1/\beta - 1 + \delta > da_1^{1/b}$ , the equilibrium converges to  $\hat{k} = 0$ .

If  $b > 0$  and  $1/\beta - 1 + \delta < da_1^{1/b}$ , the economy grows without bound, and the equilibrium converges to a sustained growth path.

In every other case, the equilibrium converges to a steady state in which  $f_K(\hat{k}, 1) = 1/\beta - 1 + \delta$ .

The 2 sectors matter a lot for disaggregating the integrated equilibrium!

In particular, we cannot solve for the equilibrium values of the variables for one of the countries by solving an optimal growth problem for that country in isolation.

Instead, the equilibrium path for  $k_t^i$  and the steady state value of  $\hat{k}^i$  depends on  $\bar{k}_0^i$  as well as on the path for  $k_t$  and the steady state value of  $\hat{k}$ .

**Proposition:** Let  $y_t^i = p_{1t}y_{1t}^i + p_{2t}y_{2t}^i = r_t k_t^i + w_t$ . Suppose that  $x_t^i > 0$  for all  $i$  and all  $t$ . Then

$$\frac{y_{t+1}^i - y_{t+1}}{y_{t+1}} = \frac{r_{t+1}c_t / y_{t+1}}{r_t c_{t-1} / y_t} \left( \frac{y_t^i - y_t}{y_t} \right)$$

If  $\delta = 1$ ,

$$\frac{y_{t+1}^i - y_{t+1}}{y_{t+1}} = \frac{s_{t+1}}{s_t} \left( \frac{y_t^i - y_t}{y_t} \right)$$

where  $s_t = c_t / y_t$ .

**Proof:** The first-order conditions from the consumers' problems are

$$\frac{c_t^i}{c_{t-1}^i} = \frac{c_t}{c_{t-1}} = \beta(1 + r_t - \delta).$$

The demand functions are

$$c_t^i = (1 - \beta) \left[ \sum_{s=t}^{\infty} \left( \prod_{\tau=t+1}^s \frac{1}{1 + r_{\tau} - \delta} \right) w_s + (1 + r_t - \delta) k_t^i \right]$$

$$c_t^i - c_t = (1 - \beta)(1 + r_t - \delta)(k_t^i - k_t).$$

The budget constraint implies that

$$c_t^i - c_t + k_{t+1}^i - k_{t+1} = (1 + r_t - \delta)(k_t^i - k_t).$$

Combining these conditions, we obtain

$$k_{t+1}^i - k_{t+1} = \frac{c_t}{c_{t-1}} (k_t^i - k_t).$$



The difference between a country's income per worker and the world's income per worker can be written as

$$y_{t+1}^i - y_{t+1} = r_{t+1}(k_{t+1}^i - k_{t+1}).$$

Using the expression for  $k_{t+1}^i - k_{t+1}$  found above and operating, we obtain:

$$\frac{y_{t+1}^i - y_{t+1}}{y_{t+1}} = \frac{r_{t+1}c_t / y_{t+1}}{r_t c_{t-1} / y_t} \left( \frac{y_t^i - y_t}{y_t} \right).$$

In the case  $\delta = 1$  this becomes (using  $c_{t+1} / c_t = \beta r_{t+1}$ ),

$$\frac{y_{t+1}^i - y_{t+1}}{y_{t+1}} = \frac{s_{t+1}}{s_t} \left( \frac{y_t^i - y_t}{y_t} \right),$$

where  $s_t = c_t / y_t$ . ■

$$\frac{y_t^i - y_t}{y_t} = \frac{s_t}{s_0} \left( \frac{y_0^i - y_0}{y_0} \right)$$

**Proposition.** Suppose that  $\delta = 1$ , that  $0 < k_0 < \hat{k}$ , and that  $x_t^i > 0$  for all  $i$  and all  $t$ . Then

if  $b > 0$ , differences in relative income levels decrease over time;

if  $b = 0$ , differences in relative income levels stay constant over time;  
and

if  $b < 0$ , differences in relative income levels increase over time.

$$\frac{y_t^i - y_t}{y_t} = \frac{s_t}{s_0} \left( \frac{y_0^i - y_0}{y_0} \right)$$

**Proposition.** Suppose that  $\delta = 1$ , that  $0 < k_0 < \hat{k}$ , and that  $x_t^i > 0$  for all  $i$  and all  $t$ . Then

if  $b > 0$ , differences in relative income levels decrease over time;

if  $b = 0$ , differences in relative income levels stay constant over time;  
and

if  $b < 0$ , differences in relative income levels increase over time.

**Notice contrast with convergence results for world of closed economies!**

## What about corner solutions in investment?

If  $x_t^i > 0$  for all  $i$  and all  $t$ , then

$$\frac{k_{t+1}^i - k_{t+1}}{k_{t+1}} = \frac{c_t / k_{t+1}}{c_{t-1} / k_t} \left( \frac{k_t^i - k_t}{k_t} \right) = \frac{z_{t+1}}{z_t} \left( \frac{k_t^i - k_t}{k_t} \right)$$

where  $z_t = c_{t-1} / k_t$  and  $z_0 = c_0 / (\beta r_0 k_0)$ .

The sequence  $z_t$  has the same monotonicity properties as the sequence  $s_t = c_t / y_t$ .

**Proposition:** Suppose that the sequence  $s_t = c_t / y_t$  in the equilibrium of the integrated economy is constant or strictly decreasing. There exists an equilibrium where  $x_t^i > 0$  for all  $i$  and all  $t$ .

**Proposition:** Suppose that the sequence  $s_t = c_t / y_t$  in the equilibrium of the integrated economy is strictly increasing. Let

$$\hat{z} = \lim_{t \rightarrow \infty} \frac{c_{t-1}}{k_t},$$

and let  $\bar{k}_0^{i_{min}} \leq \bar{k}_0^i$ ,  $i = 1, \dots, n$ . If

$$\frac{\hat{z}}{z_0} \left( \frac{\bar{k}_0^{i_{min}} - \bar{k}_0}{\bar{k}_0} \right) \geq -1,$$

then there exists an equilibrium where  $x_t^i > 0$  for all  $i$  and all  $t$ .

Otherwise, there is no equilibrium where  $x_t^i > 0$  for all  $i$  and all  $t$ . When there exists an equilibrium with no corner solutions in investment, it is the unique such equilibrium.

**Numerical example 1:** Two countries.  $\beta = 0.95$ ,  $\delta = 1$ , and  $L^1 = L^2 = 10$ .

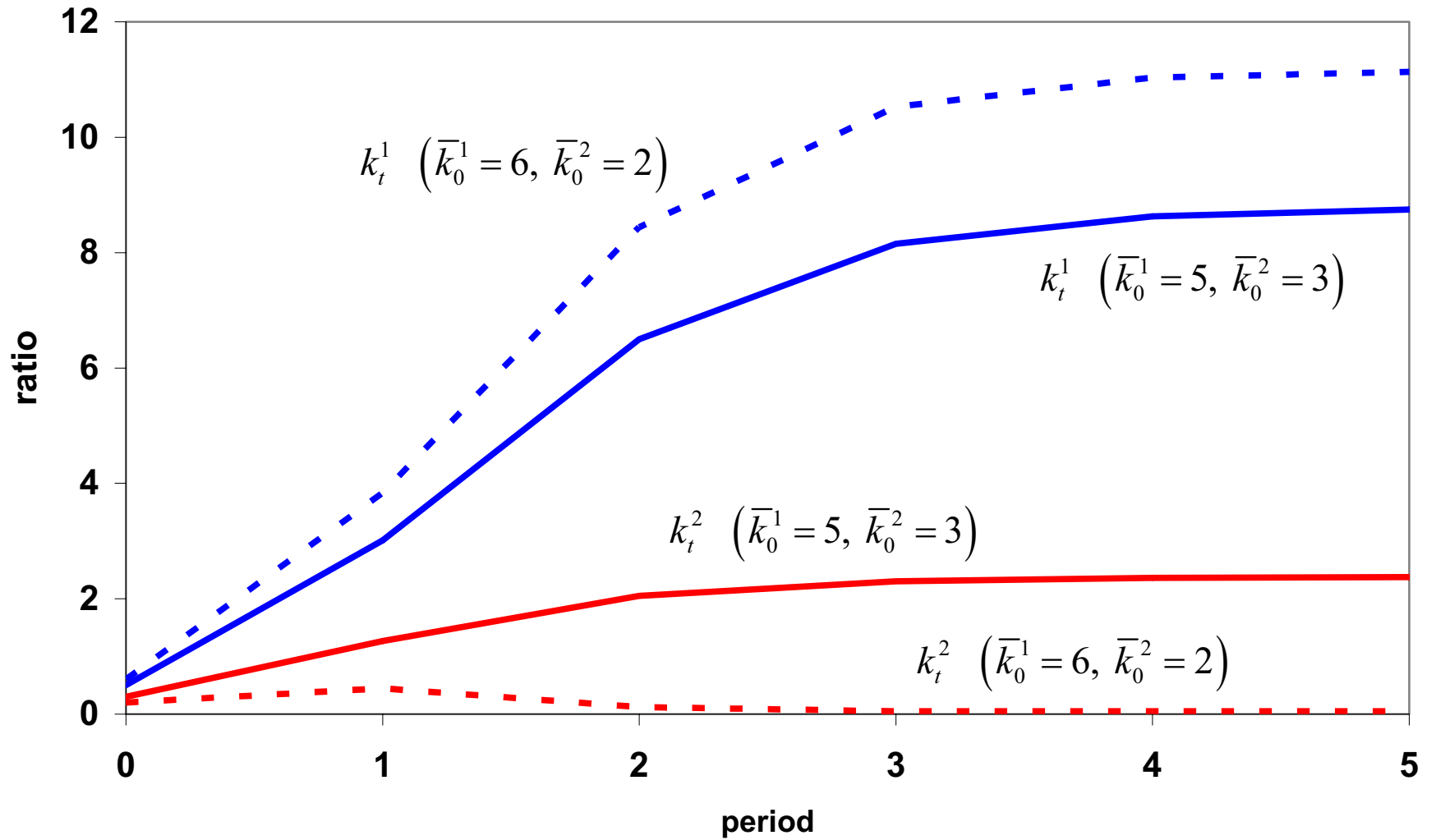
$$f(x_1, x_2) = 10 \left( 0.5x_1^{-0.5} + 0.5x_2^{-0.5} \right)^{-2}.$$

We contrast two different worlds:

In the first world,  $\bar{k}_0^1 = 5$  and  $\bar{k}_0^2 = 3$ . Here there is an equilibrium with no corner solutions for investment.

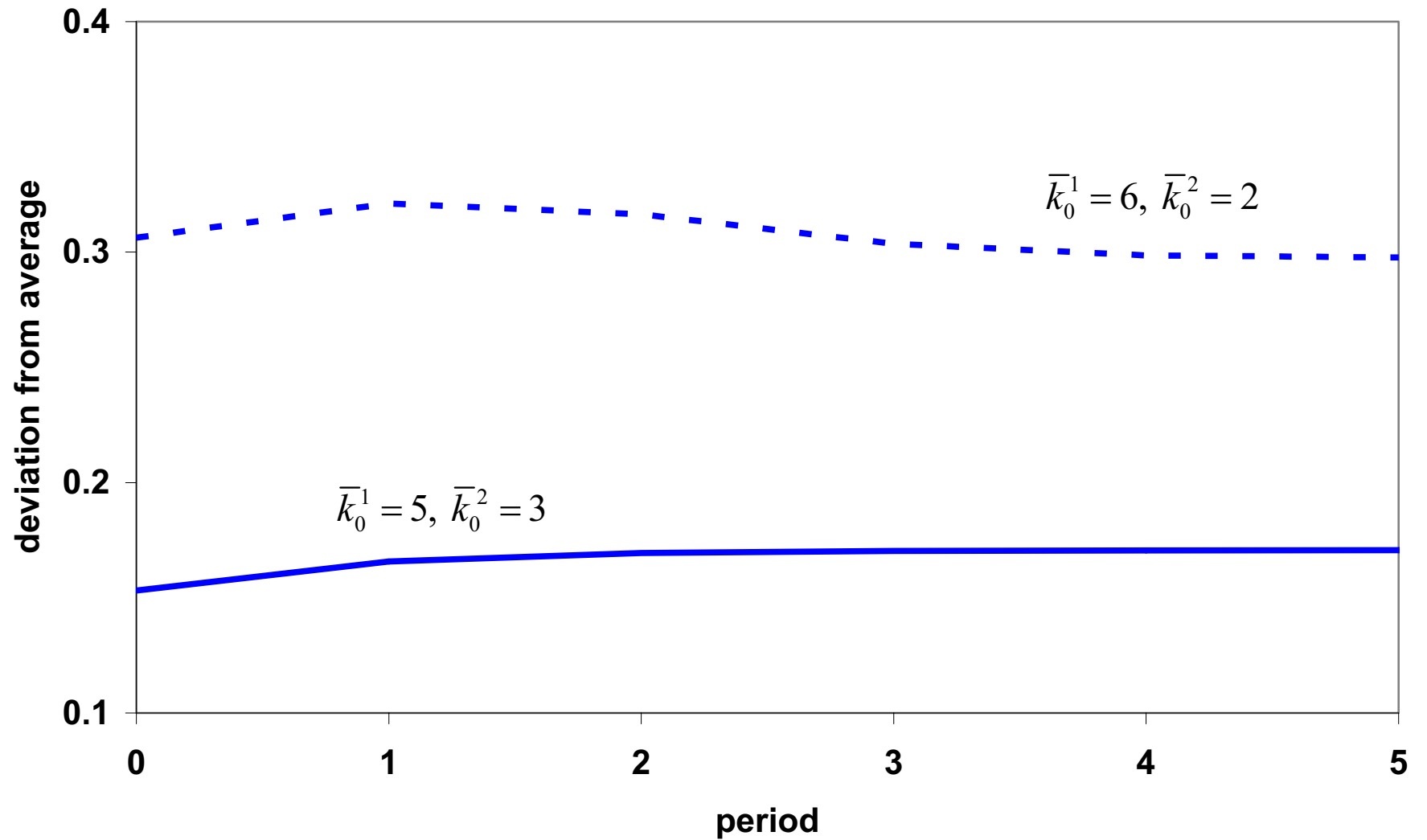
In the second world,  $\bar{k}_0^1 = 6$  and  $\bar{k}_0^2 = 2$ . Country 2 has  $x_t^i = k_t^i = 0$  starting in period 3.

# Example 1: Capital-labor ratios





## Example 1: Relative income in country 1



## Generalized Ventura Model

$u(c_1, c_2) = v(f(c_1, c_2)) = \log(f(c_1, c_2))$ , and  $f$ ,  $\phi_1$ , and  $\phi_2$  are general constant-elasticity-of-substitution functions

Define

$$F(k, \ell) = \max f(y_1, y_2)$$

$$\text{s.t. } y_1 = \phi_1(k_1, \ell_1)$$

$$y_2 = \phi_2(k_2, \ell_2)$$

$$k_1 + k_2 = k$$

$$\ell_1 + \ell_2 = \ell$$

$$k_j \geq 0, \ell_j \geq 0.$$

In Ventura model  $F(k, \ell) = f(k, \ell)$ .

## C. E. S. Model

$$y_1 = \phi_1(k_1, \ell_1) = \theta_1 \left( \alpha_1 k_1^b + (1 - \alpha_1) \ell_1^b \right)^{1/b}$$

$$y_2 = \phi_2(k_2, \ell_2) = \theta_2 \left( \alpha_2 k_2^b + (1 - \alpha_2) \ell_2^b \right)^{1/b}$$

$$f(y_1, y_2) = d \left( a_1 y_1^b + a_2 y_2^b \right)^{1/b}$$

(All elasticities of substitution are equal.)

In this case,

$$F(k, \ell) = D \left( A_1 k^b + A_2 \ell^b \right)^{1/b}$$

where

$$A_1 = \frac{\left[ \left( a_1 \alpha_1 \theta_1^b \right)^{\frac{1}{1-b}} + \left( a_2 \alpha_2 \theta_2^b \right)^{\frac{1}{1-b}} \right]^{1-b}}{\left[ \left( a_1 \alpha_1 \theta_1^b \right)^{\frac{1}{1-b}} + \left( a_2 \alpha_2 \theta_2^b \right)^{\frac{1}{1-b}} \right]^{1-b} + \left[ \left( a_1 (1 - \alpha_1) \theta_1^b \right)^{\frac{1}{1-b}} + \left( a_2 (1 - \alpha_2) \theta_2^b \right)^{\frac{1}{1-b}} \right]^{1-b}}$$

$$A_2 = 1 - A_1$$

$$D = d \left\{ \left[ \left( a_1 \alpha_1 \theta_1^b \right)^{\frac{1}{1-b}} + \left( a_2 \alpha_2 \theta_2^b \right)^{\frac{1}{1-b}} \right]^{1-b} + \left[ \left( a_1 (1 - \alpha_1) \theta_1^b \right)^{\frac{1}{1-b}} + \left( a_2 (1 - \alpha_2) \theta_2^b \right)^{\frac{1}{1-b}} \right]^{1-b} \right\}^{\frac{1}{b}} .$$

The cone of diversification for the integrated economy has the form  $\bar{\kappa}_1 k_t \geq k_t^i \geq \bar{\kappa}_2 k_t$ .

$$\bar{\kappa}_i = \left( \frac{\alpha_i}{1 - \alpha_i} \right)^{\frac{1}{1-b}} \frac{\left( a_1 (1 - \alpha_1) \theta_1^b \right)^{\frac{1}{1-b}} + \left( a_2 (1 - \alpha_2) \theta_2^b \right)^{\frac{1}{1-b}}}{\left( a_1 \alpha_1 \theta_1^b \right)^{\frac{1}{1-b}} + \left( a_2 \alpha_2 \theta_2^b \right)^{\frac{1}{1-b}}}.$$

The cone of diversification for the integrated economy has the form  $\bar{\kappa}_1 k_t \geq k_t^i \geq \bar{\kappa}_2 k_t$ .

$$\bar{\kappa}_i = \left( \frac{\alpha_i}{1 - \alpha_i} \right)^{\frac{1}{1-b}} \frac{\left( a_1 (1 - \alpha_1) \theta_1^b \right)^{\frac{1}{1-b}} + \left( a_2 (1 - \alpha_2) \theta_2^b \right)^{\frac{1}{1-b}}}{\left( a_1 \alpha_1 \theta_1^b \right)^{\frac{1}{1-b}} + \left( a_2 \alpha_2 \theta_2^b \right)^{\frac{1}{1-b}}}.$$

This is not the cone of diversification when factor prices are not equalized.

$$\kappa_1(p_2 / p_1) = \left( \frac{\alpha_1}{1 - \alpha_1} \right)^{\frac{1}{1-b}} \left[ \frac{(1 - \alpha_2)^{\frac{1}{1-b}} (\theta_2 p_2 / p_1)^{\frac{b}{1-b}} - (1 - \alpha_1)^{\frac{1}{1-b}} \theta_1^{\frac{b}{1-b}}}{\alpha_1^{\frac{1}{1-b}} \theta_1^{\frac{b}{1-b}} - \alpha_2^{\frac{1}{1-b}} (\theta_2 p_2 / p_1)^{\frac{b}{1-b}}} \right]^{\frac{1}{b}}$$

$$\kappa_1(p_2 / p_1) = \left[ \left( \frac{\alpha_2}{1 - \alpha_2} \right) \left( \frac{1 - \alpha_1}{\alpha_1} \right) \right]^{\frac{1}{1-b}} \kappa_2(p_2 / p_1).$$

## Cobb-Douglas Model

$$y_1 = \phi_1(k_1, l_1) = \theta_1 k_1^{\alpha_1} l_1^{1-\alpha_1}$$

$$y_2 = \phi_2(k_2, l_2) = \theta_2 k_2^{\alpha_2} l_2^{1-\alpha_2}$$

$$f(y_1, y_2) = d y_1^{a_1} y_2^{a_2}$$

(This is the special case of the C.E.S. model where  $b = 0$ .)

In this case

$$F(k, \ell) = Dk^{A_1} \ell^{A_2}$$

where

$$A_1 = a_1 \alpha_1 + a_2 \alpha_2$$

$$A_2 = 1 - A_1$$

$$D = \frac{d \left[ \theta_1 a_1 \alpha_1^{\alpha_1} (1 - \alpha_1)^{1 - \alpha_1} \right]^{a_1} \left[ \theta_2 a_2 \alpha_2^{\alpha_2} (1 - \alpha_2)^{1 - \alpha_2} \right]^{a_2}}{A_1^{A_1} A_2^{A_2}}$$

$$\bar{\kappa}_i = \left( \frac{\alpha_i}{1 - \alpha_i} \right) \frac{A_2}{A_1}.$$



**Proposition:** In the Cobb-Douglas model with  $\delta = 1$ , suppose that factor price equalization occurs at period  $T$ . Then factor price equalization occurs at all  $t \geq T$ . Furthermore, the equilibrium capital stocks can be solved for as

$$k_t^i = \gamma^i k_t$$

where  $\gamma^i = k_T^i / k_T$  and  $k_{t+1} = \beta A_1 D k_t^{A_1}$  for  $t \geq T$ .

**Proposition:** In the C.E.S. model with  $\delta = 1$ , suppose that the sequence  $s_t = c_t / y_t$  in the equilibrium of the integrated economy is weakly decreasing. Suppose that factor price equalization occurs in period  $T$ . Then there exists an equilibrium in which factor price equalization occurs at all  $t \geq T$ . Furthermore, this equilibrium is the only such equilibrium.

**Proposition:** In the C.E.S. model with  $\delta = 1$ , suppose that the sequence  $s_t = c_t / y_t$  in the equilibrium of the integrated economy is strictly increasing. Again let  $z_t = c_{t-1} / k_t$ ,  $z_0 = c_0 / (\beta r_0 k_0)$ , and  $\hat{z} = \lim_{t \rightarrow \infty} c_{t-1} / k_t$ . Let  $\bar{k}_0^{i_{min}} \leq \bar{k}_0^i \leq \bar{k}_0^{i_{max}}$ ,  $i = 1, \dots, n$ . If

$$\frac{\hat{z}}{z_0} \left( \frac{\bar{k}_0^{i_{min}} - \bar{k}_0}{\bar{k}_0} \right) \geq \kappa_2 - 1, \quad \frac{\hat{z}}{z_0} \left( \frac{\bar{k}_0^{i_{max}} - \bar{k}_0}{\bar{k}_0} \right) \leq \kappa_1 - 1,$$

then there exists an equilibrium with factor price equalization in every period. If, however, either of these conditions is violated, there is no equilibrium with factor price equalization in every period. When there exists an equilibrium with factor price equalization in every period, it is the unique such equilibrium.

**Numerical example 2:** Two countries.  $\beta = 0.95$ ,  $\delta = 1$ , and

$$L^1 = L^2 = 10.$$

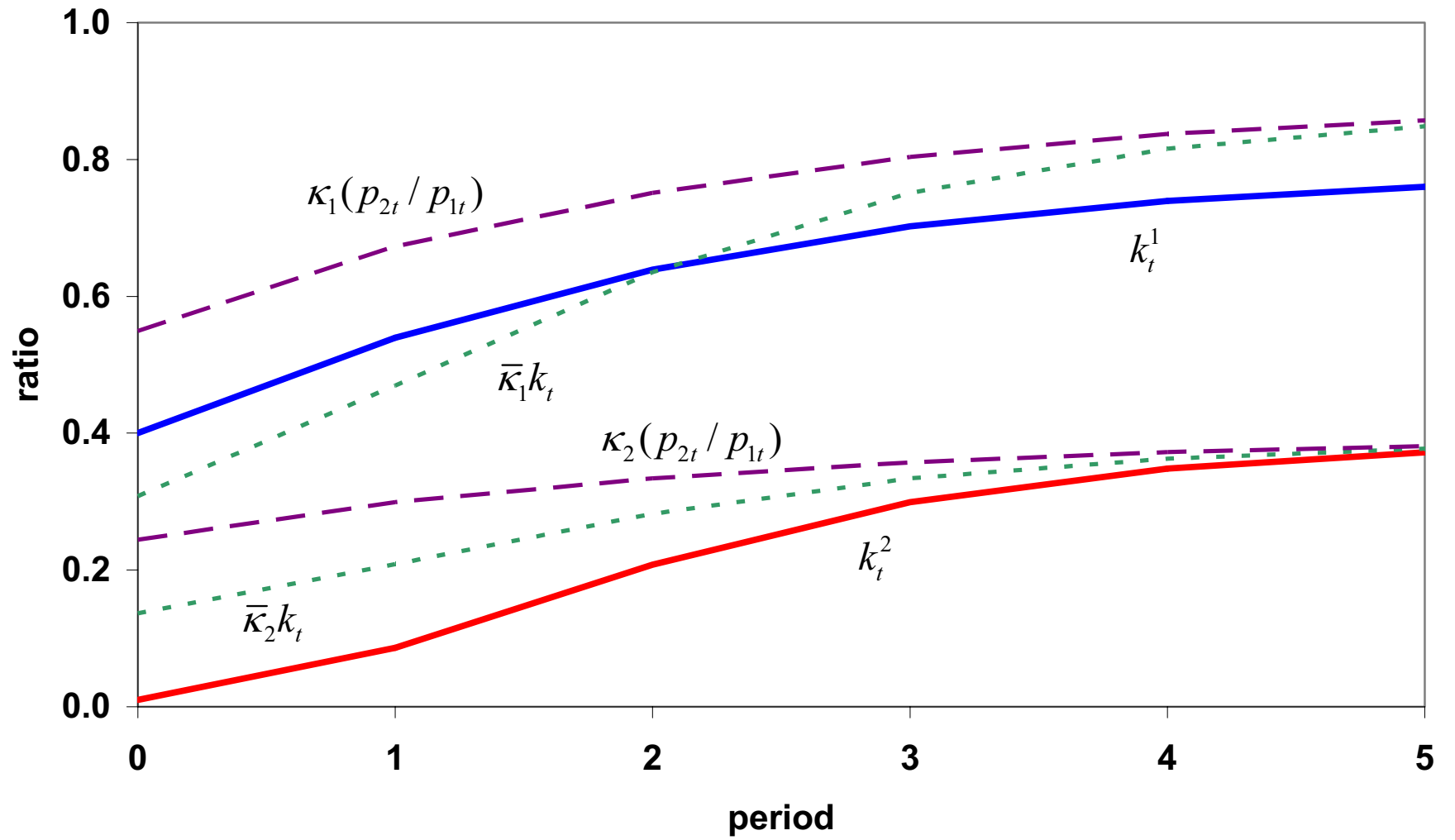
$$\phi_1(k, \ell) = 10k^{0.6}\ell^{0.4}$$

$$\phi_2(k, \ell) = 10k^{0.4}\ell^{0.6}$$

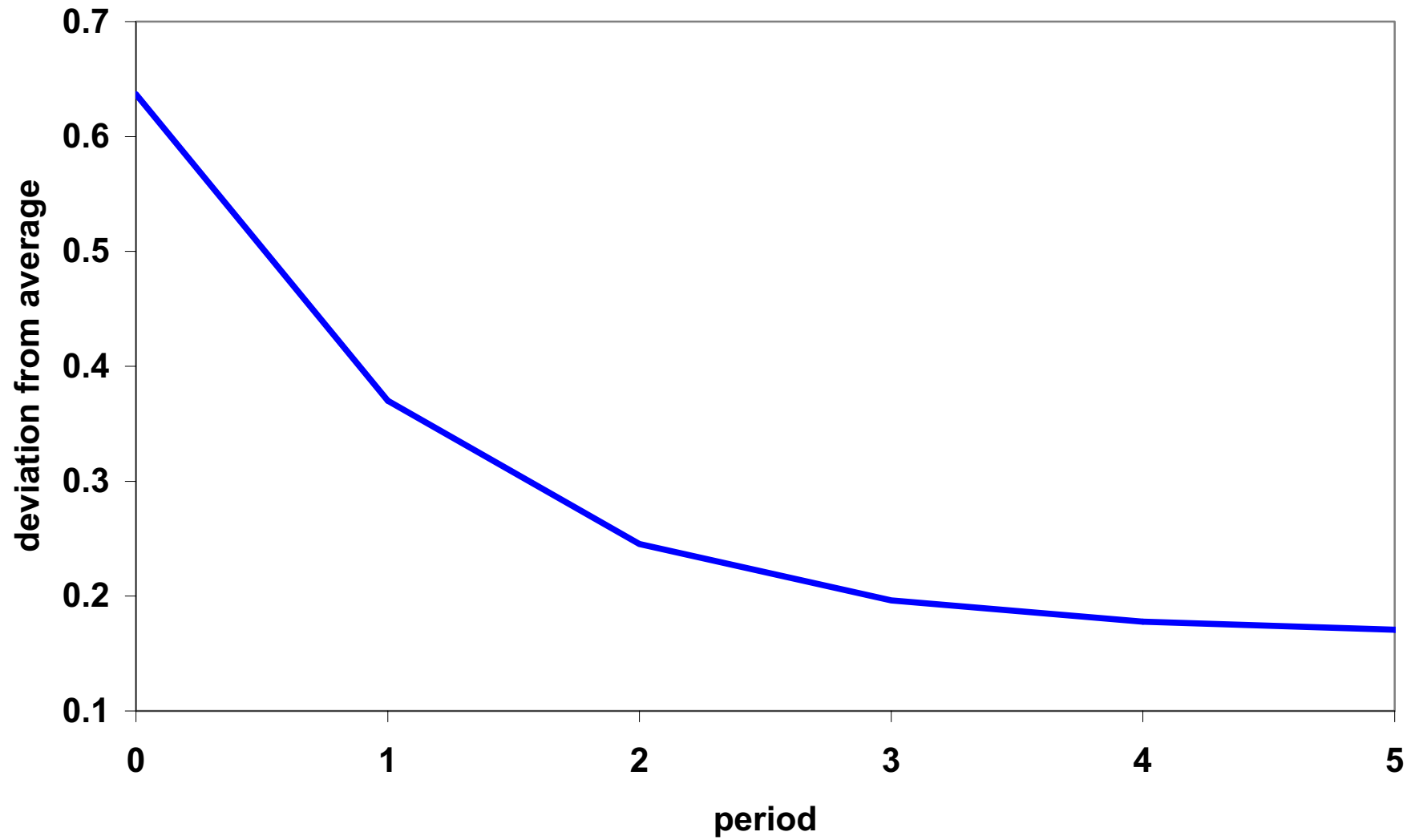
$$f(x_1, x_2) = x_1^{0.5}x_2^{0.5}$$

$$\bar{k}_0^1 = 4, \bar{k}_0^2 = 0.1.$$

## Example 2: Capital-labor ratios



## Example 2: Relative income in country 1



**Numerical example 3:** Two countries.  $\beta = 0.95$ ,  $\delta = 1$ , and

$$L^1 = L^2 = 10.$$

$$\phi_1(k, \ell) = 10 \left( 0.8k^{-0.5} + 0.2\ell^{-0.5} \right)^{-2}$$

$$\phi_2(k, \ell) = 10 \left( 0.2k^{-0.5} + 0.8\ell^{-0.5} \right)^{-2}$$

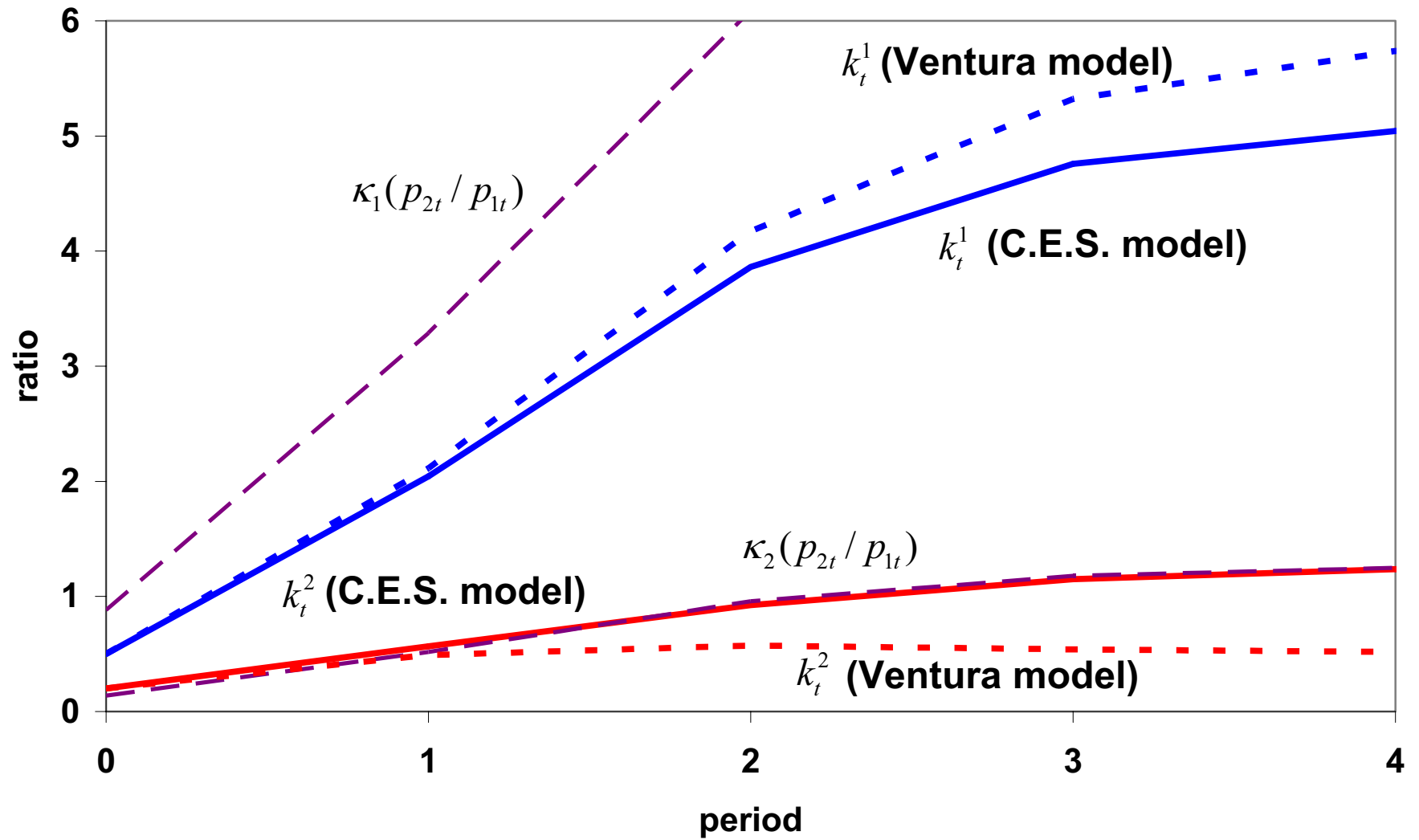
$$f(x_1, x_2) = \left( 0.5x_1^{-0.5} + 0.5x_2^{-0.5} \right)^{-2}$$

$$\bar{k}_0^1 = 5, \bar{k}_0^2 = 2.$$

Contrast with the Ventura model with the same integrated equilibrium:

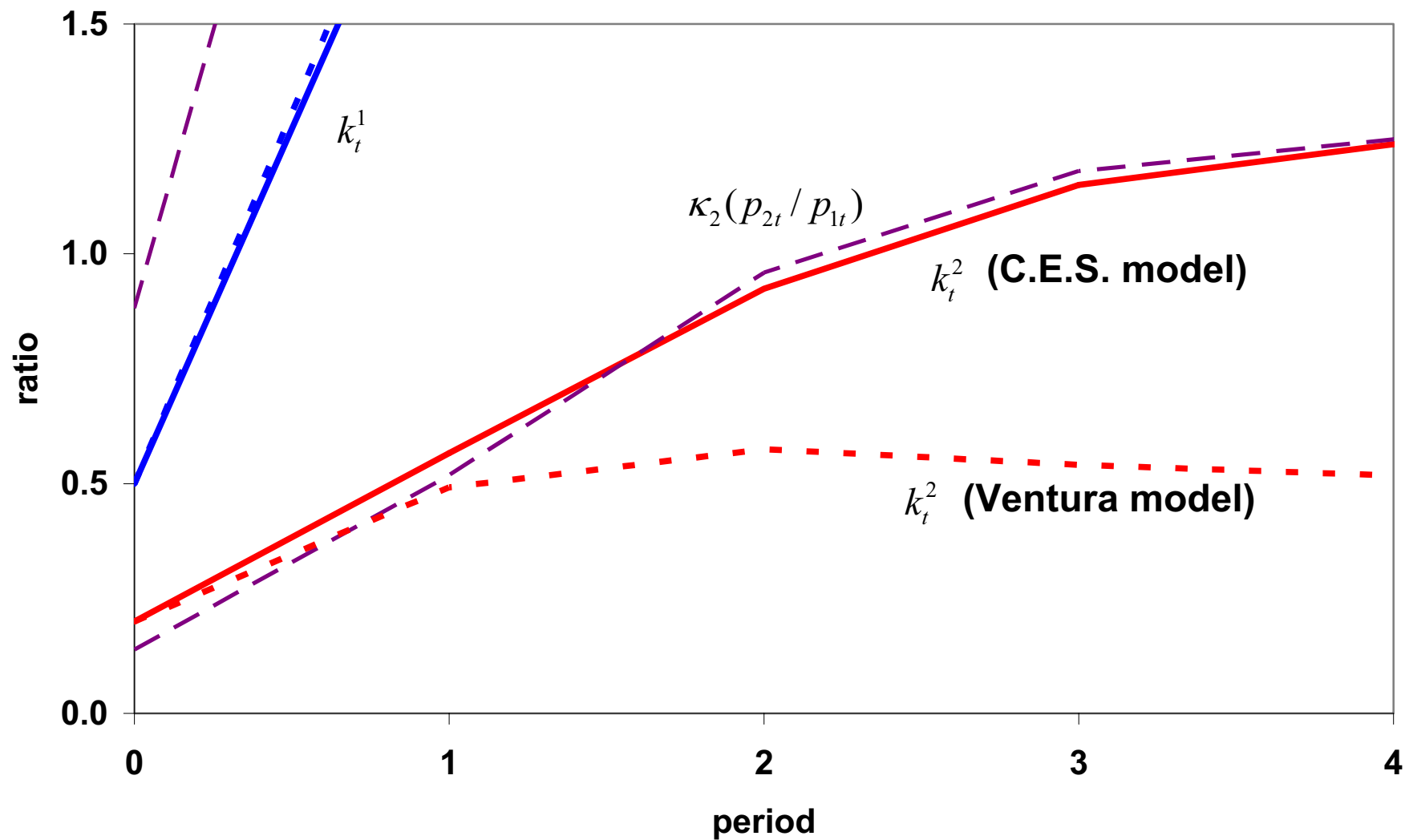
$$f(x_1, x_2) = 5.7328 \left( 0.5x_1^{-0.5} + 0.5x_2^{-0.5} \right)^{-2}.$$

### Example 3: Capital labor ratios

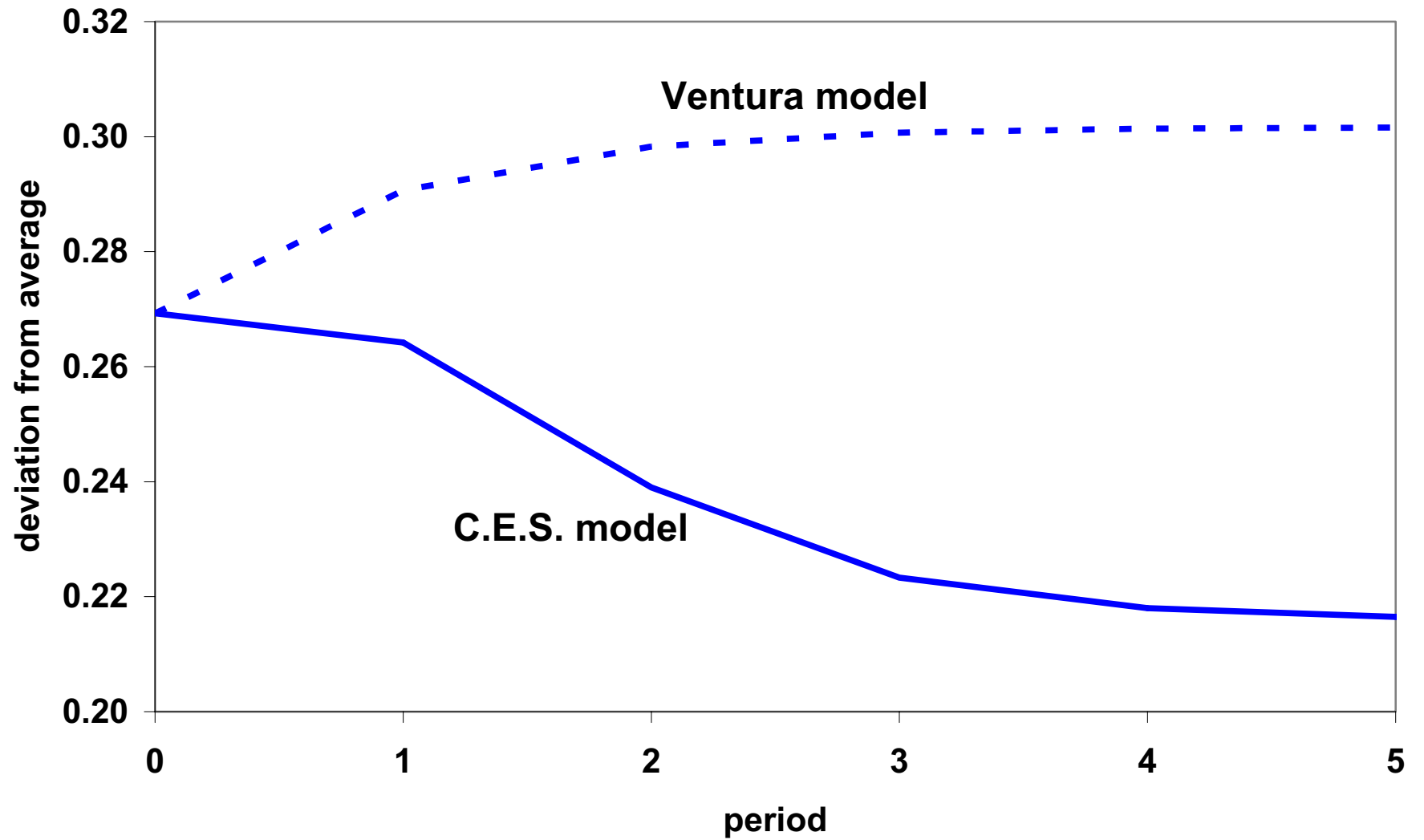




### Example 3: Capital labor ratios (detail)



### Example 3: Relative income in country 1



**8. Favorable changes in the terms of trade and/or reductions in tariffs make it easier to import intermediate goods. Although this is often interpreted as an increase in productivity, it does not show up as such in productivity measures that use real GDP as a measure of output.**

T. J. Kehoe and K. J. Ruhl, “Are Shocks to the Terms of Trade Shocks to Productivity?” Federal Reserve Bank of Minneapolis, 2007.

A deterioration in the terms of trade makes it expensive for an economy to import intermediate goods.

We can think of international trade as part of the production technology. Exports are inputs, imports are outputs. A deterioration in the terms of trade corresponds to a negative technology shock.

Can this negative “technology shock” account for the drop in TFP during the crisis?

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Can this negative “technology shock” account for the drop in TFP during the crisis?

**No!**

**Standard national income accounting (SNA, NIPA) implies that terms of trade shocks have no first-order effects on real output (GDP, GNP)**

# A simple model with intermediate goods

Labor

$$l_t = \bar{l}$$

Final good

$$y_t = f(\bar{l}, m_t)$$

Intermediate good

$$m_t = \frac{x_t}{a_t}$$

Feasibility

$$c_t + x_t = y_t$$

Real GDP

$$c_t = y_t - x_t = f(\bar{l}, m_t) - a_t m_t$$

Competitive economy solves

$$\max_{m_t} f(\bar{\ell}, m_t) - a m_t$$

$$f_m(\bar{\ell}, m(a_t)) \equiv a_t$$

$$f_{mm}(\bar{\ell}, m(a_t)) m'(a_t) = 1$$

$$m'(a_t) = \frac{1}{f_{mm}(\bar{\ell}, m(a_t))} < 0$$

How does real GDP change with an increase in  $a$  — a negative shock to the intermediate goods producing technology?

$$Y(a_t) \equiv f(\bar{\ell}, m(a_t)) - a_t m(a_t)$$

$$Y'(a_t) = f_m(\bar{\ell}, m(a_t)) m'(a_t) - a_t m'(a_t) - m(a_t) = -m(a_t) < 0$$

## A model with international trade

Suppose now that

$m$  is imported intermediate inputs,

$x$  is exports,

$p = a$  is terms of trade (real exchange rate)

Balanced trade

$$p_t m_t = x_t$$

Real GDP

$$c_t + x_t - p_0 m_t = y_t - p_0 m_t = f(\bar{\ell}, m_t) - p_0 m_t$$

where  $p_0$  is price of imports in the base year.



Competitive economy continues to solve

$$\max_{m_t} f(\bar{\ell}, m_t) - p_t m_t$$

$$f_m(\bar{\ell}, m(p_t)) \equiv p_t$$

$$m'(p_t) = \frac{1}{f_{mm}(\bar{\ell}, m(p_t))} < 0$$

How does real GDP change with an increase in  $p_t$  — a deterioration in the terms of trade (depreciation in the real exchange rate)?

$$Y(p_t) \equiv f(\bar{\ell}, m(p_t)) - p_0 m(p_t)$$

$$Y'(p_t) = f_m(\bar{\ell}, m(p_t))m'(p_t) - p_0 m'(p_t) = (p_t - p_0)m'(p_t)$$

$$p_0 \approx p_t \Rightarrow Y'(p_t) \approx 0$$

## **Alternative accounting concepts**

- Diewert and Morrison (1974, 1986)
- Kohli (1983, 1996)
- U.S. Bureau of Economic Analysis (Command Basis GDP)
- United Nations Statistics Division (Gross National Income)
- GNP, GDP (SNA, NIPA) do not.

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**Terms of trade shocks are worse than you think!**

## **Extensions**

Chain weighted price indices

Changes in tariffs

Endogenous labor

**9. In models with heterogeneous firms (for example, Melitz, Chaney), trade liberalization can cause resources to shift from less productive firms to more productive firms. Although this is often interpreted as an increase in productivity, it does not show up as such in productivity measures that use real GDP as a measure of output.**

M. J. Gibson, “Trade Liberalization, Reallocation, and Productivity,” University of Minnesota, 2006.

<http://www.econ.umn.edu/~tkehoe/papers/Gibson.pdf>.

Some countries experience aggregate productivity increases following trade liberalization

What is the economic mechanism through which this occurs?

Does trade liberalization increase aggregate productivity through reallocation toward more productive firms or through productivity increases at individual firms?

## **Reallocation mechanism**

Technology of each firm is fixed

Trade liberalization results in a reallocation of resources:

The least efficient firms exit

Resources are moved toward more efficient firms, particularly exporters

## **Main findings**

Reallocation following trade liberalization has no first-order effect on productivity, but it matters for welfare

Productivity gains must primarily come from firm-level productivity increases

Gibson studies a technology adoption mechanism in which firms can upgrade to a better technology, but it is costly to do so. Trade liberalization encourages technology adoption.



## Model

*I* symmetric countries, each with an *ad valorem* tariff on imports

Monopolistically competitive firms that are heterogeneous in technological efficiency

Sunk cost of entering export markets — only the most efficient firms export

Fixed cost of production — not all firms choose to operate

No aggregate uncertainty

## Consumer's problem

$$\max \sum_{t=0}^{\infty} \beta^t \log \left( \int_{z \in Z_t} c_t(z)^\rho dz \right)^{1/\rho}$$

$$\text{s.t.} \quad \int_{z \in Z_t^d} p_t(z) c_t(z) dz + (1 + \tau_t) \int_{z \in Z_t^x} p_t(z) c_t(z) dz = \bar{N} + \Pi_t + T_t$$

## Aggregation

Ideal real income index:

$$C_t = \left( \int_{z \in Z_t} c_t(z)^\rho dz \right)^{1/\rho}$$

Ideal price index:

$$P_t = \left( \int_{z \in Z_t^d} p_t(z)^{\frac{-\rho}{1-\rho}} dz + (1 + \tau_t)^{\frac{-\rho}{1-\rho}} \int_{z \in Z_t^x} p_t(z)^{\frac{-\rho}{1-\rho}} dz \right)^{\frac{-(1-\rho)}{\rho}}$$

Budget constraint again:

$$P_t C_t = \bar{N} + \Pi_t + T_t$$

## Demand functions

Firms take the consumer's demand functions as given

Demand for domestically produced goods:

$$\tilde{c}_t^d(p) = \left( \frac{P_t}{p} \right)^{\frac{1}{1-\rho}} C_t$$

Demand for imported goods:

$$\tilde{c}_t^x(p) = \left( \frac{P_t}{(1 + \tau_t) p} \right)^{\frac{1}{1-\rho}} C_t$$

## **Firms: Timing within a period**

Entrants learn their efficiencies

Each firm decides whether to operate or exit — producing requires paying a fixed cost of  $f^p$  units of labor

Non-exporters decide whether to pay the sunk cost of entering export markets,  $f^x$  units of labor

After producing, each firm faces exogenous probability of death  $\delta$

## Technologies

A firm of type  $a$  has the increasing-returns technology

$$y(n; a) = \max \left[ a(n - f^p), 0 \right]$$

$a \in [1, \infty)$  is the firm's technology draw from Pareto distribution

$$F(a) = 1 - a^{-\eta}$$

$f^p$  is the fixed cost, in units of labor, of producing

## Firm's static problem: Maximize period profits

Non-exporters:

$$\begin{aligned}\pi_t^d(a) &= \max_{p,n} p\tilde{c}_t^d(p) - n \\ \text{s.t. } a(n - f^p) &= \tilde{c}_t^d(p)\end{aligned}$$

Exporters:

$$\begin{aligned}\pi_t^x(a) &= \max_{p,n} p\left(\tilde{c}_t^d(p) + (I-1)\tilde{c}_t^x(p)\right) - n \\ \text{s.t. } a(n - f^p) &= \tilde{c}_t^d(p) + (I-1)\tilde{c}_t^x(p)\end{aligned}$$

## Prices

The profit-maximizing price is a constant markup over marginal cost:

$$p(a) = \frac{1}{\rho a}$$

The price of a good is inversely related to the efficiency with which it is produced



## Exporter's dynamic problem

$$v_t^x(a) = \max \left[ 0, \pi_t^x(a) + \frac{1-\delta}{1+r_{t+1}} v_{t+1}^x(a) \right]$$

## Non-exporter's dynamic problem

$$v_t^d(a) = \max \left[ 0, \pi_t^d(a) + \frac{1-\delta}{1+r_{t+1}} \max \left[ v_{t+1}^d(a), v_{t+1}^x(a) - \frac{1+r_{t+1}}{1-\delta} f^x \right] \right]$$

Outer maximization: Whether to operate

Inner maximization: Whether to devote  $f^x$  units of labor to enter export markets

## Firm entry

There is free entry of firms, and firms enter as non-exporters

The cost of a technology draw from probability distribution  $F$  is  $f^e$  units of labor

The measure of draws taken,  $e_t$ , is determined endogenously through a free-entry condition:

$$\frac{1}{1+r_{t+1}} \int v_{t+1}^d(a) F(da) - f^e \leq 0, \quad = 0 \text{ if } e_t > 0$$

The inequality reflects the constraint that  $e_t \geq 0$

## Distributions of firms by efficiency

Suppose that at the beginning of period  $t$  the distribution of non-exporters is  $m_t^d$  and the distribution of exporters is  $m_t^x$

To obtain the distributions of firms that choose to operate, apply the decision rules:

$$\mu_t^x(a) = \int_1^a \chi_t^x(\alpha) m_t^x(d\alpha)$$

$$\mu_t^d(a) = \int_1^a \chi_t^d(\alpha) m_t^d(d\alpha)$$

Distributions evolve in response to firm entry,  $e_t$  and changes in export status,  $\chi_t^e$

## Labor market clearing

The supply of labor is fixed at  $\bar{N}$  and is allocated among 3 activities: production, entering export markets, and entering the domestic market

$$\sum_s \int (n_t^d(a) \mu_t^d(da) + n_t^x(a) \mu_t^x(da) + f^x \chi_t^e(a) \mu_t^d(da)) + f^e e_t = \bar{N}.$$

## Measuring productivity

Labor productivity in the data is a measure of real value added per worker or per hour

Standard way of calculating real value added is to use base-period prices

## Measuring real value added per worker

Value added at current prices:

$$y_t = \int_{z \in Z_t^d} p_t(z) y_t(z) dz$$

Value added at base-period (period-0) prices:

$$Y_t = \int_{z \in Z_t^d} p_0(z) y_t(z) dz$$

Real value added per worker is  $Y_t / \bar{N}$

## What if a good was not produced in the base period?

This is an issue in the data as well

The standard recommendation for obtaining a proxy for the base-period price is to deflate the current price by the price index for a basket of goods that were produced in both periods, say  $\tilde{Z}$ :

$$\tilde{P}_t = \frac{\int_{\tilde{Z}} p_t(z) y_0(z) dz}{\int_{\tilde{Z}} p_0(z) y_0(z) dz}$$

Proxy for the period-0 price of a good not produced in period 0:

$$p_0(z) = \frac{p_t(z)}{\tilde{P}_t}$$

## Measuring social welfare

Ideal real income index:

$$\frac{\bar{N} + \Pi_t + T_t}{P_t} = C_t = \left( \int_{z \in Z_t} c_t(z)^\rho dz \right)^{1/\rho}$$

The ideal price index  $P_t$  takes into account changes in variety and the consumer's elasticity of substitution — in contrast to price indices in the data

## **To what extent can reallocation following trade liberalization account for long-term productivity gains?**

To determine the long-term effects of trade liberalization, we compare stationary equilibria of the model

two versions of the model:

Static version with  $\beta \rightarrow 1$  (similar to Melitz (2003)): analytical result

Dynamic version with  $0 < \beta < 1$ : illustrative numerical example



## Static model: An analytical finding

Proposition: In a stationary equilibrium with  $\beta \rightarrow 1$ , real value added per worker does not depend on the level of the tariff

To see why:

With  $\beta \rightarrow 1$ ,  $\Pi = 0$ , so the budget constraint gives

$$\int_{z \in Z_t^d} p_t(z) c_t(z) dz + \int_{z \in Z_t^x} p_t(z) c_t(z) dz = \bar{N}$$

The balanced trade condition is

$$\int_{z \in Z_t^d} p_t(z) (y_t(z) - c_t(z)) dz = \int_{z \in Z_t^x} p_t(z) c_t(z)$$

Add them together to get

$$\int_{z \in Z_t^d} p_t(z) y_t(z) dz = \bar{N}$$

So value added at current prices is constant, does not depend on  $\tau$

What about base-period prices? Without technology adoption, the price of each good in the economy is constant:  $p(z; a) = 1/(\rho a)$

So base-period prices are equal to current prices and the prices of new goods do not get deflated

Result:

$$Y_t = \int_{z \in Z_t^d} p_0(z) y_t(z) dz = \int_{z \in Z_t^d} p_t(z) y_t(z) dz = \bar{N}$$

## **Intuition for the result**

Reallocation following trade liberalization has no long-term effect on measured productivity

Why? Two factors:

Prices — they are inversely related to the efficiency with which a good is produced

General equilibrium effects — changes in the real wage (partial equilibrium analysis would predict a substantial increase in measured productivity)

## Parameterization for illustrative numerical experiment

$\bar{N} = 1$  Normalization

$\rho = 0.5$  Elasticity of substitution of 2 (Ruhl 2003)

$\eta = 1.5$

$\delta = 0.05$

$f^e = 1$

$f^x$  20 percent of firms export initially

$f^p$  Efficiency cutoff for operating is 1 initially

## Illustrative numerical experiment in the static model

$$\beta \rightarrow 1$$

Policy experiment: Eliminate a 10 percent tariff between 2 countries

Compare stationary equilibria to assess long-term effects of trade liberalization:

Percent change in measured productivity	0.0
Percent change in welfare	0.5

## **A note on the welfare increase**

The increase in welfare following trade liberalization is not due to an increase in variety — the measure of varieties available to the consumer decreases

Reallocation toward more efficient firms drives the welfare increase

This is in sharp contrast to trade models with homogeneous firms, in which the increase in welfare is driven by an increase in variety

Main point: Reallocation matters for welfare but not for measured productivity

## Illustrative numerical experiment in the dynamic model

To what extent can the fully dynamic model account for measured productivity gains?

$\beta = 0.96$       Real interest rate of 4 percent

Same numerical experiment:

Percent change in measured productivity	0.7
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Percent change in welfare	1.8
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