Are Shocks to the Terms of Trade Shocks to Productivity?

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Evidence on terms of trade, GDP, and TFP

\[ \rho(tot, tfp) = -0.46 \quad \rho(tot, gdp) = -0.30 \]
Evidence on terms of trade, GDP, and TFP

\[ \rho(tot, tfp) = -0.73 \quad \rho(tot, gdp) = -0.75 \]
Evidence on terms of trade, GDP, and TFP

Argentina

\[ \rho(tot, tfp) = -0.39 \quad \rho(tot, gdp) = -0.26 \]
Evidence on terms of trade, GDP, and TFP

\[ \rho(tot, tfp) = -0.37 \]

\[ \rho(tot, gdp) = -0.17 \]
Terms of trade volatility in the world

<table>
<thead>
<tr>
<th></th>
<th>std(terms of trade)</th>
<th>std(TFP)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Developing countries</td>
<td>0.132</td>
<td>0.026</td>
</tr>
<tr>
<td>Developed countries</td>
<td>0.053</td>
<td>0.017</td>
</tr>
<tr>
<td>Ratio</td>
<td>2.49</td>
<td>1.53</td>
</tr>
</tbody>
</table>

Hodrick-Prescott filtered annual data. source: Sengul (2006)
International trade as a production technology

“For small open economies, adverse terms of trade shocks can have much the same effect as negative technology shocks, and this is one of the important differences between macroeconomics in these economies and that which underlies some of the traditional closed economy models.”

Easterly, Islam, and Stiglitz (2001)
International trade as a production technology

Inputs are exports and outputs are imports.

\[ p_t M_t = X_t \implies M_t = \frac{1}{p_t} X_t \]

A deterioration in the terms of trade (an increase in \( p_t \)) acts as a productivity shock.
International trade as a production technology

Inputs are exports and outputs are imports.

\[ p_t M_t = X_t \implies M_t = \frac{1}{p_t} X_t \]

A deterioration in the terms of trade (an increase in \( p_t \)) acts as a productivity shock.

Or does it?
Overview of results

- Changes in $p_t$ have no first order effect on chain weighted GDP or measured productivity.

- With fixed proportions production, result is exact even for large shocks. (Forget about calculus!)

- Without chain weighting, effect involves $p_t - p_0$. (Effect goes either way!)

- With elastically supplied factors of production, effect goes either way.

- Results generalize to changes in tariffs and other trade barriers.
What drives the correlation between $p_t$ and real GDP and TFP?
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Not the mechanism we have discussed!
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Not the mechanism we have discussed!

These ideas are well understood by economists interested in index numbers and national income accounting.

Diewert and Morrison (1986)

Roadmap

1. Simple closed economy
2. Model reinterpreted as an open economy
3. Chain weighting
4. Elasticity of substitution
5. Extension: endogenous labor choice
6. Extension: taxes and tariffs
7. Quantitative effects in Mexico
Simple model: Closed economy

\[ l_t = \bar{l} \]
\[ y_t = f(l_t, m_t) \]
\[ m_t = \frac{x_t}{a_t} \]
\[ c_t + x_t = y_t \]

Normalize the price of the \( y \) good to be 1.

\[ p_t = a_t \]
Real GDP:

expenditure side

\[ Y_t = c_t = y_t - x_t \]

output side

\[ Y_t = (y_t + p_0 m_t) - (p_0 m_t + x_t) = y_t - x_t \]

where \( p_0 = a_0 \)
Firms solve

\[
\max f(\ell, m_t) - a_t m_t
\]

\[
f_m(\ell, m_t) = a_t
\]

\[
m'(a_t) = \frac{1}{f_{mm}(\ell, m(a_t))} < 0
\]

With fixed proportions, \( y_t = \min[\ell_t, m_t / b], \)

\[
m'(a_t) = 0
\]
How does real GDP change?

\[ Y(a_{t+1}) - Y(a_t) \approx Y'(a_t)(a_{t+1} - a_t) \]

where

\[ Y(a_t) = f(\bar{\ell}, m(a_t)) - a_t m(a_t) \]

\[ Y'(a_t) = f_m(\bar{\ell}, m(a_t))m'(a_t) - a_t m'(a_t) - m(a_t) = -m(a_t) < 0. \]

With fixed proportions, \( y_t = \min[\ell_t, m_t / b] \),

\[ Y(a_t) = \bar{\ell} - a_t b \ell \]

\[ Y'(a_t) = -b \bar{\ell} = -m_t. \]

Real GDP and productivity decline.
Simple model: Open economy

$m_t$ is an imported intermediate input

$x_t$ are exports of the $y$ good

$p_t$ is the terms of trade

we assume balanced trade,

$$p_t m_t = x_t$$
Real GDP

\[ Y_t = c_t + x_t - p_0 m_t = y_t - p_0 m_t = f(\bar{\ell}, m_t) - p_0 m_t \]

An increase in \( p_t \) has the identical impact on consumption and welfare as the decline in productivity in the closed economy.

But what happens to real GDP and productivity?

\[ Y(p_t) = f(\bar{\ell}, m(p_t)) - p_0 m(p_t) \]

\[ Y'(p_t) = f_m(\bar{\ell}, m(p_t))m'(p_t) - p_0 m'(p_t) = (p_t - p_0)m'(p_t) \]
With fixed proportions,

\[ Y(p_t) = \bar{\ell} - p_0 b \bar{\ell} \]

\[ Y'(p_t) = 0, \]

but

\[ c(p_t) = (1 - p_t b) \bar{\ell}. \]

This is the case where consumption, and therefore welfare, falls the most in response to a deterioration in the terms of trade.
Chain weighted real GDP

NIPA: Fisher chain weights, (UN SNA: Laspeyres chain weights)

\[ Y_t(p_t) = \frac{f(\ell, m(p_t)) - p_t m(p_t)}{P_t} \]

\[ P_{t+1} = \left( \frac{f(\ell, m(p_{t+1})) - p_{t+1} m(p_{t+1})}{f(\ell, m(p_{t+1})) - p_t m(p_{t+1})} \right)^{1/2} \left( \frac{f(\ell, m(p_t)) - p_{t+1} m(p_t)}{f(\ell, m(p_t)) - p_t m(p_t)} \right)^{1/2} P_t \]

\[ Y(p_{t+1}) = \left( \frac{f(\ell, m(p_{t+1})) - p_{t+1} m(p_{t+1})}{f(\ell, m(p_t)) - p_{t+1} m(p_t)} \right)^{1/2} \left( \frac{f(\ell, m(p_{t+1})) - p_t m(p_{t+1})}{f(\ell, m(p_t)) - p_t m(p_t)} \right)^{1/2} Y(p_t) \]
How does real GDP change with \( p \)?

\[
Y(p_{t+1}) - Y(p_t) \approx Y'(p_t)(p_{t+1} - p_t)
\]

\[
\frac{d \log Y(p_{t+1})}{dp_{t+1}} = -\frac{m(p_{t+1})}{2(f(\ell, m(p_t)) - p_t m(p_t))} + \frac{m(p_t)}{2(f(\ell, m(p_t)) - p_{t+1} m(p_t))}
\]

\[
+ \frac{(p_{t+1} - p_t)m'(p_{t+1})}{2(f(\ell, m(p_{t+1})) - p_t m(p_{t+1}))}
\]

\[
\frac{d \log Y(p_t)}{dp_{t+1}} = 0
\]

With any method of chaining, effect involving \( p_t - p_0 \) disappears.
Elasticity of substitution

\[ f(\ell_t, m_t) = \left( (1 - \beta) \ell_t^\rho + \beta m_t^\rho \right)^{\frac{1}{\rho}} \]

For each value of \( \rho \), choose \( \beta \) so that

\[ \frac{m_t}{\left( (1 - \beta) \ell_t^\rho + \beta m_t^\rho \right)^{\frac{1}{\rho}}} = 0.08 \]

Real GDP and the elasticity of substitution

\[ \sigma = 0.33 \]
\[ \sigma = 2.0 \]
\[ \sigma = 6.67 \]
Consumption and the elasticity of substitution

\[ \sigma = 6.67 \]

\[ \sigma = 2.0 \]

\[ \sigma = 0.33 \]
Extensions to the simple model

Variable labor supply

\[
\max u(c_t, \bar{l} - l_t)
\]

s.t. \( c_t = w_t \bar{l} \)

where \( w_t = f_\ell(\ell_t, m_t) \).

\[
w_t u_c(c_t, \bar{l} - l_t) = u_z(c_t, \bar{l} - l_t)
\]

which implicitly defines the function \( \ell(w) \):

\[
w_t u_c(w_t \ell(w_t), \bar{l} - \ell(w_t)) = u_z(w_t \ell(w_t), \bar{l} - \ell(w_t))
\]

\[
\ell'(w_t) = -\frac{u_c(c_t, \bar{l} - l_t) + u_{cc}(c_t, \bar{l} - l_t)w_t \ell_t - u_{cz}(c_t, \bar{l} - l_t)\ell_t}{u_{cc}(c_t, \bar{l} - lv)w_t^2 - 2u_{cz}(c_t, \bar{l} - l_t)w_t + u_{zz}(c_t, \bar{l} - l_t)}.
\]
C. E. S. case

\[ u(c, z) = \begin{cases} 
\frac{(c^\rho + \gamma z^\rho - 1 - \gamma)}{\rho} & \text{for } \rho \leq 1, \rho \neq 0 \\
\log c + \gamma \log z & \text{for } \rho = 0 
\end{cases} \]

\[ \ell'(w) = \frac{\rho c^{\rho-1}}{(1 - \rho)(w^2 c^{\rho-2} + \gamma (\bar{\ell} - \ell)^{\rho-2})} \]

\ell'(w) has same sign as \( \rho \).
How do $w$ and $m$ vary with $p$?

$$f_{\ell}(\ell(w(p)), m(p)) = w(p)$$

$$f_{m}(\ell(w(p)), m(p))) = p$$

$$f_{\ell\ell}(\ell, m)\ell'(w)w'(p) + f_{\ell m}(\ell, m)m'(p) = w'(p)$$

$$f_{\ell m}(\ell, m)\ell'(w)w'(p) + f_{mm}(\ell, m)m'(p) = 1$$

$$w'(p) = \frac{f_{\ell m}(\ell, m)}{f_{mm}(\ell, m) - (f_{mm}(\ell, m)f_{\ell\ell}(\ell, m) - f_{\ell m}(\ell, m)^2)\ell'(w)}$$

$$m'(p) = \frac{1 - f_{\ell\ell}(\ell, m)\ell'(w)}{f_{mm}(\ell, m) - (f_{mm}(\ell, m)f_{\ell\ell}(\ell, m) - f_{\ell m}(\ell, m)^2)\ell'(w)}$$
Consumer welfare:

\[ c(p) = f(\ell(w(p)), m) - pm(p) \]

\[ \frac{d}{dp} u(c(p_t), \bar{\ell} - \ell(w(p_t))) = -u_c(c_t, \bar{\ell} - \ell_t)m_t < 0. \]

Real GDP:

\[ Y(p_t) = f(\ell(w(p_t)), m(p_t)) - p_0m(p_t) \]

\[ Y'(p_t) = f'(\ell_t, m_t)\ell'(w_t)w'(p_t) + (p_t - p_0)m'(p_t) \]

Real GDP can either rise or fall with \( p_t \)

If \( \ell'(w_t) > 0 \), which implies that \( w'(p_t) < 0 \), and if \( (p_t - p_0)m'(p_t) \) is small, real GDP falls.
Productivity:

\[
Y(p_t) / \ell(w(p_t))
\]

\[
\frac{d}{dp_t} \frac{Y(p_t)}{\ell(w(p_t))} = \frac{\ell(w_t)Y'(p_t) - Y(p_t)\ell'(w_t)w'(p_t)}{\ell(w_t)^2}
\]

\[
\frac{d}{dp_t} \frac{Y(p_t)}{\ell(w(p_t))} = \frac{(p_t - p_0)(\ell_t m'(p_t) - m_{t-1} \ell'(w_t)w'(p_t))}{\ell_t^2}
\]

with fixed proportions case,

\[
Y(p_t) = (1 - p_0b)\ell(w(p_t))
\]
Tariffs

\[
\max f(\ell, m_t) - (1 + \tau_t) p_t m_t
\]

Real GDP:

\[
Y'(p_t) = ((1 + \tau) p_t - p_0) m'(p_t)
\]

\[
Y'(\tau_t) = ((1 + \tau_t) p_t - p_0) m'(\tau_t)
\]

\[\approx 0 \text{ if } (1 + \tau_t) p_t - p_0 \approx 0 \text{ or if } f \text{ is close to fixed proportions.}\]

Welfare:

\[
c'(p_t) = \tau p_t m'(1 + \tau) p_t - m(p_t(1 + \tau)) p_t
\]

\[
c'(\tau_t) = \tau_t p_t m'(1 + \tau_t) p_t
\]
Alternative income measures

U.S. NIPA: command-basis GDP

U.N. SNA: Gross Domestic Income

\[
GDP_t = \frac{C_t}{P_t^C} + \frac{I_t}{P_t^I} + \frac{G_t}{P_t^G} + \frac{X_t}{P_t^X} - \frac{M_t}{P_t^M}
\]

\[
GDI_t = \frac{C_t}{P_t^C} + \frac{I_t}{P_t^I} + \frac{G_t}{P_t^G} + \frac{X_t - M_t}{P_t^M}.
\]

or deflate \( X_t - M_t \) by \( P_t^Y \) or deflate \( X_t - M_t \) by \( P_t^X \), or…
Mexico

Index (1994=100)

Terms of trade

Real GDP

Command GDP

Years: 1990 to 1998
Terms of trade shocks are worse than you thought!
Quantitative Example

Price of Imports/Price of Exports in Mexico

![Graph showing the price of imports/price of exports in Mexico from 1980 to 1990. The index is normalized to 2000 as the base year. The graph demonstrates a trend with fluctuations, indicating changes in trade dynamics over the specified period.](image-url)
Open Economy Model

Two kinds of goods:

- Imports \((m \text{ – goods})\)
- Domestically produced goods \((d \text{ – goods})\)

Domestic good is the numeraire

- The terms of trade, \(p_m\), is exogenous

Add 3 exogenous variables

- Terms of trade, \(p_{m,t}\)
- Productivity – not TFP!!
- Investment-consumption good productivity, \(D_t\)
Open Economy Model

Households

$$\max \sum_{t=T_0}^{\infty} \beta^t \left( \gamma \log (C_t) + (1 - \gamma) \log (\bar{h}N_t - L_t) \right)$$

s.t. $$q_t C_t + q_t (K_{t+1} - (1 - \delta) K_t) = w_t L_t + r_t K_t$$

Domestic Good Technology

$$Z_t + X_t = A_t K_t^\alpha L_t^{1-\alpha}$$

Feasibility

$$C_t + K_{t+1} - (1 - \delta) K_t = D_t \left( \omega Z_t^\rho + (1 - \omega) M_t^\rho \right)^{1/\rho}$$
The firm’s problem

\[
\min_{Z_t, M_t} Z_t + p_{m,t} M_t
\]

s.t. \( \bar{Y}_t \leq D_t \left( \omega Z_t^\rho + (1 - \omega) M_t^\rho \right)^{\frac{1}{\rho}} \)

Investment-consumption good price

\[
q_t = D_t^{-1} \left( \omega^{\frac{1}{1-\rho}} + (1 - \omega)^{\frac{1}{1-\rho}} p_{m,t}^{\frac{-\rho}{1-\rho}} \right)^{\frac{1-\rho}{-\rho}}
\]
Open Economy Model Calibration

Exogenous processes

- Terms of trade, $p_{m,t}$, from data
- Productivity in investment-consumption sector, $D_t$, from data
- Productivity in the domestic sector, $A_t$

Exogenous productivity is

$$ A_t = \frac{1}{\omega^\rho \left( (C_t + I_t)^\rho D_t^{-\rho} - (1 - \omega) M_t^\rho \right)^{1/\rho}} + X_t $$

TFP is calculated with real GDP:

$$ \hat{Y}_t = q_{\bar{T}} (C_t + I_t) + X_t - p_{m,\bar{T}} M_t $$
Solve the model two ways:

1. Model with terms of trade shocks
2. Model without terms of trade shocks (do not recalibrate)
Real GDP per working age person in Mexico

model with terms of trade shocks

model without terms of trade shocks
TFP in Mexico, base year = 2000

Index (1982=100)

Data

Model without terms of trade shocks

Model with terms of trade shocks

TFP in Mexico, base year = 1982

The graph shows the Total Factor Productivity (TFP) in Mexico from 1980 to 1990, with the base year set to 1982. The index is expressed as a percentage of the base year, with 1982 indexed at 100.

Two models are compared:
- Model without terms of trade shocks
- Model with terms of trade shocks
TFP in Mexico, chain weighted

The chart illustrates the Total Factor Productivity (TFP) in Mexico from 1980 to 1990, with two models compared:

- **Model without terms of trade shocks**
- **Model with terms of trade shocks**

The index values are on a scale from 70 to 110, with 1982 as the base year (1982=100). The graph shows a decline in TFP over the period, with the model with terms of trade shocks exhibiting a more pronounced decline compared to the model without terms of trade shocks.
Conclusion

Base period prices: terms of trade have an ambiguous effect on TFP

Chain weighting: terms of trade have no effect on TFP

Terms of trade shocks can increase GDP volatility, but only by changing factor inputs, not productivity.