Are Shocks to the Terms of Trade Shocks to Productivity?

Timothy J. Kehoe
University of Minnesota and Federal Reserve Bank of Minneapolis

Kim J. Ruhl
University of Texas at Austin

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www.econ.umn.edu/~tkehoe
Evidence on terms of trade, GDP, and TFP
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Mexican Economy

- Terms of trade
- Real GDP
- Total factor productivity

Graph showing the growth rates from 1980 to 2000.
International trade is often thought of as a production technology.

Inputs are exports and outputs are imports.

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A deterioration in the terms of trade (an increase in \( p_t \)) acts as a productivity shock.

**Or does it?**
GDP in current prices

\[ Y_t = (C_t + I_t + G_t + X_t) - p_t M_t \]

\[ p_t \uparrow \Rightarrow M_t \downarrow \Rightarrow (C_t + I_t + G_t + X_t) \downarrow \]

Real GDP in base period prices

\[ Y_t = (C_t + I_t + G_t + X_t) - p_0 M_t \]

\[ p_t \uparrow \Rightarrow M_t \downarrow \Rightarrow (C_t + I_t + G_t + X_t) \downarrow \Rightarrow Y_t? \]
In a simple model, changes in $p_t$ have no first order effect on chain weighted GDP or measured productivity.

With fixed proportions production, result is exact even for large shocks. (Forget about calculus!)

Without chain weighting, effect involves $p_t - p_0$. (Effect goes either way!)

With elastically supplied factors of production, effect goes either way.

Results generalize to changes in tariffs and other trade barriers.
What drives the correlation between $p_t$ and real GDP and TFP?
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These ideas are well understood by economists interested in index numbers and national income accounting.

Diewert and Morrison (1986)

Simple model: Closed economy

\[ \ell_t = \bar{\ell} \]

\[ y_t = f(\ell_t, m_t) \]

\[ m_t = \frac{x_t}{a_t} \]

\[ c_t + x_t = y_t \]

Normalize the price of the \( y \) good to be 1.

\[ p_t = a_t \]
Real GDP:

expenditure side

\[ Y_t = c_t = y_t - x_t \]

output side

\[ Y_t = (y_t + p_0m_t) - (p_0m_t + x_t) = y_t - x_t \]

where \( p_0 = a_0 \)
Firms solve

\[ \max f(\ell, m_t) - a_t m_t \]

\[ f_m(\ell, m_t) = a_t \]

\[ m'(a_t) = \frac{1}{f_{mm}(\ell, m(a_t))} < 0 \]

With fixed proportions, \( y_t = \min[\ell_t, m_t/b] \),

\[ m'(a_t) = 0 \]
How does real GDP change?

\[ Y(a_{t+1}) - Y(a_t) \approx Y'(a_t)(a_{t+1} - a_t) \]

where

\[ Y(a_t) = f(\ell, m(a_t)) - a_t m(a_t) \]

\[ Y'(a_t) = f_m(\ell, m(a_t))m'(a_t) - a_t m'(a_t) - m(a_t) = -m(a_t) < 0. \]

With fixed proportions, \( y_t = \min[\ell_t, m_t/b] \),

\[ Y(a_t) = \ell - a_t \bar{b} \ell \]

\[ Y'(a_t) = -b \bar{\ell} = -m_t. \]

Real GDP and productivity decline.
Simple model: Open economy

\( m_t \) is an imported intermediate input

\( x_t \) are exports of the \( y \) good

\( p_t \) is the terms of trade

we assume balanced trade,

\[ p_t m_t = x_t \]
Real GDP

\[ Y_t = c_t + x_t - p_0 m_t = y_t - p_0 m_t = f(\bar{\ell}, m_t) - p_0 m_t \]

An increase in \( p_t \) has the identical impact on consumption and welfare as the decline in productivity in the closed economy.

But what happens to real GDP and productivity?

\[ Y(p_t) = f(\bar{\ell}, m(p_t)) - p_0 m(p_t) \]

\[ Y'(p_t) = f_m(\bar{\ell}, m(p_t))m'(p_t) - p_0 m'(p_t) = (p_t - p_0)m'(p_t) \]
With fixed proportions,

\[ Y(p_t) = \bar{\ell} - p_0 b \bar{\ell} \]

\[ Y'(p_t) = 0, \]

but

\[ c(p_t) = (1 - p_t b) \bar{\ell}. \]

This is the case where consumption, and therefore welfare, falls the most in response to a deterioration in the terms of trade.
Extensions to the simple model

Variable labor supply

\[
\max \ u(c_t, \bar{\ell} - \ell_t) \\
\text{s.t. } c_t = w_t \bar{\ell}
\]

where \( w_t = f_\ell(\ell_t, m_t) \).

\[
w_t u_c(c_t, \bar{\ell} - \ell_t) = u_z(c_t, \bar{\ell} - \ell_t)
\]

which implicitly defines the function \( \ell(w) \):

\[
w_t u_c(w_t \ell(w_t), \bar{\ell} - \ell(w_t)) = u_z(w_t \ell(w_t), \bar{\ell} - \ell(w_t))
\]

\[
\ell'(w_t) = -\frac{u_c(c_t, \bar{\ell} - \ell_t) + u_{cc}(c_t, \bar{\ell} - \ell_t)w_t \ell_t - u_{cz}(c_t, \bar{\ell} - \ell_t)\ell_t}{u_{cc}(c_t, \bar{\ell} - \ell v)w_t^2 - 2u_{cz}(c_t, \bar{\ell} - \ell_t)w_t + u_{zz}(c_t, \bar{\ell} - \ell_t)}.
\]
C. E. S. case

\[ u(c, z) = \begin{cases} 
\left( c^\rho + \gamma z^\rho - 1 - \gamma \right) / \rho & \text{for } \rho \leq 1, \ \rho \neq 0 \\
\log c + \gamma \log z & \text{for } \rho = 0 
\end{cases} \]

\[ \ell'(w) = \frac{\rho c^{\rho-1}}{(1 - \rho)(w^2 c^{\rho-2} + \gamma (\bar{\ell} - \ell)^{\rho-2})} \]

\( \ell'(w) \) has same sign as \( \rho \).
How do $w$ and $m$ vary with $p$?

$$f_{\ell}(\ell(w(p)), m(p)) = w(p)$$

$$f_{m}(\ell(w(p)), m(p))) = p$$

$$f_{\ell\ell}(\ell, m)\ell'(w)w'(p) + f_{\ell m}(\ell, m)m'(p) = w'(p)$$

$$f_{\ell m}(\ell, m)\ell'(w)w'(p) + f_{m m}(\ell, m)m'(p) = 1$$

$$w'(p) = \frac{f_{\ell m}(\ell, m)}{f_{m m}(\ell, m) - (f_{m m}(\ell, m)f_{\ell\ell}(\ell, m) - f_{\ell m}(\ell, m)^2)\ell'(w)}$$

$$m'(p) = \frac{1 - f_{\ell\ell}(\ell, m)\ell'(w)}{f_{m m}(\ell, m) - (f_{m m}(\ell, m)f_{\ell\ell}(\ell, m) - f_{\ell m}(\ell, m)^2)\ell'(w)}$$
Consumer welfare:

\[ c(p) = f(\ell(w(p)), m) - pm(p) \]

\[ \frac{d}{dp} u(c(p_t), \bar{\ell} - \ell(w(p_t))) = -u_c(c_t, \bar{\ell} - \ell_t)m_t < 0. \]

Real GDP:

\[ Y(p_t) = f(\ell(w(p_t)), m(p_t)) - p_0m(p_t) \]

\[ Y'(p_t) = f'_\ell(\ell_t, m_t)\ell'(w_t)w'(p_t) + (p_t - p_0)m'(p_t) \]

Real GDP can either rise or fall with \( p_t \)

If \( \ell'(w_t) > 0 \), which implies that \( w'(p_t) < 0 \), and if \((p_t - p_0)m'(p_t)\) is small, real GDP falls.
Productivity:

\[
Y(p_t) / \ell(w(p_t))
\]

\[
\frac{d}{dp_t} \frac{Y(p_t)}{\ell(w(p_t))} = \frac{\ell(w_t)Y'(p_t) - Y(p_t)\ell'(w_t)w'(p_t)}{\ell(w_t)^2}
\]

\[
\frac{d}{dp_t} \frac{Y(p_t)}{\ell(w(p_t))} = \frac{(p_t - p_0)(\ell_t m'(p_t) - m_{t-1}\ell''(w_t)w'(p_t))}{\ell_t^2}
\]

with fixed proportions case,

\[
Y(p_t) = (1 - p_0b)\ell(w(p_t))
\]
Tariffs

$$\max f(\ell, m_t) - (1 + \tau_t) p_t m_t$$

Real GDP:

$$Y'(p_t) = ((1 + \tau) p_t - p_0) p_t m'(p_t)$$

$$Y'(\tau_t) = ((1 + \tau_t) p_t - p_0) a m'(\tau_t)$$

$$\approx 0 \text{ if } (1 + \tau_t) p_t - p_0 \approx 0 \text{ or if } f \text{ is close to fixed proportions.}$$

Welfare:

$$c'(p_t) = ((1 + \tau) p_t - p_0) p_t m'(p_t) - m(p_t)$$

$$c'(\tau_t) = ((1 + \tau_t) p_t - p_0) p_t m'(\tau_t)$$
Chain weighted real GDP


(also Statistics Canada)

\[ Y_t(p_t) = \frac{f(\bar{\ell}, m(p_t)) - p_t m(p_t)}{P_t} \]

\[ P_{t+1} = \left(\frac{f(\bar{\ell}, m(p_{t+1})) - p_{t+1} m(p_{t+1})}{f(\bar{\ell}, m(p_{t+1})) - p_t m(p_{t+1})}\right)^{\frac{1}{2}} \left(\frac{f(\bar{\ell}, m(p_t)) - p_{t+1} m(p_t)}{f(\bar{\ell}, m(p_t)) - p_t m(p_t)}\right)^{\frac{1}{2}} P_t \]

\[ Y(p_{t+1}) = \left(\frac{f(\bar{\ell}, m(p_{t+1})) - p_{t+1} m(p_{t+1})}{f(\bar{\ell}, m(p_t)) - p_{t+1} m(p_t)}\right)^{\frac{1}{2}} \left(\frac{f(\bar{\ell}, m(p_{t+1})) - p_{t+1} m(p_{t+1})}{f(\bar{\ell}, m(p_t)) - p_t m(p_t)}\right)^{\frac{1}{2}} Y(p_t) \]
U.N. Statistics Division — System of National Accounts (SNA):
Laspeyres chain weights (although Fisher and Paasche are allowed)

\[ Y_t(p_t) = \frac{f(\ell, m(p_t)) - p_t m(p_t)}{p_t} \]

\[ P_{t+1} = \frac{f(\ell, m(p_{t+1})) - p_{t+1} m(p_{t+1})}{f(\ell, m(p_{t+1})) - p_t m(p_{t+1})} \]

\[ Y(p_{t+1}) = \frac{f(\ell, m(p_{t+1})) - p_t m(p_{t+1})}{f(\ell, m(p_t)) - p_t m(p_t)} Y(p_t) \]

With any method of chaining, effect involving \( p_t - p_0 \) disappears.
Elasticity of substitution

\[ f(\ell_t, m_t) = \left( (1 - \beta) \ell_t^\rho + \beta m_t^\rho \right)^{\frac{1}{\rho}} \]

For each value of \( \rho \), choose \( \beta \) so that

\[ \frac{m_t}{\left( (1 - \beta) \ell_t^\rho + \beta m_t^\rho \right)^{\frac{1}{\rho}}} = 0.08 \]

Real GDP and the elasticity of substitution

\[ \sigma = 0.33 \]

\[ \sigma = 2.0 \]

\[ \sigma = 6.67 \]
Consumption and the elasticity of substitution

\[ \sigma = 6.67 \]
\[ \sigma = 2.0 \]
\[ \sigma = 0.33 \]
Alternative income measures

U.S. NIPA: command-basis GDP

U.N. SNA: Gross Domestic Income

\[
GDP_t = \frac{C_t}{P_t^C} + \frac{I_t}{P_t^I} + \frac{G_t}{P_t^G} + \frac{X_t}{P_t^X} - \frac{M_t}{P_t^M}
\]

\[
GDI_t = \frac{C_t}{P_t^C} + \frac{I_t}{P_t^I} + \frac{G_t}{P_t^G} + \frac{X_t - M_t}{P_t^M}.
\]

or deflate \( X_t - M_t \) by \( P_t^Y \) or deflate \( X_t - M_t \) by \( P_t^X \), or…
Hodrick-Prescott filtered data

United States

deviations from trend

command GDP

real GDP

Hodrick-Prescott filtered data

Switzerland

deviations from trend

real GDP

command GDP

Conclusion

Terms of trade shocks are worse than you thought!