Growth Accounting Framework

The growth accounting framework used in this course is based on the aggregate production function taking the Cobb-Douglas form:

$$Y_t = A_t K_t^{\alpha} L_t^{1-\alpha},$$

where Y_t is output, A_t is total factor productivity (TFP), K_t is capital input, L_t is labor input, and $1-\alpha$ is labor's share of income. In order to obtain a decomposition for output per working-age person, we rewrite the above equation as follows:

$$\frac{Y_t}{N_t} = A_t^{\frac{1}{1-\alpha}} \left(\frac{K_t}{Y_t} \right)^{\frac{\alpha}{1-\alpha}} \left(\frac{L_t}{N_t} \right).$$

Written in this form, output per working-age person, $\frac{Y_t}{N_t}$, decomposes into a productivity factor, $A_t^{\frac{1}{1-\alpha}}$; a capital factor, $\left(\frac{K_t}{Y_t} \right)^{\frac{\alpha}{1-\alpha}}$; and a labor factor, $\frac{L_t}{N_t}$. The decomposition requires collecting data for the series of output, capital stock, working-age population, and hours worked. A value for labor's share of income must also be chosen. Once collected, these series and labor's share of income allow for the calculation of the TFP series:

$$A_t = \frac{Y_t}{K_t^{\alpha} L_t^{1-\alpha}}.$$

Data Sources

Y_t, real GDP: Common sources for real GDP include the *Detailed Tables of Main Aggregates* in the Organisation for Economic Co-operation and Development's (OECD) *Annual National Accounts* available at www.sourceoecd.com, the International Monetary Fund's *International Financial Statistics* (IFS), and Angus Maddison's *Historical Statistics for the World Economy: 1-2006 AD* online at www.ggdc.net/Maddison.

L_t, total annual hours worked: Depending on the time period and country in question, data on hours worked can be difficult to find. Two sources of data are the OECD's *Population and Labour Force Statistics* at www.sourceoecd.com and the Conference Board and Groningen Growth and Development Centre's *Total Economy Database*, which is available at www.conference-board.org/economics.

N_t, working-age (15-64) population: The OECD's *Population and Labour Force Statistics* and the World Bank's *World Development Indicators* provide data on working-age population.

$1-\alpha$, labor's share of income: The labor income share is defined as unambiguous labor income divided by the sum of unambiguous labor income and unambiguous capital

income. Therefore, GDP needs to be adjusted by subtracting off the ambiguous portions of income, namely, the household's net operating surplus and mixed income and net taxes. The labor income share for the aggregate economy is then defined as:

$$1-\alpha = \frac{CE_t}{GDP_t - (OSMI_t - CF_t) - T_t},$$

where CE_t denotes "Compensation of Employees," $OSMI_t$ denotes "Household gross operating surplus and mixed income," CF_t denotes "Household consumption of fixed capital," and T_t denotes "Taxes less subsidies," all of which are available from the OECD's *Annual National Accounts*. Unfortunately, the household data are not available for many countries over long periods. When this is the case, an assigned value of $\alpha = 0.3$ is often used.

K_t, real capital stock: National accounts generally do not report a series for the capital stock, so the capital stock must be constructed using the perpetual inventory method:

$$K_{t+1} = (1-\delta)K_t + X_t,$$

where δ is a constant depreciation rate of capital and X_t is real investment. The perpetual inventory method requires data on the series of real investment, a value for δ, and a value for the initial capital stock, K_0.

The OECD's *Annual National Accounts* and IFS both provide data on investment. When using the OECD, the category "Gross Capital Formation" forms investment. When using the IFS, the category "Gross Fixed Capital Formation" plus "Changes in Inventories" forms investment. When calculating real investment, it is best to collect data on nominal investment and then deflate the series by a GDP deflator. This procedure ensures the real investment series, and, thus, the capital stock series, and the real GDP series are deflated by the same price index.

The initial capital stock is chosen so the capital-output ratio in the initial period equals the average capital-output ratio over a reference period. For example, in calculating the capital stock series for, say, Austria, 1960-2005, one could choose the initial capital stock such that the capital-output ratio in 1960 matched the average capital-output ratio, 1961-1970:

$$\frac{K_{1960}}{Y_{1960}} = \frac{1}{10}\sum_{1961}^{1970}\frac{K_t}{Y_t}.$$

The value of δ is chosen such that the average ratio of depreciation to GDP using the constructed capital stock series matches the average ratio of depreciation to GDP in the data over the calibration period. The OECD's *Annual National Accounts* report depreciation in the data as "Consumption of Fixed Capital." Using the example of Austria, the choice of δ matches the average ratio of depreciation to GDP in the data over the calibration period 1970-2005:

$$\frac{1}{36}\sum_{t=1970}^{2005}\frac{\delta K_t}{Y_t} = 0.1388.$$

The above three equations (perpetual inventory method, capital-output ratio, and depreciation-GDP ratio) form a system used to solve for the initial capital stock, K_0; the depreciation rate, δ; and the capital stock series, K_t.