International Trade and Finance

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Outline:

1. Standard theory (hybrid Heckscher-Ohlin/New Trade Theory) does not well when matched with the data on the growth and composition of trade.

2. Applied general equilibrium models that put the standard theory to work do not well in predicting the impact of trade liberalization experiences like NAFTA.

3. Much of the growth of trade after a trade liberalization experience is growth on the extensive margin. Models need to allow for corner solutions or fixed costs.

4. Fixed costs seem better than Ricardian corner solutions for reconciling time series data on real exchange rate fluctuations with data on trade growth after liberalization experiences.
5. Models of trade with heterogeneous firms typically impose fixed costs on firms that decide to export. The focus is on the decision to export. The theory and the data indicate that there is a lot of room for focusing on the decision to import.

6. Models with uniform fixed cost across firms with heterogeneous productivity have implications that are sharply at odds with micro data. A model with increasing costs of accessing a fraction of a market has many of features of models with fixed costs without these undesirable properties.

7. Growth theory needs to be reconsidered in the light of trade theory. In particular, a growth model that includes trade can have the opposite convergence properties from a model of closed economies.
8. Favorable changes in the terms of trade and/or reductions tariffs make it easier to import intermediate goods. Although this is often interpreted as an increase in productivity, it does not show up as such in productivity measures that use real GDP as a measure of output.

9. In models with heterogeneous firms (for example, Melitz, Chaney), trade liberalization can cause resources to shift from less productive firms to more productive firms. Although this is often interpreted as an increase in productivity, it does not show up as such in productivity measures that use real GDP as a measure of output.
TRADE THEORY AND TRADE FACTS

- Some recent trade facts
- A “New Trade Theory” model
- Accounting for the facts
- Intermediate goods?
- Policy?

How important is the quantitative failure of the New Trade Theory?

Where should trade theory and applications go from here?
SOME RECENT TRADE FACTS

- The ratio of trade to product has increased. World trade/world GDP increased by 59.3 percent 1961-1990. OECD-OECD trade/OECD GDP increased by 111.5 percent 1961-1990.

- Trade has become more concentrated among industrialized countries. OECD-OECD trade/OECD-RW trade increased by 87.1 percent 1961-1990.

- Trade among industrialized countries is mostly intraindustry trade. Grubel-Lloyd index for OECD-OECD trade in 1990 is 68.4. Grubel-Lloyd index for OECD-RW trade in 1990 is 38.1.
Helpman and Krugman (1985):
“These....empirical weaknesses of conventional trade theory...become understandable once economies of scale and imperfect competition are introduced into our analysis.”

Markusen, Melvin, Kaempfer, and Maskus (1995):
“Thus, nonhomogeneous demand leads to a decrease in North-South trade and to an increase in intraindustry trade among the northern industrialized countries. These are the stylized facts that were to be explained.”

Goal: To measure how much of the increase in the ratio of trade to output in the OECD and of the concentration of world trade among OECD countries can be accounted for by the “New Trade Theory.”
PUNCHLINE

In a calibrated general equilibrium model, the New Trade Theory cannot account for the increase in the ratio of trade to output in the OECD.
Back-of-the-envelope calculations:

Suppose that the world consists of the OECD and the only trade is manufactures.

With Dixit-Stiglitz preferences, country $j$ exports all of its production of manufactures $Y_m^j$ except for the fraction $s^j = Y^j / Y_{oe}$ that it retains for domestic consumption.

World imports:

$$M = \sum_{j=1}^{n} \left( 1 - s^j \right) Y_m^j.$$

World trade/GDP:

$$\frac{M}{Y_{oe}} = \frac{M}{Y_{oe}} \frac{Y_{oe}}{Y_{oe}} = \left( 1 - \sum_{j=1}^{n} (s^j)^2 \right) \frac{Y_m}{Y_{oe}}.$$
World trade/GDP:

\[ \frac{M}{Y^{oe}} = \frac{M}{Y^{oe}} \frac{Y^{oe}_{m}}{Y^{oe}} = \left(1 - \sum_{j=1}^{n} (s^{j})^2\right) \frac{Y^{oe}_{m}}{Y^{oe}}. \]

\( \left(1 - \sum_{j=1}^{n} (s^{j})^2\right) \) goes from 0.663 in 1961 to 0.827 in 1990.

\( Y^{oe}_{m} / Y^{oe} \) goes from 0.295 in 1961 to 0.222 in 1990.

\[ 0.663 \times 0.295 = 0.196 \approx 0.184 = 0.827 \times 0.222. \]

Effects cancel!
A “NEW TRADE THEORY” MODEL

Environment:

- Static: endowments of factors are exogenous
- 2 regions: OECD and rest of world
- 2 traded goods: homogeneous — primaries (CRS) and differentiated — manufactures (IRS)
- 1 nontraded good — services (CRS)
- 2 factors: (effective) labor and capital
- Identical technologies and preferences (love for variety) across regions
- Primaries are inferior to manufactures

We only consider merchandise trade in both the data and in the model.
Key Features of the Model

Consumers' problem:

$$\max \frac{\beta_p (c_p^j + \gamma_p)^{\eta} + \beta_m (\int_{D^p} c_m^j(z)^{\rho} dz_p)^{\eta/\rho} + \beta_s (c_s^j + \gamma_s)^{\eta}}{\eta} - 1$$

s.t. $$q_p c_p^j + \int_{D^p} q_m(z)c_m^j(z)dz_p + q_s^j c_s^j \leq r^j k^j + w^j h^j.$$
Firms' problems

Primaries and Services: Standard CRS problems.

\[ Y_p^j = \theta_p (K_p^j)^{\alpha_p} (H_p^j)^{1-\alpha_p} \]
\[ Y_s^j = \theta_s (K_s^j)^{\alpha_s} (H_s^j)^{1-\alpha_s} \]

Manufactures: Standard (Dixit-Stiglitz) monopolistically competitive problem:

- Fixed cost.

\[ Y_m(z) = \max \left[ \theta_m K_m(z)^{\alpha_m} H_m(z)^{1-\alpha_m} - F, 0 \right] \]
• Firm \( z \) sets its price \( q_m(z) \) to max profits given all of the other prices.

\[
Y_m(z) = \sum_{j=1}^{n} C^j_m(z) + C^r_w(z).
\]

\[
C^j_m(z) = \beta_m^{\frac{1}{1-\eta}} \left( r^j K^j + w^j H^j + q_p \gamma_p N_j + q_s \gamma_s N_j \right) q_m(z)^{\frac{1}{1-p}} \left[ \int_{D^w} q_m(z')^{\frac{-\rho}{1-p}} dz' \right]^{\frac{\rho-\eta}{\rho(1-\eta)}} \Delta
\]

\[
\Delta = \beta_p^{\frac{1}{l-\eta}} q_p^{\frac{-\eta}{l-\eta}} + \beta_m^{\frac{1}{l-\eta}} \left[ \left( \int_{D^w} q_m(z')^{\frac{-\rho}{1-p}} dz' \right)^{\frac{-\eta}{\rho}} + \beta_s^{\frac{1}{l-\eta}} q_s^{\frac{-\eta}{l-\eta}} \right]^{\frac{-\eta}{l-\eta}}
\]

• Every firm is uniquely associated with only one variety (symmetry).
• Free entry.
• \( D^w = [0, d^w] \) with \( d^w \) finite and endogenously determined.
Volume of Trade

Let $s^j$ be the share of country $j$, $j = 1,...,n$, $rw$, in the world production of manufactures,

$$s^j = \frac{\int_{D^j} Y_m(z)dz}{\int_{D^w} Y_m(z)dz} = Y_m^j / Y_m^w.$$ 

The imports by country $j$ from the OECD are

$$M_{oe}^j = (1 - s^{rw} - s^j)C_m^j$$

$$M_{oe}^{rw} = (1 - s^{rw})C_m^{rw}.$$ 

Total imports in the OECD from the other OECD countries are

$$M_{oe}^o = \sum_{j=1}^{n} M_{oe}^j (1 - s^{rw} - \sum_{j=1}^{n} (s^j)^2 / (1 - s^{rw}))C_m^{oe}.$$
### OECD in 1990

<table>
<thead>
<tr>
<th>Country</th>
<th>Share of GDP %</th>
<th>Country</th>
<th>Share of GDP %</th>
</tr>
</thead>
<tbody>
<tr>
<td>Australia</td>
<td>1.79</td>
<td>Japan</td>
<td>18.04</td>
</tr>
<tr>
<td>Austria</td>
<td>0.97</td>
<td>Netherlands</td>
<td>1.72</td>
</tr>
<tr>
<td>Belgium-Lux</td>
<td>1.26</td>
<td>New Zealand</td>
<td>0.26</td>
</tr>
<tr>
<td>Canada</td>
<td>3.45</td>
<td>Norway</td>
<td>0.70</td>
</tr>
<tr>
<td>Denmark</td>
<td>0.78</td>
<td>Portugal</td>
<td>0.41</td>
</tr>
<tr>
<td>Finland</td>
<td>0.81</td>
<td>Spain</td>
<td>3.00</td>
</tr>
<tr>
<td>France</td>
<td>7.26</td>
<td>Sweden</td>
<td>1.40</td>
</tr>
<tr>
<td>Germany</td>
<td>9.96</td>
<td>Switzerland</td>
<td>0.17</td>
</tr>
<tr>
<td>Greece</td>
<td>0.50</td>
<td>Turkey</td>
<td>0.91</td>
</tr>
<tr>
<td>Iceland</td>
<td>0.04</td>
<td>United Kingdom</td>
<td>5.92</td>
</tr>
<tr>
<td>Ireland</td>
<td>0.28</td>
<td>United States</td>
<td>33.72</td>
</tr>
<tr>
<td>Italy</td>
<td>6.64</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
ACCOUNTING FOR THE FACTS

Compare the changes that the model predicts for 1961-1990 with what actually took place.

Focus on key variables:
- OECD-OECD Trade/OECD GDP
- OECD-OECD Trade/OECD-RW Trade
- OECD Manufacturing GDP/OECD GDP

Calibrate to 1990 data.

Backcast to 1961 by imposing changes in parameters:
- relative sizes of countries in the OECD populations
- sectoral productivities
- endowments
# ACCOUNTING FOR THE FACTS

Benchmark 1990 OECD Data Set  
(Billion U.S. dollars)

<table>
<thead>
<tr>
<th></th>
<th>Primaries</th>
<th>Manufactures</th>
<th>Services</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>$H_{i}^{oe}$</td>
<td>228</td>
<td>2,884</td>
<td>8,644</td>
<td>11,756</td>
</tr>
<tr>
<td>$K_{i}^{oe}$</td>
<td>441</td>
<td>775</td>
<td>3,497</td>
<td>4,713</td>
</tr>
<tr>
<td>$Y_{i}^{oe}$</td>
<td>669</td>
<td>3,659</td>
<td>12,141</td>
<td>16,469</td>
</tr>
<tr>
<td>$C_{i}^{oe}$</td>
<td>862</td>
<td>3,466</td>
<td>12,141</td>
<td>16,469</td>
</tr>
<tr>
<td>$Y_{i}^{oe} - C_{i}^{oe}$</td>
<td>-193</td>
<td>193</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>
**ACCOUNTING FOR THE FACTS**

Benchmark 1990 Rest of the World Data Set  
(Billion U.S. dollars)

<table>
<thead>
<tr>
<th>Primaries</th>
<th>Manufactures</th>
<th>Services</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Y_i^{rw}$</td>
<td>1,223</td>
<td>1,159</td>
<td>3,447</td>
</tr>
<tr>
<td>$C_i^{rw}$</td>
<td>1,030</td>
<td>1,352</td>
<td>3,447</td>
</tr>
<tr>
<td>$Y_i^{rw} - C_i^{rw}$</td>
<td>193</td>
<td>-193</td>
<td>0</td>
</tr>
</tbody>
</table>
ACCOUNTING FOR THE FACTS

- \( N^{oe} = 854, \ N^{rw} = 4,428. \)
- \( \sum_{i=p,m,s} Y^{rw}_i = \sum_{i=p,m,s} C^{rw}_i = 5,829. \)
- Set \( q_p = q_m(z) = q_s = w = r = 1 \) (quantities are 1990 values).
- \( \rho = 1/1.2 \) (Morrison 1990, Martins, Scarpetta, and Pilat 1996).
- Normalize \( d^w = 100. \)
- Calibrate \( H^{rw}, K^{rw} \) so that benchmark data set is an equilibrium.

- Alternative calibrations of utility parameters \( \gamma_p, \gamma_s, \) and \( \eta. \)
### OECD in 1961

<table>
<thead>
<tr>
<th>Country</th>
<th>Share of GDP %</th>
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<th>Share of GDP %</th>
</tr>
</thead>
<tbody>
<tr>
<td>Austria</td>
<td>0.75</td>
<td>Netherlands</td>
<td>1.37</td>
</tr>
<tr>
<td>Belgium-Lux</td>
<td>1.25</td>
<td>Norway</td>
<td>0.60</td>
</tr>
<tr>
<td>Canada</td>
<td>4.22</td>
<td>Portugal</td>
<td>0.32</td>
</tr>
<tr>
<td>Denmark</td>
<td>0.70</td>
<td>Spain</td>
<td>1.38</td>
</tr>
<tr>
<td>France</td>
<td>6.99</td>
<td>Sweden</td>
<td>1.62</td>
</tr>
<tr>
<td>Germany</td>
<td>9.71</td>
<td>Switzerland</td>
<td>1.07</td>
</tr>
<tr>
<td>Greece</td>
<td>0.50</td>
<td>Turkey</td>
<td>0.83</td>
</tr>
<tr>
<td>Iceland</td>
<td>0.03</td>
<td>United Kingdom</td>
<td>8.08</td>
</tr>
<tr>
<td>Ireland</td>
<td>0.21</td>
<td>United States</td>
<td>55.74</td>
</tr>
<tr>
<td>Italy</td>
<td>4.64</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Numerical Experiments

Calculate equilibrium in 1961:

\[
\begin{align*}
\theta_{p,1961} &= \theta_{p,1990} \\
\theta_{m,1961} &= \frac{\theta_{m,1990}}{1.014^{29}}, \quad F_{1961} = \frac{F_{1990}}{1.014^{29}} \\
\theta_{s,1961} &= \frac{\theta_{s,1990}}{1.005^{29}} \text{ (Echevarria 1997)}
\end{align*}
\]

\[N^{oe} = 536, \quad N^{rw} = 2,545\]
Numerical Experiments

Choose $H_{1961}^{oe}, K_{1961}^{oe}, H_{1961}^{rw}, K_{1961}^{rw}$ so that

\[
\frac{\sum_{i=p,m,s} Y_{i,1990}^{oe} / N_{1990}^{oe}}{\sum_{i=p,m,s} Y_{i,1961}^{oe} / N_{1961}^{oe}} = 2.400
\]

\[
\frac{\sum_{i=p,m,s} Y_{i,1961}^{rw} / N_{1961}^{rw}}{\sum_{i=p,m,s} Y_{i,1990}^{rw} / N_{1990}^{rw}} = 2.055
\]

\[
\frac{K_{1961}^{oe}}{K_{1990}^{oe}} = \frac{H_{1961}^{oe}}{H_{1990}^{oe}}
\]

\[
q_{p,1961} (Y_{p,1961}^{rw} - C_{p,1961}^{rw}) \frac{\sum_{i=p,m,s} q_{i,1961} Y_{i,1961}^{rw}}{\sum_{i=p,m,s} Y_{i,1961}^{rw}} = 0.050
\]
How Can the Model Work in Matching the Facts?

• The ratio of trade to product has increased:

  The size distribution of countries has become more equal (Helpman-Krugman).

• Trade has become more concentrated among industrialized countries:

  OECD countries have comparative advantage in manufactures, while the RW has comparative advantage in primaries. Because they are inferior to manufactures, primaries become less important in trade as the world becomes richer (Markusen).

How Can the Model Work in Matching the Facts?
• Trade among industrialized countries is largely intraindustry trade:

OECD countries export manufactures. Because of taste for variety, every country consumes some manufactures from every other country (Dixit-Stiglitz).

• The different total factor productivity growth rates across sectors imply that the price of manufactures relative to primaries and services has fallen sharply between 1961 and 1990. If price elasticities of demand are not equal to one, a lot can happen.
Experiment 1

\[ \gamma_p = \gamma_s = \eta = 0 \]

<table>
<thead>
<tr>
<th>Data</th>
<th>1961</th>
<th>1990</th>
<th>Change</th>
</tr>
</thead>
<tbody>
<tr>
<td>OECD-OECD Trade/OECD GDP</td>
<td>0.053</td>
<td>0.112</td>
<td>111.5%</td>
</tr>
<tr>
<td>OECD-OECD Trade/OECD-RW Trade</td>
<td>0.844</td>
<td>1.579</td>
<td>87.1%</td>
</tr>
<tr>
<td>OECD Manf GDP/OECD GDP</td>
<td>0.295</td>
<td>0.222</td>
<td>−24.6%</td>
</tr>
</tbody>
</table>

1. \[ \gamma_p = 0, \gamma_s = 0, \eta = 0 \]

<table>
<thead>
<tr>
<th>Data</th>
<th>1961</th>
<th>1990</th>
<th>Change</th>
</tr>
</thead>
<tbody>
<tr>
<td>OECD-OECD Trade/OECD GDP</td>
<td>0.108</td>
<td>0.136</td>
<td>25.8%</td>
</tr>
<tr>
<td>OECD-OECD Trade/OECD-RW Trade</td>
<td>0.893</td>
<td>1.169</td>
<td>30.9%</td>
</tr>
<tr>
<td>OECD Manf GDP/OECD GDP</td>
<td>0.223</td>
<td>0.222</td>
<td>−0.4%</td>
</tr>
</tbody>
</table>
Experiment 2

\[ \gamma_p = -169.5, \ \gamma_s = 314.7 \] to match consumption in RW in 1990,
\[ \eta = 0 \]

<table>
<thead>
<tr>
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<th>Change</th>
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</tbody>
</table>

2. \[ \gamma_p = -169.5, \ \gamma_s = 314.7, \ \eta = 0 \]

<table>
<thead>
<tr>
<th>Data</th>
<th>1961</th>
<th>1990</th>
<th>Change</th>
</tr>
</thead>
<tbody>
<tr>
<td>OECD-OECD Trade/OECD GDP</td>
<td>0.103</td>
<td>0.132</td>
<td>28.1%</td>
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<tr>
<td>OECD-OECD Trade/OECD-RW Trade</td>
<td>0.739</td>
<td>1.060</td>
<td>43.6%</td>
</tr>
<tr>
<td>OECD Manf GDP/OECD GDP</td>
<td>0.225</td>
<td>0.222</td>
<td>-1.4%</td>
</tr>
</tbody>
</table>
Experiment 3

\[ \gamma_p = -169.5, \gamma_s = 314.7, \]
\[ \eta = 0.559 \] to match growth in OECD-OECD Trade/OECD GDP

<table>
<thead>
<tr>
<th>Data</th>
<th>1961</th>
<th>1990</th>
<th>Change</th>
</tr>
</thead>
<tbody>
<tr>
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<td>87.1%</td>
</tr>
<tr>
<td>OECD Manf GDP/OECD GDP</td>
<td>0.295</td>
<td>0.222</td>
<td>-24.6%</td>
</tr>
</tbody>
</table>

3. \[ \gamma_p = -169.5, \gamma_s = 314.7, \eta = 0.559 \]

<table>
<thead>
<tr>
<th>Data</th>
<th>1961</th>
<th>1990</th>
<th>Change</th>
</tr>
</thead>
<tbody>
<tr>
<td>OECD-OECD Trade/OECD GDP</td>
<td>0.063</td>
<td>0.132</td>
<td>111.5%</td>
</tr>
<tr>
<td>OECD-OECD Trade/OECD-RW Trade</td>
<td>0.738</td>
<td>1.060</td>
<td>43.7%</td>
</tr>
<tr>
<td>OECD Manf GDP/OECD GDP</td>
<td>0.137</td>
<td>0.222</td>
<td>62.7%</td>
</tr>
</tbody>
</table>
Experiments 4 and 5

$\gamma_p = -169.5, \gamma_s = 314.7$, reasonable values of $\eta$ ($0.5 \geq 1/(1-\eta) \geq 0.1$)

<table>
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<tr>
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<th>1990</th>
<th>Change</th>
</tr>
</thead>
<tbody>
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</tr>
<tr>
<td>OECD Manf GDP/OECD GDP</td>
<td>0.295</td>
<td>0.222</td>
<td>−24.6%</td>
</tr>
</tbody>
</table>

4. $\gamma_p = -169.5, \gamma_s = 314.7, \eta = -1$

<table>
<thead>
<tr>
<th>Data</th>
<th>1961</th>
<th>1990</th>
<th>Change</th>
</tr>
</thead>
<tbody>
<tr>
<td>OECD-OECD Trade/OECD GDP</td>
<td>0.118</td>
<td>0.132</td>
<td>11.7%</td>
</tr>
<tr>
<td>OECD-OECD Trade/OECD-RW Trade</td>
<td>0.739</td>
<td>1.060</td>
<td>43.5%</td>
</tr>
<tr>
<td>OECD Manf GDP/OECD GDP</td>
<td>0.259</td>
<td>0.222</td>
<td>−14.1%</td>
</tr>
</tbody>
</table>

5. $\gamma_p = -169.5, \gamma_s = 314.7, \eta = -9$

<table>
<thead>
<tr>
<th>Data</th>
<th>1961</th>
<th>1990</th>
<th>Change</th>
</tr>
</thead>
<tbody>
<tr>
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<td>0.132</td>
<td>1.6%</td>
</tr>
<tr>
<td>OECD-OECD Trade/OECD-RW Trade</td>
<td>0.739</td>
<td>1.060</td>
<td>43.5%</td>
</tr>
<tr>
<td>OECD Manf GDP/OECD GDP</td>
<td>0.284</td>
<td>0.222</td>
<td>−21.8%</td>
</tr>
</tbody>
</table>
Sensitivity Analysis:  
Alternative Calibration Methodologies

- Alternative specifications of nonhomogeneity
- Gross imports calibration
- Alternative RW endowment calibration
- Alternative RW growth calibration
- Intermediate goods
**INTERMEDIATE GOODS?**

\[
Y_p^j = \min \left[ \frac{X_{pp}^j}{a_{pp}}, \int_{D^w} X_{mp}^j(z)dz, \frac{X_{sp}^j}{a_{sp}}, \theta_p \left( K_p^j \right)^{\alpha_p} \left( H_p^j \right)^{1-\alpha_p} \right]
\]

\[
Y_m(z) = \min \left[ \frac{X_{pm}^j(z)}{a_{pm}}, \int_{D^w} X_{mm}^j(z, z')dz', \frac{X_{sm}^j(z)}{a_{sm}}, \theta_m \left( K_m(z) \right)^{\alpha_m} \left( H_m(z) \right)^{1-\alpha_m} - F \right]
\]

\[
Y_s^j = \min \left[ \frac{X_{ps}^j}{a_{ps}}, \int_{D^w} X_{ms}^j(z)dz, \frac{X_{ss}^j}{a_{ss}}, \theta_s \left( K_s^j \right)^{\alpha_s} \left( H_s^j \right)^{1-\alpha_s} \right]
\]
### Results for Model with Intermediate Goods

<table>
<thead>
<tr>
<th>Data</th>
<th>1961</th>
<th>1990</th>
<th>Change</th>
</tr>
</thead>
<tbody>
<tr>
<td>OECD-OECD Trade/OECD GDP</td>
<td>0.053</td>
<td>0.112</td>
<td>111.5%</td>
</tr>
<tr>
<td>OECD-OECD Trade/OECD-RW Trade</td>
<td>0.844</td>
<td>1.579</td>
<td>87.1%</td>
</tr>
<tr>
<td>OECD Manf GDP/OECD GDP</td>
<td>0.295</td>
<td>0.222</td>
<td>−24.6%</td>
</tr>
</tbody>
</table>

4. $\gamma_p = -307.8$, $\gamma_s = 262.2$, $\eta = -1$

| OECD-OECD Trade/OECD GDP                              | 0.323 | 0.370 | 14.5%    |
| OECD-OECD Trade/OECD-RW Trade                         | 0.994 | 1.305 | 31.3%    |
| OECD Manf GDP/OECD GDP                                | 0.263 | 0.222 | −15.6%   |

5. $\gamma_p = -307.8$, $\gamma_s = 262.2$, $\eta = -9$

| OECD-OECD Trade/OECD GDP                              | 0.337 | 0.370 | 9.7%     |
| OECD-OECD Trade/OECD-RW Trade                         | 0.933 | 1.305 | 39.9%    |
| OECD Manf GDP/OECD GDP                                | 0.307 | 0.222 | −27.5%   |
In a version of our model with \( n \) OECD countries, a manufacturing sector, and a uniform ad valorem tariff \( \tau \), the ratio of exports to income is given by

\[
\frac{M}{Y} = \frac{(n-1)C_f}{Y} = \frac{n-1}{n-1+(1+\tau)^{1/(1-\rho)}}
\]

Fixing \( n \) to replicate the size distribution of national incomes in the OECD, and setting \( \rho = 1/1.2 \), a fall in \( \tau \) from 0.45 to 0.05 produces an increase in the ratio of trade to output as seen in the data.
2. Applied general equilibrium models that put the standard theory to work do not well in predicting the impact of trade liberalization experiences like NAFTA.

Applied general equilibrium models were the only analytical game in town when it came to analyzing the impact of NAFTA in 1992-1993.

Typical sort of model: Static applied general equilibrium model with large number of industries and imperfect competition (Dixit-Stiglitz or Eastman-Stykolt) and finite number of firms in some industries. In some numerical experiments, new capital is placed in Mexico owned by consumers in the rest of North America to account for capital flows.

Examples:
Brown-Deardorff-Stern model of Canada, Mexico, and the United States
Cox-Harris model of Canada
Sobarzo model of Mexico

Research Agenda:

- Compare results of numerical experiments of models with data.

- Determine what shocks — besides NAFTA policies — were important.

- Construct a simple applied general equilibrium model and perform experiments with alternative specifications to determine what was wrong with the 1992-1993 models.
Applied GE Models Can Do a Good Job!

Spain: Kehoe-Polo-Sancho (1992) evaluation of the performance of the Kehoe-Manresa-Noyola-Polo-Sancho-Serra MEGA model of the Spanish economy: A Shoven-Whalley type model with perfect competition, modified to allow government and trade deficits and unemployment (Kehoe-Serra). Spain’s entry into the European Community in 1986 was accompanied by a fiscal reform that introduced a value-added tax (VAT) on consumption to replace a complex range of indirect taxes, including a turnover tax applied at every stage of the production process. What would happen to tax revenues? Trade reform was of secondary importance.


Other changes besides policy changes are important!
## Changes in Consumer Prices in the Spanish Model
(Percent)

<table>
<thead>
<tr>
<th>sector</th>
<th>data 1985-1986</th>
<th>model policy only</th>
<th>model shocks only</th>
<th>model policy&amp;shocks</th>
</tr>
</thead>
<tbody>
<tr>
<td>food and nonalcoholic beverages</td>
<td>1.8</td>
<td>-2.3</td>
<td>4.0</td>
<td>1.7</td>
</tr>
<tr>
<td>tobacco and alcoholic beverages</td>
<td>3.9</td>
<td>2.5</td>
<td>3.1</td>
<td>5.8</td>
</tr>
<tr>
<td>clothing</td>
<td>2.1</td>
<td>5.6</td>
<td>0.9</td>
<td>6.6</td>
</tr>
<tr>
<td>housing</td>
<td>-3.3</td>
<td>-2.2</td>
<td>-2.7</td>
<td>-4.8</td>
</tr>
<tr>
<td>household articles</td>
<td>0.1</td>
<td>2.2</td>
<td>0.7</td>
<td>2.9</td>
</tr>
<tr>
<td>medical services</td>
<td>-0.7</td>
<td>-4.8</td>
<td>0.6</td>
<td>-4.2</td>
</tr>
<tr>
<td>transportation</td>
<td>-4.0</td>
<td>2.6</td>
<td>-8.8</td>
<td>-6.2</td>
</tr>
<tr>
<td>recreation</td>
<td>-1.4</td>
<td>-1.3</td>
<td>1.5</td>
<td>0.1</td>
</tr>
<tr>
<td>other services</td>
<td>2.9</td>
<td>1.1</td>
<td>1.7</td>
<td>2.8</td>
</tr>
<tr>
<td>weighted correlation with data</td>
<td></td>
<td>-0.08</td>
<td>0.87</td>
<td>0.94</td>
</tr>
<tr>
<td>variance decomposition of change</td>
<td></td>
<td>0.30</td>
<td>0.77</td>
<td>0.85</td>
</tr>
<tr>
<td>regression coefficient $a$</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>regression coefficient $b$</td>
<td>-0.08</td>
<td>0.54</td>
<td>0.67</td>
<td></td>
</tr>
</tbody>
</table>
Measures of Accuracy of Model Results

1. Weighted correlation coefficient.

2. Variance decomposition of the (weighted) variance of the changes in the data:

\[
\text{vardec}(y_{\text{data}}, y_{\text{model}}) = \frac{\text{var}(y_{\text{model}})}{\text{var}(y_{\text{model}}) + \text{var}(y_{\text{data}} - y_{\text{model}})}.
\]

3, 4. Estimated coefficients \(a\) and \(b\) from the (weighted) regression

\[
x_{i}^{\text{data}} = a + bx_{i}^{\text{model}} + e_{i}.
\]
## Changes in Value of Gross Output/GDP in the Spanish Model (Percent)

<table>
<thead>
<tr>
<th>sector</th>
<th>data 1985-1986</th>
<th>model policy only</th>
<th>model shocks only</th>
<th>model policy&amp;shocks</th>
</tr>
</thead>
<tbody>
<tr>
<td>agriculture</td>
<td>-0.4</td>
<td>-1.1</td>
<td>8.3</td>
<td>6.9</td>
</tr>
<tr>
<td>energy</td>
<td>-20.3</td>
<td>-3.5</td>
<td>-29.4</td>
<td>-32.0</td>
</tr>
<tr>
<td>basic industry</td>
<td>-9.0</td>
<td>1.6</td>
<td>-1.8</td>
<td>-0.1</td>
</tr>
<tr>
<td>machinery</td>
<td>3.7</td>
<td>3.8</td>
<td>1.0</td>
<td>5.0</td>
</tr>
<tr>
<td>automobile industry</td>
<td>1.1</td>
<td>3.9</td>
<td>4.7</td>
<td>8.6</td>
</tr>
<tr>
<td>food products</td>
<td>-1.8</td>
<td>-2.4</td>
<td>4.7</td>
<td>2.1</td>
</tr>
<tr>
<td>other manufacturing</td>
<td>0.5</td>
<td>-1.7</td>
<td>2.3</td>
<td>0.5</td>
</tr>
<tr>
<td>construction</td>
<td>5.7</td>
<td>8.5</td>
<td>1.4</td>
<td>10.3</td>
</tr>
<tr>
<td>commerce</td>
<td>6.6</td>
<td>-3.6</td>
<td>4.4</td>
<td>0.4</td>
</tr>
<tr>
<td>transportation</td>
<td>-18.4</td>
<td>-1.5</td>
<td>1.0</td>
<td>-0.7</td>
</tr>
<tr>
<td>services</td>
<td>8.7</td>
<td>-1.1</td>
<td>5.8</td>
<td>4.5</td>
</tr>
<tr>
<td>government services</td>
<td>7.6</td>
<td>3.4</td>
<td>0.9</td>
<td>4.3</td>
</tr>
<tr>
<td>weighted correlation with data</td>
<td>0.16</td>
<td>0.80</td>
<td>0.77</td>
<td></td>
</tr>
<tr>
<td>variance decomposition of change</td>
<td>0.11</td>
<td>0.73</td>
<td>0.71</td>
<td></td>
</tr>
<tr>
<td>regression coefficient $a$</td>
<td>-0.52</td>
<td>-0.52</td>
<td>-0.52</td>
<td></td>
</tr>
<tr>
<td>regression coefficient $b$</td>
<td>0.44</td>
<td>0.75</td>
<td>0.67</td>
<td></td>
</tr>
</tbody>
</table>
## Changes in Trade/GDP in the Spanish Model (Percent)

<table>
<thead>
<tr>
<th>direction of exports</th>
<th>data 1985-1986</th>
<th>model policy only</th>
<th>model shocks only</th>
<th>model policy&amp;shocks</th>
</tr>
</thead>
<tbody>
<tr>
<td>Spain to rest of E.C.</td>
<td>-6.7</td>
<td>-3.2</td>
<td>-4.9</td>
<td>-7.8</td>
</tr>
<tr>
<td>Spain to rest of world</td>
<td>-33.2</td>
<td>-3.6</td>
<td>-6.1</td>
<td>-9.3</td>
</tr>
<tr>
<td>rest of E.C. to Spain</td>
<td>14.7</td>
<td>4.4</td>
<td>-3.9</td>
<td>0.6</td>
</tr>
<tr>
<td>rest of world to Spain</td>
<td>-34.1</td>
<td>-1.8</td>
<td>-16.8</td>
<td>-17.7</td>
</tr>
</tbody>
</table>

- weighted correlation with data: 0.69, 0.77, 0.90
- variance decomposition of change: 0.02, 0.17, 0.24

- regression coefficient $a$: -12.46, 2.06, 5.68
- regression coefficient $b$: 5.33, 2.21, 2.37
<table>
<thead>
<tr>
<th>variable</th>
<th>data 1985-1986</th>
<th>model policy only</th>
<th>model shocks only</th>
<th>model policy &amp; shocks</th>
</tr>
</thead>
<tbody>
<tr>
<td>wages and salaries</td>
<td>-0.53</td>
<td>-0.87</td>
<td>-0.02</td>
<td>-0.91</td>
</tr>
<tr>
<td>business income</td>
<td>-1.27</td>
<td>-1.63</td>
<td>0.45</td>
<td>-1.24</td>
</tr>
<tr>
<td>net indirect taxes and tariffs</td>
<td>1.80</td>
<td>2.50</td>
<td>-0.42</td>
<td>2.15</td>
</tr>
<tr>
<td>correlation with data</td>
<td>0.998</td>
<td>-0.94</td>
<td>0.99</td>
<td></td>
</tr>
<tr>
<td>variance decomposition of change</td>
<td>0.93</td>
<td>0.04</td>
<td>0.96</td>
<td></td>
</tr>
<tr>
<td>regression coefficient (a)</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>regression coefficient (b)</td>
<td>0.73</td>
<td>-3.45</td>
<td>0.85</td>
<td></td>
</tr>
<tr>
<td>private consumption</td>
<td>-0.81</td>
<td>-1.23</td>
<td>-0.51</td>
<td>-1.78</td>
</tr>
<tr>
<td>private investment</td>
<td>1.09</td>
<td>1.81</td>
<td>-0.58</td>
<td>1.32</td>
</tr>
<tr>
<td>government consumption</td>
<td>-0.02</td>
<td>-0.06</td>
<td>-0.38</td>
<td>-0.44</td>
</tr>
<tr>
<td>government investment</td>
<td>-0.06</td>
<td>-0.06</td>
<td>-0.07</td>
<td>-0.13</td>
</tr>
<tr>
<td>exports</td>
<td>-3.40</td>
<td>-0.42</td>
<td>-0.69</td>
<td>-1.07</td>
</tr>
<tr>
<td>-imports</td>
<td>3.20</td>
<td>-0.03</td>
<td>2.23</td>
<td>2.10</td>
</tr>
<tr>
<td>correlation with data</td>
<td>0.40</td>
<td>0.77</td>
<td>0.83</td>
<td></td>
</tr>
<tr>
<td>variance decomposition of change</td>
<td>0.20</td>
<td>0.35</td>
<td>0.58</td>
<td></td>
</tr>
<tr>
<td>regression coefficient (a)</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td></td>
</tr>
<tr>
<td>regression coefficient (b)</td>
<td>0.87</td>
<td>1.49</td>
<td>1.24</td>
<td></td>
</tr>
</tbody>
</table>
## Public Finances in the Spanish Model
(Percent of GDP)

<table>
<thead>
<tr>
<th>variable</th>
<th>data 1985-1986</th>
<th>model policy only</th>
<th>model shocks only</th>
<th>model policy&amp;shocks</th>
</tr>
</thead>
<tbody>
<tr>
<td>indirect taxes and subsidies</td>
<td>2.38</td>
<td>3.32</td>
<td>-0.38</td>
<td>2.98</td>
</tr>
<tr>
<td>tariffs</td>
<td>-0.58</td>
<td>-0.82</td>
<td>-0.04</td>
<td>-0.83</td>
</tr>
<tr>
<td>social security payments</td>
<td>0.04</td>
<td>-0.19</td>
<td>-0.03</td>
<td>-0.22</td>
</tr>
<tr>
<td>direct taxes and transfers</td>
<td>-0.84</td>
<td>-0.66</td>
<td>0.93</td>
<td>0.26</td>
</tr>
<tr>
<td>government capital income</td>
<td>-0.13</td>
<td>-0.06</td>
<td>0.02</td>
<td>-0.04</td>
</tr>
<tr>
<td>correlation with data</td>
<td>0.99</td>
<td>-0.70</td>
<td>0.92</td>
<td>0.92</td>
</tr>
<tr>
<td>variance decomposition of change</td>
<td>0.93</td>
<td>0.08</td>
<td>0.86</td>
<td>0.86</td>
</tr>
<tr>
<td>regression coefficient $a$</td>
<td>-0.06</td>
<td>0.35</td>
<td>-0.17</td>
<td></td>
</tr>
<tr>
<td>regression coefficient $b$</td>
<td>0.74</td>
<td>-1.82</td>
<td>0.80</td>
<td></td>
</tr>
</tbody>
</table>
Models of NAFTA Did Not Do a Good Job!

Ex-post evaluations of the performance of applied GE models are essential if policy makers are to have confidence in the results produced by this sort of model.

Just as importantly, they help make applied GE analysis a scientific discipline in which there are well-defined puzzles and clear successes and failures for alternative hypotheses.
For the past three decades, the tool of choice for analyzing the impact of trade policy has been the multisectoral applied GE model.

At a conference organized by the U.S. International Trade Commission, held in February 1992 at the request of the U.S. Congress, to which all economists studying the impact of NAFTA had been invited, 10 of the 12 studies presented used applied GE models.

This is still the sort of model used to analyze policies like the U.S.-Central America-Dominican Republic Free Trade Agreement and the U.S.-Korea Free Trade Agreement.

Typical sort of model: Static applied general equilibrium model with 20-40 industries, imperfect competition (Dixit-Stiglitz or Eastman-Stykolt) in some industries, perfect competition and Armington aggregators in others.

Brown-Deardorff-Stern model of Canada, Mexico, and the United States

Cox-Harris model of Canada

Sobarzo model of Mexico
Methodology for comparing results of models with data  

For exports by country $i$ of industry $j$ to country $k$, $x_{ij}^k$, 

$$z_{ij}^k = 100 \left( \frac{x_{ijT_1}^k / y_{iT_1}^k}{x_{ijT_0}^k / y_{iT_0}^k} - 1 \right)$$

where $y_{it}$ is the current price GDP in country $i$ in year $t$

To compare the predictions of the model with the changes in data, we calculate the weighted correlation coefficient and the coefficients $a$ and $b$ from the weighted regression

$$\min_{a,b} \sum_{j=1}^{n} \alpha_j \left( a + bz_{j}^{model} - z_{j}^{data} \right)^2$$
How to turn least traded products data into predictions:

Predict that least traded products will have their exports grow \( a + b \) percent faster than the exporting country’s GDP and other products grow \( a \) percent faster.

For a particular industry, this prediction becomes

\[
a(1 - s_i) + (a + b)s_i = a + bs_i,
\]

where \( s_i \) is the fraction of the industry’s exports made up of least traded products in the base year.

Notice that this method does not make use of the products with 0 recorded trade.
Product-industry detail

Use concordance between the ISIC and the SITC based on those in the World Bank's Trade and Production Database (Nicita and Olarreaga, 2001, 2006)

Canadian exports of chemicals to the United States grew 113.5 percent while the BDS model predicted −3.1 percent.

88 complete 4-digit SITC categories — 80 least traded.

Parts of another 27 categories — 11 least traded.

Compared to Canadian GDP, the 22 percent of 1988 exports of chemicals that are least traded increase by 212 percent, while the other 78 percent increase by only 32 percent.
Compared to Canadian GDP

Exports of 5823 (Alkyds and Other Polyesters) increases by 1,285 percent

Exports of 5832 (Polypropylene) increases by 738 percent

Exports of 5121 (Acyclic Alcohols and their Halogenated and Derivatives) decreases by 37 percent.
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>agriculture</td>
<td>166.3</td>
<td>3.1</td>
<td>0.05</td>
<td>37.3</td>
</tr>
<tr>
<td>mining and quarrying</td>
<td>-17.9</td>
<td>-0.3</td>
<td>0.00</td>
<td>257.2</td>
</tr>
<tr>
<td>food</td>
<td>160.1</td>
<td>2.2</td>
<td>0.07</td>
<td>69.8</td>
</tr>
<tr>
<td>textiles</td>
<td>180.8</td>
<td>-0.9</td>
<td>1.00</td>
<td>131.5</td>
</tr>
<tr>
<td>clothing</td>
<td>1478.9</td>
<td>1.3</td>
<td>1.00</td>
<td>76.3</td>
</tr>
<tr>
<td>leather products</td>
<td>72367.2</td>
<td>1.4</td>
<td>1.00</td>
<td>-0.4</td>
</tr>
<tr>
<td>footwear</td>
<td>—</td>
<td>3.7</td>
<td>—</td>
<td>37.8</td>
</tr>
<tr>
<td>wood products</td>
<td>2128.6</td>
<td>4.7</td>
<td>1.00</td>
<td>33.9</td>
</tr>
<tr>
<td>furniture and fixtures</td>
<td>2009.4</td>
<td>2.7</td>
<td>1.00</td>
<td>125.8</td>
</tr>
<tr>
<td>paper products</td>
<td>56.6</td>
<td>-4.3</td>
<td>0.05</td>
<td>-48.8</td>
</tr>
<tr>
<td>printing and publishing</td>
<td>1186.7</td>
<td>-2.0</td>
<td>1.00</td>
<td>43.1</td>
</tr>
<tr>
<td>chemicals</td>
<td>371.5</td>
<td>-7.8</td>
<td>0.15</td>
<td>113.5</td>
</tr>
<tr>
<td>petroleum and products</td>
<td>8453.5</td>
<td>-8.5</td>
<td>0.99</td>
<td>161.9</td>
</tr>
<tr>
<td>rubber products</td>
<td>14.8</td>
<td>-1.0</td>
<td>0.57</td>
<td>71.5</td>
</tr>
<tr>
<td>nonmetal mineral products</td>
<td>103.2</td>
<td>-1.8</td>
<td>1.00</td>
<td>28.9</td>
</tr>
<tr>
<td>glass products</td>
<td>-66.7</td>
<td>-2.2</td>
<td>1.00</td>
<td>32.8</td>
</tr>
<tr>
<td>iron and steel</td>
<td>210.1</td>
<td>-15.0</td>
<td>0.03</td>
<td>30.7</td>
</tr>
<tr>
<td>nonferrous metals</td>
<td>2019.0</td>
<td>-64.7</td>
<td>1.00</td>
<td>28.7</td>
</tr>
<tr>
<td>metal products</td>
<td>232.9</td>
<td>-10.0</td>
<td>0.51</td>
<td>74.8</td>
</tr>
<tr>
<td>nonelectrical machinery</td>
<td>218.8</td>
<td>-8.9</td>
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<td>0.62</td>
<td>34.7</td>
<td>43.2</td>
</tr>
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</table>

**weighted correlation with data**

| Mexico to Canada | 0.45 | 0.24 | 0.14 | 0.003 |
| Mexico to United States | 100.02 | 136.33 | 49.49 | 73.29 |
| regression coefficient $a$ | 1.80 | 313.80 | 0.10 | 1.56 |
### Changes in U.S. Exports relative to U.S. GDP (percent)

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<th>sector</th>
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<th>United States to Mexico</th>
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<td>2006 data</td>
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<td>1.26</td>
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<td></td>
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<td>95.50</td>
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## Changes in North American trade relative to exporter’s GDP

<table>
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<td></td>
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<td>(a)</td>
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<tr>
<td>Canada to Mexico</td>
<td>88–06</td>
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<td>178.63</td>
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<tr>
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<td>88–06</td>
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<td>64.83</td>
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<td>100.02</td>
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<td>U.S. to Mexico</td>
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<tr>
<td><strong>pooled regression</strong></td>
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<td>0.02</td>
<td>46.19</td>
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</table>
In 1993 Ross Perot said

The reason that most U.S. policymakers are so blind to the job shifting that will occur if NAFTA is ratified is that they rely on dozens of “reputable” academic studies that say it won’t happen. Yet these studies are based on unrealistic assumptions and flawed mathematical models…Let’s be clear about this: these studies certainly do not provide a basis on which Congress can make an informed decision about NAFTA.

What Do We Learn from these Evaluations?

The Spanish model seems to have been far more successful in predicting the consequences of policy changes than the three models of NAFTA, but

- Kehoe, Polo, and Sancho (KPS) knew the structure of their model well enough to precisely identify the relationships between the variables in their model with those in the data;

- KPS were able to use the model to carry out numerical exercises to incorporate the impact of exogenous shocks.

KPS had an incentive to show their model in the best possible light.
Armington aggregator

\[ x_i^{mex} = \theta_i\left( \alpha_{i,can}^{mex} x_{i,can}^{mex} \rho + \alpha_{i,mex}^{mex} x_{i,mex}^{mex} \rho + \alpha_{i,us}^{mex} x_{i,us}^{mex} \rho + \alpha_{i,rw}^{mex} x_{i,rw}^{mex} \rho \right)^{1/\rho} \]

Dixit-Stiglitz/Ethier aggregator

\[ x_i^{mex} = \theta_i\left( \sum_{j=1}^{n_i} x_{i,j}^{mex} \rho \right)^{1/\rho} \]

modified to allow for home country bias

\[ x_i^{mex} = \theta_i\left( \alpha_{i,can}^{mex} \sum_{j=1}^{n_i,can} x_{i,j,can}^{mex} \rho + \alpha_{i,mex}^{mex} \sum_{j=1}^{n_i,mex} x_{i,j,mex}^{mex} \rho \\
+ \alpha_{i,us}^{mex} \sum_{j=1}^{n_i,us} x_{i,j,us}^{mex} \rho + \alpha_{i,rw}^{mex} \sum_{j=1}^{n_i,rw} x_{i,j,rw}^{mex} \rho \right)^{1/\rho} \]
3. Much of the growth of trade after a trade liberalization experience is growth on the extensive margin. Models need to allow for corner solutions or fixed costs.


What happens to the least-traded goods:

Over the business cycle?
During trade liberalization?

Indirect evidence on the extensive margin
Evidence on the Extensive Margin

- Data
  - 4 digit SITC bilateral trade data (OECD)
  - 789 codes in revision 2

- Least Traded Goods
  - Look 5 years before trade agreement
  - Rank codes from lowest value of exports to highest based on average of first 3 years in sample
  - Lowest decile of codes = least-traded goods

- Two Episodes
  - Canada-Mexico during NAFTA
  - United States-Germany in 1990s
Composition of Exports: Mexico to Canada

![Graph showing the composition of exports from Mexico to Canada as a cumulative fraction of 1989 exports.](image-url)

- 736.6
- 28.8
- 10.3
- 5.3
- 1.8
- 1.9
- 1.5
- 0.8
- 1.4
- 0.8

Cumulative Fraction of 1989 Exports
Composition of Exports: Mexico to Canada

Cumulative Fraction of 1989 Exports

Fraction of 1999 Exports

Cumulative Fraction of 1989 Exports
Exports: Mexico to the Canada

Fraction of Total Export Value

Export Value	0.00	0.10	0.20	0.30
Lessons from data

Trade liberalization increases trade on the extensive margin, business cycle fluctuations do not.

Structural changes may increase trade on the extensive margin.

A country increasing its exports on the extensive margin because of trade liberalization may increase its exports on the extensive margin to other countries.
Composition of Exports: Chile to the United States

Fraction of 1984 export value vs. Cumulative fraction of 1974 export value.

- The highest bar represents 780.6, corresponding to a cumulative fraction of 0.1.
- The second highest bar represents 6.4, corresponding to a cumulative fraction of 0.2.
- Lower bars correspond to cumulative fractions of 0.3.

The graph shows the distribution of export values from Chile to the United States between 1974 and 1984.
Exports: Chile to the United States

Fraction of Total Export Value vs Year

- Fraction of Total Export Value: 0.00, 0.05, 0.10, 0.15, 0.20, 0.25, 0.30, 0.35, 0.40
Exports: United States to Chile

Year

Fraction of Total Export Value

Composition of Exports: China to the United States

cumulative fraction of 1991 export value

fraction of 2004 export value

690.3
46.4
23
11.2
7.9
4.7
2.9
1.2
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0

cumulative fraction of 1991 export value
Composition of Exports: United States to China

Cummulative fraction of 1991 export value

Fraction of 2004 export value
Exports: United States to China

Fraction of Total Export Value

Year

Composition of Exports: United Kingdom to Canada

The bar chart represents the composition of exports from the United Kingdom to Canada, showing the fraction of 2004 export value at different cumulative fractions of the 1989 export value.
Ricardian model with a continuum of goods \( x \in [0,1] \)

production technologies \( y(x) = \ell(x)/a(x) \), \( y^*(x) = \ell^*(x)/a^*(x) \)

ad valorem tariffs \( \tau \), \( \tau^* \)

\[
(1 + \tau^*)wa(x) < w^*a^*(x) \iff \frac{a(x)}{a^*(x)} < \frac{w^*}{(1 + \tau^*)w}
\]

\( \Rightarrow \) home country produces good and exports it to the foreign country.

\[
\frac{a(x)}{a^*(x)} > \frac{(1 + \tau)w^*}{w}
\]

\( \Rightarrow \) foreign country produces good and exports it to the home country.
\[
\frac{(1 + \tau)w^*}{w} > \frac{a(x)}{a^*(x)} > \frac{w^*}{(1 + \tau^*)w}
\]

\(\Rightarrow\) good is not traded.

**Lowering tariffs generates trade in previously nontraded goods.**
4. Fixed costs seem better than Ricardian corner solutions for reconciling time series data on real exchange rate fluctuations with data on trade growth after liberalization experiences.

The “Armington” Elasticity

- Elasticity of substitution between domestic and foreign goods
- Crucial elasticity in international economic models

- International Real Business Cycle (IRBC) models:
  - Terms of trade volatility
  - Net exports and terms of trade co-movements

- Applied General Equilibrium (AGE) Trade models:
  - Trade response to tariff changes
The Elasticity Puzzle

- Time series (Business Cycles):
  - Estimates are low
  - Relative prices volatile
  - Quantities less volatile

- Panel studies (Trade agreement):
  - Estimates are high
  - Small change in tariffs (prices)
  - Large change in quantities
### Time Series Estimates: Low Elasticity (1.5)

<table>
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<tr>
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<tbody>
<tr>
<td>Reinert and Roland Holst (1992)</td>
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<td>Reinert and Shiells (1993)</td>
<td>[0.1, 1.5]</td>
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<tr>
<td>Gallaway et al. (2003)</td>
<td>[0.2, 4.9]</td>
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</table>

### Trade Liberalization Estimates: High Elasticity (9.0)

<table>
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<td>Clausing (2001)</td>
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<td>Head and Reis (2001)</td>
<td>[7.9, 11.4]</td>
</tr>
<tr>
<td>Romalis (2002)</td>
<td>[4.0, 13.0]</td>
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</tbody>
</table>
Why do the Estimates Differ?

- Time series – no liberalization:
  - Change in trade volume from goods already traded
  - Change mostly on the intensive margin

- Trade liberalization:
  - Change in intensive margin plus
  - New types of goods being traded
  - Change on the extensive margin
Modeling the Extensive Margin

- Model: extensive margin from export entry costs

- Empirical evidence of entry costs
  - Roberts and Tybout (1997)
  - Bernard and Wagner (2001)
  - Bernard and Jensen (2003)
The Effects of Entry Costs

- Business cycle shocks:
  - Small extensive margin effect

- Trade liberalization:
  - Big extensive margin effect

- Asymmetry creates different empirical elasticities
Model Overview

- Two countries: \( \{h, f\} \), with labor \( L \)
- Infinitely lived consumers
- No international borrowing/lending
- Continuum of traded goods plants in each country
  - Differentiated goods
  - Monopolistic competitors
  - Heterogeneous productivity
- Export entry costs
  - Differs across plants: second source of heterogeneity
- Non-traded good, competitive market: \( A \)
- Tariff on traded goods (iceberg): \( \tau \)
Uncertainty

- At date $t$, $H$ possible events, $\eta_t = 1,...,H$
- Each event is associated with a vector of productivity shocks:

\[ z_t = \left[ z_h(\eta_t), z_f(\eta_t) \right] \]

- First-order Markov process with transition matrix $\Lambda$

\[ \lambda_{\eta\eta'} = \text{pr}(\eta_{t+1} = \eta' | \eta_t = \eta) \]
Traded Good Plants

- Traded good technology:
  \[ y(\phi, \kappa) = z\phi l \]

- Plant heterogeneity \((\phi, \kappa)\)
  - constant, idiosyncratic productivity: \(\phi\)
  - export entry cost: \(\kappa\)
  - plant of type \((\phi, \kappa)\)

- \(\nu\) plants born each period with distribution \(F(\phi, \kappa)\)

- Fraction \(\delta\) of plants exogenously die each period
Timing

\[ \mu_{hx}(\phi, \kappa) : \text{plants of type } (\phi, \kappa) \text{ who paid entry cost} \]

\[ \mu_{hd}(\phi, \kappa) : \text{plants of type } (\phi, \kappa) \text{ who have not paid entry cost} \]

\[ \mu = \left( \mu_{hd}, \mu_{hx}, \mu_{fd}, \mu_{fx} \right) \]
Consumers

\[
\max_{q,c_h^h(i),c_f^h(i)} \gamma \log(C) + (1 - \gamma) \log(A)
\]

s.t.

\[
C = \left[ \int_{i \in I_h^h(\mu)} c_h^h(i) \rho \, dt + \int_{i \in I_f^h(\mu)} c_f^h(i) \rho \, dt \right]^\frac{1}{\rho}
\]

\[
\int_{i \in I_h^h(\mu)} p_h^h(i) c_h^h(i) \, dt + \int_{i \in I_f^h(\mu)} (1 + \tau) p_f^h(i) c_f^h(i) \, dt + p_{hA} A = L + \Pi_h
\]
Non-traded Good

\[
\max \ p_{hA}(\eta, \mu) A - l \\
\text{s.t.} \ A = z_h(\eta)l
\]

Normalize \( w_h = 1 \), implying \( p_{hA}(\eta, \mu) = z_h(\eta) \)
Traded Goods: Static Profit Maximization

\[ \pi_d \left( p^h, l; \phi, \kappa, \eta, \mu \right) = \max_{p^h, l} p^h z(\eta) \phi l - l \]

s.t. \hspace{1em} z(\eta) \phi l = \tilde{c}^h_{h} \left( p^h; \eta, \mu \right) \]

\[ \pi_x \left( p^f, l; \phi, \kappa, \eta, \mu \right) = \max_{p^f, l} p^f z(\eta) \phi l - l \]

s.t. \hspace{1em} z(\eta) \phi l = \tilde{c}^f_{h} \left( p^f; \eta, \mu \right) \]

Pricing rules:

\[ p^h \left( \phi, \kappa, \eta, \mu \right) = p^f \left( \phi, \kappa, \eta, \mu \right) = \frac{1}{\rho \phi z(\eta)} \]
Dynamic Choice: Export or Sell Domestically

- Exporter’s Value Function:

\[ V_x(\phi, \kappa, \eta, \mu) = d(\eta, \mu)(\pi_d(\phi, \kappa, \eta, \mu) + \pi_x(\phi, \kappa, \eta, \mu)) + (1 - \delta) \beta \sum_{\eta'} V_x(\phi, \kappa, \eta', \mu') \lambda_{\eta \eta'} \]

s.t. \( \mu' = M(\eta, \mu) \)

- \( d(\eta, \mu) \) = multiplier on budget constraint
• Non-exporter’s Value Function:

\[ V_d(\phi, \kappa, \eta, \mu) = \max \left\{ \pi_d(\phi, \kappa, \eta, \mu) d(\eta, \mu) + \beta (1 - \delta) \sum_{\eta'} V_d(\phi, \kappa, \eta', \mu') \lambda_{\eta\eta'}, \right. \]

\[ \left. \left[ \pi_d(\phi, \kappa, \eta, \mu) - \kappa \right] d(\eta, \mu) + \beta (1 - \delta) \sum_{\eta'} V_x(\phi, \kappa, \eta', \mu') \lambda_{\eta\eta'} \right\} \]

s.t. \( \mu' = M(\eta, \mu) \)
Equilibrium

• Cutoff level of productivity for each value of the entry cost

• For a plant of type \((\phi, \kappa)\)
  
  \[
  \text{If } \phi \geq \hat{\phi}_\kappa (\eta, \mu) \text{ export and sell domestically}
  \]
  
  \[
  \text{If } \phi < \hat{\phi}_\kappa (\eta, \mu) \text{ only sell domestically}
  \]

• In Equilibrium
  
  ○ “Low” productivity/“high” entry cost plants sell domestic
  
  ○ “High” productivity/“low” entry cost plants also export
  
  ○ Similar to Melitz (2003)
Determining Cutoffs

- For the cutoff plant:
  - entry cost = discounted, expected value of exporting

- \( \hat{\phi}_\kappa (\eta, \mu) \) is the level of productivity, \( \phi \), that solves:

\[
d (\eta, \mu) \kappa = (1-\delta) \beta \left[ \sum_{\eta'} V_x (\phi, \kappa, \eta', \mu') \lambda_{\eta\eta'} - \sum_{\eta'} V_d (\phi, \kappa, \eta', \mu') \lambda_{\eta\eta'} \right]
\]

entry cost \hspace{1cm} expected value of exporting
Finding the Cutoff Producer

Value of Exporting: Steady State

Entry Cost

Firm Productivity ($\phi$)

Non-Exporters

Exporters
Choosing Parameters

- Set $\sigma = \frac{1}{1 - \rho} = 2$ and $\tau = 0.15$

- Calibrate to the United States (1987) and a symmetric partner.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>Annual real interest rate (4%)</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>Share of manufactures in GDP (18%)</td>
</tr>
<tr>
<td>$\delta$</td>
<td>Annual loss of jobs from plant deaths as percentage of employment (Davis et. al., 1996) (6%)</td>
</tr>
</tbody>
</table>
Other Parameters

• Distribution over new plants:

\[ F_\kappa (\phi) = \frac{1}{\phi^{\theta_\phi}} \quad F_\phi (\kappa) = \frac{1}{(\bar{\kappa} - \kappa)^{\theta_\kappa}} \]

• \( \bar{\kappa}, \bar{\phi}, \nu, \theta_\phi, \theta_\kappa \) jointly determine:

  • Average plant size (12 employees)
  • Standard deviation of plant sizes (892)
  • Average exporting plant size (15 employees)
  • Standard deviation of exporting plant sizes (912)
  • Fraction of production that is exported (9%)
Productivity Process

- Two shocks, low and high:

\[ z_i = 1 - \varepsilon \]
\[ z_i = 1 + \varepsilon \]

- Countries have symmetric processes with Markov Matrix

\[ \Lambda_i = \begin{bmatrix} \bar{\lambda} & 1 - \bar{\lambda} \\ 1 - \bar{\lambda} & \bar{\lambda} \end{bmatrix} \]

- \( \varepsilon \): standard deviation of the U.S. Solow Residuals (1.0%)
- \( \bar{\lambda} \): autocorrelation of the U.S. Solow Residuals (0.90)
How does Trade Liberalization Differ from Business Cycles?

- Trade liberalization
  - Permanent changes
  - Large magnitudes

- Business cycles
  - Persistent, but not permanent changes
  - Small magnitudes
Developing Intuition: Persistent vs. Permanent Shocks

• 1% positive productivity shock in foreign country
  • Shock is persistent – autocorrelation of 0.90

• 1% decrease in tariffs
  • Change in tariffs is permanent
Response to 1% Productivity Shock
Autocorrelation = 0.90

Value of Exporting: 1% Productivity Shock
Value of Exporting: Steady State
Entry Cost

Costs and Benefits

Firm Productivity ($\phi$)

$\hat{\phi}_{bc}$ $\hat{\phi}_{ss}$
Response to a 1% Foreign Productivity Shock

Increase in imports on intensive margin = 1.89%
Increase in imports on extensive margin = 0.16%
Total increase in imports = 2.05%
Change in consumption of home goods = -0.10%

\[
\frac{\text{% Change Imports/Dom. Cons.}}{\text{% Change Price}} = \frac{2.17}{0.99} = 2.19
\]
Response to 1% Permanent Decrease in Tariffs

Value of Exporting: 1% Decrease in Tariffs
Value of Exporting: 1% Productivity Shock
Value of Exporting: Steady State

Costs and Benefits

Firm Productivity ($\phi$)

Entry Cost

$\phi_{tar}$ $\phi_{bc}$ $\phi_{ss}$
Response to a 1% Tariff Reduction

Increase in imports on intensive margin  =  1.42%
Increase in imports on extensive margin  =  3.04%

Total increase in imports  =  4.46%

Change in consumption of home goods  =  -0.33%

\[
\frac{\text{% Change Imports/Dom. Cons.}}{\text{% Change Tariff}} = \frac{4.81}{1.00} = 4.81
\]
Quantitative Results

- Two experiments

- Trade liberalization
  - Eliminate 15% tariff
  - Compute elasticity across tariff regimes

- Time series regressions
  - Use model to generate simulated data
  - Estimate elasticity as in the literature
## Trade Liberalization Elasticity

<table>
<thead>
<tr>
<th>Variable</th>
<th>Entry Costs (% change)</th>
<th>No Entry Costs (% change)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Exports</td>
<td>87.1</td>
<td>30.5</td>
</tr>
<tr>
<td>Imports/Dom. Cons.</td>
<td>93.0</td>
<td>32.2</td>
</tr>
<tr>
<td>Exporting Plants</td>
<td>37.7</td>
<td>0.0</td>
</tr>
<tr>
<td>Implied Elasticity</td>
<td>6.2</td>
<td>2.1</td>
</tr>
</tbody>
</table>
Elasticity in the Time Series

- Simulate: produce price/quantity time series
- Regress:

\[
\log \left( \frac{C_{f,t}}{C_{h,t}} \right) = \alpha + \sigma \log \left( \frac{p_{h,t}}{p_{f,t}} \right) + \varepsilon_t
\]

<table>
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<tr>
<th>Parameter</th>
<th>Estimate</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \alpha )</td>
<td>-0.015</td>
</tr>
<tr>
<td>(standard error)</td>
<td>(6.36e-04)</td>
</tr>
<tr>
<td>( \sigma )</td>
<td>1.39</td>
</tr>
<tr>
<td>(standard error)</td>
<td>(0.06)</td>
</tr>
<tr>
<td>R- squared</td>
<td>0.30</td>
</tr>
</tbody>
</table>
Conclusion

• Gap between dynamic macro models and trade models
  ○ Partially closes the gap
  ○ Modeling firm behavior as motivated by the data
  ○ Step towards better modeling of trade policy

• Single model can account for the elasticity puzzle
  ○ Time series elasticity of 1.4
  ○ Trade liberalization elasticity of 6.2
5. Models of trade with heterogeneous firms imposed fixed costs on firms that decide to export. The focus is on the decision to export. The theory and the data indicate that there is a lot of room for focusing on the decision to import.

Motivation

Dynamics of international trade flows

Long-run: Large, gradual changes  
(tariff reform)

Short-run: Small changes  
(fluctuations in relative prices)

Standard Theory: does not capture difference

Constant elasticity of substitution between imports and domestic goods
Question

What accounts for slow-moving dynamics of international trade flows?

This Paper’s Answer

Trade in intermediate inputs
Costly, irreversible importing decision at producer-level
Previous Literature’s Answers

Lags or costs of adjustment: contracting / distribution
Parameterize to generate slow-moving dynamics

This paper’s contribution:
Model mechanism based on micro-level evidence

Quantitative test of theory:
Endogenous aggregate dynamics in line with data

Significance of Results

Effects of trade reform
1. Timing and magnitude of trade growth
2. Welfare gains
Data: Aggregate Dynamics

Armington (1969) elasticity: elasticity of substitution between aggregate imported and domestic goods

Low estimates from time-series data (< 2)

High estimates from trade liberalization (> 6)
Data: Aggregate Dynamics
Gradual increase in trade after liberalization
NAFTA (Jan 1, 1994)
Data: Plant-level

Cross-section
Not all plants use imported intermediate inputs
Importing plants larger than non-importing plants

Panel
Reallocation between importers / non-importers is significant
### Data: Plant-level Cross-section

<table>
<thead>
<tr>
<th></th>
<th>% use imports</th>
<th>Avg. size ratio to non-importers</th>
</tr>
</thead>
<tbody>
<tr>
<td>Chile 1979-86</td>
<td>24.1</td>
<td>3.4</td>
</tr>
<tr>
<td>Chile average</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Chile 1979-86</td>
<td></td>
<td></td>
</tr>
<tr>
<td>US 1992</td>
<td>23.8</td>
<td>2.3</td>
</tr>
</tbody>
</table>

(Kurz, 2006)
Data: Plant-level Dynamics

Decompose changes in aggregate trade volumes

e.g., increase in aggregate imported/total inputs due to:

1. Importers increase ratio (Within) +
2. Importers expand, non-importers shrink (Between) +
3. Interaction between the two (Cross) +
4. Non-importers switch to importing (Switch) +
5. Higher proportion of new entrants are importers (Entry)

Baily, Hulten, Campbell (1992): productivity growth
### Data: Plant-level Dynamics

Imported / Total Intermediate Inputs: Chile, 1979-1986

<table>
<thead>
<tr>
<th></th>
<th>TOTAL</th>
<th>Fraction of Total (%)</th>
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<tr>
<td></td>
<td></td>
<td>Within</td>
</tr>
<tr>
<td>Avg of 1-year changes</td>
<td>-18%</td>
<td>79</td>
</tr>
<tr>
<td>7-year change</td>
<td>-77%</td>
<td>74</td>
</tr>
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</table>
Model

Heterogeneous Plants

Produce using intermediate inputs
Importing costly, irreversible
Trade growth through *Between* and *Entry* margins

2-country, 2-good real business cycle model

Technology shocks: short-run changes
Tariff reduction: long-run changes
Time and Uncertainty

Dates $t = 0, 1, 2, ...$

Event at date $t$: $s_t$. State at date $t$: $s^t = (s_0, s_1, \ldots, s_t)$.

$\Pr(s_t \mid s^{t-1}) = \phi(s_t \mid s_{t-1})$

$\tilde{\phi}(s^t) = \phi(s_t \mid s_{t-1})\phi(s_{t-1} \mid s_{t-2}) \cdots \phi(s_1 \mid s_0)$

Commodities and prices are functions $x(s^t) \rightarrow x_t$

Technology shocks $A(s^t), A^*(s^t)$
Representative Consumer

Preferences:

\[ E \sum_{t=0}^{\infty} \beta^t U(C_t, 1 - N_t) = \sum_{t=0}^{\infty} \sum_{s^t} \beta^t \tilde{\phi}(s^t) U(C(s^t), 1 - N(s^t)) \]

Budget constraint:

\[ C_t + \sum_{s_{t+1}} Q(s^t, s_{t+1}) B(s^t, s_{t+1}) \leq w_t N_t + B(s^t) + \Pi_t + T_t \]

Consumer owns plants
Plants

Heterogeneous in inherent efficiency \( z \).
Aggregate technology shocks \( A_t \).

Within each country, produce homogeneous output
Perfectly competitive, decreasing returns to scale technologies

Two types of decisions
1. Existing plants: static profit maximization
2. New plants: technology choice (import or not)
Plant technologies

Non-importing

\[ f_d(n, d; z) = z^{1-\alpha-\theta} d^\alpha n^\theta \]

Importing

\[ f_m(n, d, m; z) = z^{1-\alpha-\theta} \left( \gamma \min \left\{ \frac{d}{\omega}, \frac{m}{1-\omega} \right\} \right)^\alpha n^\theta \]

\[ \alpha + \theta < 1, \quad \omega < 1, \]
\[ \gamma : \text{efficiency gain from importing} \]
Static profit maximization

Non-importing plant with efficiency $z$ operating at date $t$

$$\pi_{dt}(z) = \max_{n,d} A_t f_d(n, d; z) - w_t n - d$$

Importing plant

$$\pi_{mt}(z) = \max_{n,d,m} A_t f_m(n, d, m; z) - w_t n - d - (1 + \tau) p_t m$$

No dependence on date of entry
Plant technologies, costs

Non-importing

\[ f_d(n, d; z) = z^{1-\alpha-\theta} d^\alpha n^\theta \]

Price of intermediate input: 1

Importing

\[ f_m(n, d, m; z) = z^{1-\alpha-\theta} \left( \gamma \min \left\{ \frac{d}{\omega}, \frac{m}{1-\omega} \right\} \right)^\alpha n^\theta \]

Price of composite intermediate input: \( \frac{1}{\gamma} (\omega + (1+\tau) p_i (1-\omega)) \)
Plant technologies, costs

Importing technology is more cost-efficient if

$$\gamma > \omega + (1 + \tau) p_t (1 - \omega)$$

Depends on equilibrium price $p_t$

- Estimate $\gamma$ from plant data
- Check that inequality holds along equilibrium path
Dynamic problem: Timing

Plant pays cost $\kappa_e$ to get a draw of $z$ from distribution $g$.

Decide whether to start producing or exit.

Pay sunk investment $\kappa_c$ to use non-importing technology, or $\kappa_m$ to use importing technology, with $\kappa_m > \kappa_c$.

Face static profit maximization problem each period.

Probability $\delta$ of exit after production each period.
Timing: Plant Entering at date $t$

Pay $\kappa_m$ → $\pi_{mt+1}(z)$ → $\pi_{mt+2}(z)$ → ...

Exit w/p $\delta$

Pay $\kappa_c$ → $\pi_{ct+1}(z)$ → $\pi_{ct+2}(z)$ → ...

Exit w/p $\delta$

Pay $\kappa_e$, Learn $z$ → Exit
Dynamic Problem: Plant entering at date $t$

Present values of static profits:

$$V_{dt}(z) = E_t \sum_{k=1}^{\infty} (1 - \delta)^{k-1} P_{t,t+k} \pi_{dt+k}(z)$$

$$V_{mt}(z) = E_t \sum_{k=1}^{\infty} (1 - \delta)^{k-1} P_{t,t+k} \pi_{mt+k}(z)$$

with $P_{t,t+k} = \beta^k \frac{U_{C t+k}}{U_{C t}}$ (consumer owns plants)
Technology Choice

\[ V_t(z) = \max \left\{ 0, -\kappa_c + V_{dt}(z), -\kappa_m + V_{mt}(z) \right\} \]

Produce using non-importing technology if
\[-\kappa_c + V_{dt}(z) > \max \left\{ 0, -\kappa_m + V_{mt}(z) \right\} \]

Produce using importing technology if
\[-\kappa_m + V_{mt}(z) > \max \left\{ 0, -\kappa_c + V_{dt}(z) \right\} \]

Otherwise exit
Technology Choice

$V_{dt}(z)$ and $V_{mt}(z) - V_{dt}(z)$ increasing in $z$

Cutoffs $\hat{z}_{dt}$ and $\hat{z}_{mt}$,

$$V_{dt}(\hat{z}_{dt}) = \kappa_c$$
$$V_{mt}(\hat{z}_{mt}) - V_{dt}(\hat{z}_{mt}) = \kappa_m$$

Use importing technology if $z \in [\hat{z}_{mt}, \infty)$

Use non-importing technology if $z \in [\hat{z}_{dt}, \hat{z}_{mt})$

Otherwise exit
Technology Choice: cutoffs

\[ g(z) \]

- **Exit**
- **Non-importing**
- **Importing**

\[ z_L \] \[ \hat{z}_d \] \[ \hat{z}_m \]

efficiency, z
Equilibrium Conditions: Plant Dynamics

$\mu_{dt}(z)$: Mass of non-importing plants, efficiency $z$ at date $t$.

$X_t$: Mass of entrants at date $t$ (start producing at date $t + 1$)

Dynamics of distribution:

$$
\mu_{dt+1}(z) = \begin{cases} 
(1 - \delta)\mu_{dt}(z) + X_t g(z) & \text{if } z \in [\hat{z}_{dt}, \hat{z}_{mt}] \\
(1 - \delta)\mu_{dt}(z) & \text{otherwise}
\end{cases}
$$
Equilibrium Conditions: Plant Dynamics

$\mu_{mt}(z)$: Mass of importing plants, efficiency $z$ at date $t$.

$X_t$: Mass of entrants at date $t$ (start producing at date $t+1$)

Dynamics of distribution:

$$\mu_{mt+1}(z) = \begin{cases} (1-\delta)\mu_{mt}(z) + X_t g(z) & \text{if } z > \hat{z}_{mt} \\ (1-\delta)\mu_{mt}(z) & \text{otherwise} \end{cases}$$
Equilibrium Conditions: Feasibility

Goods

\[ C_t + X_t \left( \kappa_e + \kappa_c \int_{2_{dt}}^{z_{mt}} g(z)dz + \kappa_m \int_{z_{mt}}^{\infty} g(z)dz \right) \]

\[ + \int d_{dt}(z)\mu_{dt}(z)dz + \int d_{mt}(z)\mu_{mt}(z)dz + \int m^*_t(z)\mu^*_m(z)dz \]

\[ = \int y_{dt}(z)\mu_{dt}(z)dz + \int y_{mt}(z)\mu_{mt}(z)dz \]

Labor

\[ \int n_{dt}(z)\mu_{dt}(z)dz + \int n_{mt}(z)\mu_{mt}(z)dz = N_t \]
Equilibrium Conditions: Free Entry and Asset Market

Expected value of entry is

\[ V_{et} = -\kappa_e + \int_{z_L}^{\infty} V_t(z)g(z)dz \]

Free Entry:

\[ V_{et} \leq 0, \quad \text{if } X_t > 0 \]

Asset Market Clearing:

\[ B(s^t) + B^*(s^t) = 0 \]
Aggregation

To solve equilibrium conditions, need $\mu_{dt}(\cdot)$, $\mu_{mt}(\cdot)$

For example: $\int n_{dt}(z) \mu_{dt}(z) dz$

Let $Z_{dt} = \int z \mu_{dt}(z) dz$

Plants make decisions proportional to efficiency $z$: $n_{dt}(z) = \tilde{n}_{dt} \times z$

So, $\int n_{dt}(z) \mu_{dt}(z) dz = \tilde{n}_{dt} Z_{dt}$
Aggregation

Replace $\mu_{dt}(\bullet)$ with $Z_{dt}$ as state variable:

$$
\mu_{dt+1}(z) = \begin{cases} 
(1 - \delta) \mu_{dt}(z) + X_t g(z) & \text{if } z \in [\hat{z}_{dt}, \hat{z}_{mt}] \\
(1 - \delta) \mu_{dt}(z) & \text{otherwise}
\end{cases}
$$

\[ 
\downarrow
\]

$$
Z_{dt+1} = (1 - \delta) Z_{dt} + X_t \int_{\hat{z}_{dt}}^{\hat{z}_{mt}} g(z)dz
$$

Same with $\mu_{mt}(\bullet), \mu_{dt}^{*}(\bullet), \mu_{mt}^{*}(\bullet)$
Analysis of Model

1. Aggregate imported / domestic intermediate ratio – what determines substitutability?
   
   Static allocation across plants
   Investment decisions of new plants

2. Quantitative analysis

   Parameterization
   Business Cycle simulation – short-run elasticity
   Trade Reform – long-run elasticity; speed of trade growth
Import / domestic ratio

Plant level:

Non-importing plant: fixed, zero.

Importing plant: fixed, \( \frac{m_t(z)}{d_{mt}(z)} = \frac{1-\omega}{\omega} \)
Import / domestic ratio

Aggregate:

\[
\frac{M_t}{D_{mt} + D_{dt}} = \frac{\tilde{m}_t Z_{mt}}{\tilde{d}_{mt} Z_{mt} + \tilde{d}_{dt} Z_{dt}} = \frac{1 - \omega}{\omega} \frac{\tilde{d}_{mt} Z_{mt}}{\tilde{d}_{mt} Z_{mt} + \tilde{d}_{dt} Z_{dt}}
\]

Increasing in:

\(\tilde{d}_{mt}/\tilde{d}_{dt}\): non-importing / importing plant with same \(z\);

\(Z_{mt}/Z_{dt}\): mass of importers / non-importers (\(z\)-weighted)
Effects of increase in relative price \((1 + \tau) p_t\):

1. At date \(t\): allocation between plants,

\[
\frac{\tilde{d}_{mt}}{\tilde{d}_{dt}} = \left( \frac{\gamma}{\omega + (1 + \tau) p_t (1 - \omega)} \right)^{\alpha/(1-\alpha-\theta)}
\]

Decreasing in \((1 + \tau) p_t\)

Importers less profitable; allocated less inputs in equilibrium
Effects of increase in relative price \((1 + \tau)p_t\) if persistent:

2. At date \(t + 1\): new plants entering at date \(t\),

\[
\frac{Z_{mt+1}}{Z_{dt+1}} = \frac{(1 - \delta)Z_{mt} + X_t \int_{z_{mt}}^{\hat{z}_{mt}} g(z)dz}{(1 - \delta)Z_{dt} + X_t \int_{z_{dt}}^{\hat{z}_{dt}} g(z)dz}
\]

Decreasing in \((1 + \tau)p_t\)

Importing less profitable; fewer new plants choose importing.

\(\hat{z}_{mt} \downarrow, \hat{z}_{dt} \uparrow\)
Dynamic effect of decrease in \((1 + \tau)p_t\)

Distribution of Plants, \(t\)
Dynamic effect of decrease in $(1 + \tau)p_t$

Distribution of Plants, $t$

Diagrams showing the mass of plants against efficiency ($z$) with non-importers and importers indicated.
Dynamic effect of decrease in \((1 + \tau) p_t\)

Distribution of Plants, \(t+1\)
Dynamic effect of decrease in \((1 + \tau)p_t\)

Distribution of Plants, \(t+2\)
Dynamic effect of decrease in \((1 + \tau)p_t\)

Distribution of Plants, \(t+5\)
Dynamic effect of decrease in \((1 + \tau) p_t\)

Distribution of Plants, \(t + 10\)
Dynamic effect of decrease in \((1 + \tau)p_t\)

Distribution of Plants, \(t+20\)
Dynamic effect of decrease in \((1 + \tau)p_t\)

Distribution of Plants, \(t+50\)
Dynamic effect of decrease in $(1 + \tau) p_t$

Distribution of Plants, $t+\infty$
1. Cyclical fluctuations: static reallocation dominant
   Low aggregate elasticity of substitution (~1.3)

2. Trade liberalization: gradual change in ratio of plants
   High aggregate elasticity of substitution (~7)
   Gradual increase in trade

**Conclusions**

Heterogeneity and irreversibility in importing at producer level

Slow-moving dynamics at aggregate level

Significant implications for welfare gains from trade reform
6. Models with uniform fixed cost across firms with heterogeneous productivity have implications that are sharply at odds with micro data. A model with increasing costs of accessing a fraction of a market has many of features of models with fixed costs without these undesirable properties.

Two Key Observations in Trade Data

Key Observation 1: Who exports and how much

(Eaton Kortum and Kramarz '05)

- Most firms do not export **and**
- Large fraction of firms exporting to each country sell tiny amounts there

Example

- Only 1.9% of French firms export to Portugal **and**
- More than 25% of French firms exporting to Portugal < 10K there
Example: 1.9% of French firms export to Portugal, mostly tiny amounts
Two Key Observations in Trade Data

Key Observation 1: Who exports and how much

- Most firms do not export and
- Large fraction of firms exporting to each country sell tiny amounts there

Key Observation 2: Trading decisions after a trade liberalization

(Kehoe ’05, Kehoe & Ruhl ’03)
- Large increases in trade for goods with positive but little trade
Example: Large increases in goods with positive but little trade prior NAFTA

![Bar chart showing the ratio of total imports in 1997-99 to 1990-92 for each category of US imports from Mexico for previously traded goods categorized by sales in 1990-92.](chart.png)
Existing Firm-Level Models of Trade

- Models such as those of Melitz ’03 and Chaney ’06 assume
  - Differentiated products
  - Heterogeneous productivity firms
  - Fixed market access cost of exporting

- Yield 2 puzzles related to 2 key observations
Two Puzzles for Theory with Fixed Costs

- **Puzzle 1: Fixed Cost model needs**
  - Large fixed cost for most firms not to export
  - Small fixed cost for small exporters

- **Puzzle 2: Fixed Cost model relies solely on Dixit-Stiglitz demand**
  - Predicts symmetric changes for all previously positively traded goods

- This paper points out the shortcomings of the Fixed Cost model
  - Proposes a **theory of marketing** that can resolve them
**Example:** TV channel, each ad randomly reaches 50% of consumers

<table>
<thead>
<tr>
<th></th>
<th>1st ad</th>
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<th>3rd ad</th>
</tr>
</thead>
<tbody>
<tr>
<td>fraction reached</td>
<td>50%</td>
<td></td>
<td></td>
</tr>
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<td>cost per consumer</td>
<td>2</td>
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A Theory of Marketing: The Basic Idea

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<tbody>
<tr>
<td><strong>fraction reached</strong></td>
<td>50%</td>
<td>+25%</td>
<td>+12.5%</td>
</tr>
<tr>
<td><strong>cost per consumer</strong></td>
<td>2</td>
<td>4</td>
<td>8</td>
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<td>8</td>
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</tbody>
</table>

**Properties of marketing cost per consumer**

a) Costly to reach first consumer

b) Increasing marketing cost per consumer to reach additional consumers
**A Theory of Marketing: The Basic Idea**

**Example:** TV channel, each ad randomly reaches 50% of consumers

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<td>+25%</td>
<td>+12.5%</td>
</tr>
<tr>
<td>cost per consumer</td>
<td>2</td>
<td>4</td>
<td>8</td>
</tr>
</tbody>
</table>

**Properties of marketing cost per consumer**

a) Costly to reach first consumer

b) Increasing marketing cost per consumer to reach additional consumers

*Model with a)+b) can account for observation 1, namely,*

- Most firms do not export **and**
- Large fraction of firms exporting to each country sell tiny amounts there
A Theory of Marketing: The Basic Idea

Example: TV channel, each ad randomly reaches 50% of consumers

<table>
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<tr>
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Properties of marketing cost per consumer

a) Costly to reach first consumer

b) Increasing marketing cost per consumer to reach additional consumers

c) More ads bring fewer new consumers (saturation)
A Theory of Marketing: The Basic Idea

Example: TV channel, each ad randomly reaches 50% of consumers

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Properties of marketing cost per consumer

a) Costly to reach first consumer

b) Increasing marketing cost per consumer to reach additional consumers

c) More ads bring fewer new consumers (saturation)

Model with c) can account for observation 2, namely,

- Large increases in trade for goods with positive but little trade
Model Environment

Builds on Melitz ’03 and Chaney ’06

- **Countries**
  - Index by $i$ when exporting, $j$ when importing, $i,j = 1,...,N$
  - $L_j$ consumers
  - Firms sell locally and/or export
Model Environment

Builds on Melitz ’03 and Chaney ’06

- **Representative Consumers**
  - Sell unit of labor, own shares of domestic firms
  - Symmetric CES Dixit-Stiglitz preferences over continuum of goods
  - Buy the goods they have access to

- **Firms**
  - Indexed by productivity $\phi$ (drawn from same distribution), nationality $i$
  - Each sells 1 good
  - Determine probability a consumer in a market has access to their good
Demand Faced by a Type $\phi$ Firm from Country $i$

- $n_{ij}(\phi)$: probability a type $\phi$ firm from $i$ reaches a consumer in $j$

- Large number of consumers
  - thus firm reaches fraction $n_{ij}(\phi)$ of them

- Effective demand for firm $\phi$:
  \[
  \underbrace{n_{ij}(\phi)L_j}_{\text{consumers that firm reaches}} \quad \underbrace{\frac{p_{ij}(\phi)^{-\sigma}}{P_j^{1-\sigma}}y_j}_{\text{D-S demand per consumer}}
  \]

$p_{ij}(\phi)$: price that type $\phi$ firm from $i$ charges in $j$, $y_j$: output (income) per capita

$P_j$: D-S price aggregator, $\sigma$: elasticity of substitution ($\sigma > 1$, demand is elastic)
Firm’s Problem

Type $\phi$ firm from country $i$ solves for each country $j = 1, ..., N$

$$\pi_{ij} = \max_{n_{ij}, p_{ij}, q_{ij}} p_{ij} q_{ij} - w_i \frac{\tau_{ij} q_{ij}}{\phi} - w_i f(n_{ij}, L_j)$$

s.t. \[ q_{ij} = n_{ij} L_j \frac{p_{ij}^{\sigma}}{P_j^{1-\sigma}} y_j, \quad n_{ij} \in [0, 1] \]

- Uses production function $q_{ij} = \phi l_{ij}$ to produce good
- $\tau_{ij}$: iceberg cost to ship a unit of good from $i$ to $j$ (in terms of labor)
- $f(n_{ij}, L_j)$: marketing to reach fraction $n_{ij}$ of a population with size $L_j$
Firm’s Problem

- **Result:** Price is the usual markup over unit production cost,

\[ p_{ij}(\phi) = \tilde{\sigma} \frac{\tau_{ij} w_j}{\phi}, \quad \tilde{\sigma} = \frac{\sigma}{\sigma - 1} \]

- Given price markup rule firm solves:

\[
\pi_{ij} = \max_{n_{ij}} n_{ij} L_j \phi^{\sigma - 1} \left( \frac{\tau_{ij} w_j \tilde{\sigma}}{\sigma} \right)^{1 - \sigma} \frac{y_j}{P_j^{1-\sigma}} - w_j f(n_{ij}, L_j)
\]

\[
\text{Revenue per consumer (net of labor production cost)}
\]

s.t

\[ n_{ij} \in [0, 1] \]

- Look at marginal decision of reaching additional fractions of consumers
Marginal Revenue & Cost from Reaching Additional Consumers

Marginal cost

$\phi$

MR of access for productivity $\phi$

Constant marginal cost

Increasing marginal cost

$f_1(0, L)$

Fraction of consumers reached

$n = 1$
The Market Access Cost Function

• Solve the differential equation

\[ n'(S) = [1 - n(S)]^\beta L^{1-\alpha} \frac{1}{L}, \quad \text{s.t. } n(0) = 0 \]

• Obtain Market Access Cost function
  
  • Assuming that \( \frac{1}{\psi} \) is the labor required for each ad

\[
  f(n, L) = \begin{cases} 
  \frac{L^\alpha (1-(1-n)^{-\beta+1})}{\psi (-\beta+1)} & \text{if } \beta \in [0,1) \cup (1, +\infty) \\
  -\frac{L^\alpha}{\psi} \log(1 - n) & \text{if } \beta = 1 
  \end{cases}
\]

where \( \alpha \in [0, 1] \)
The properties of the Market Access Cost function

\[ \frac{L^a}{\psi} = 1 \]

\[ \beta = 1 \]

\[ \beta = 0 \]

Fraction of consumers reached

\( n = 1 \)
The properties of the Market Access Cost function

\[ \frac{L^\alpha}{\psi} \]

\[ \beta = 1 \]

\[ \beta = 0 \]

Fraction of consumers reached

Accessing 1\textsuperscript{st} fraction of consumers costly

Costs Arkolakis: Market Access Costs & the New Consumers Margin
The properties of the Market Access Cost function

Accessing 1st fraction of consumers costly, but accessing 1st consumer cheaper for larger L (if \(\alpha \ll 1\))

Cost as Arkolakis: Market Access Costs & the New Consumers Margin
The product of the two margins: total sales per firm

Sales per firm

Productivity

(Fixed cost)

(Endogenous cost)

$\phi_{ij}^*$
Models’ predictions on which firms export

Sales per firm

Productivity

\( \phi_{ij} \)

Right prediction: Some firms don’t export

(Fixed cost)

(Endogenous cost)
Models’ predictions on how much firms export

Wrong prediction: Minimum exports to cover fixed cost

- (Fixed cost)
- (Endogenous cost)

Sales per firm

Productivity

\( \phi_{ij} \)
Models’ predictions on how much firms export

Sales per firm

Right prediction:
Export tiny amounts (few consumers)

Productivity

(Fixed cost)

(Endogenous cost)

\( \phi_{ij}^* \)
Comparing the Calibrated Model to French Data

- Look at the sales distribution for the model with $\beta = 0, 1$

- Remember: $\beta = 1$ calibrated to match higher sales in France of French firms exporting to more countries

- $\frac{1}{\psi}, \alpha$ calibrated to match number of French exporters to each country
Calibrated Endogenous Cost model accounts for large fraction of small exporters

Costs Arkolakis: Market Access Costs & the New Consumers Margin
Observation 2: Trading Decisions After Trade Liberalization

• **Data:** Large increases in trade in least traded goods, Kehoe & Ruhl ’03

• Look at US-Mexico trade liberalization; extend Kehoe-Ruhl analysis

• **Compute growth of positively traded goods prior to NAFTA**
  1. Data: US imports from Mexico ’90-’99, 6-digit HS, ≈ 5400 goods
  2. Keep goods traded throughout ’90-’92, ≈ 2900 goods
  3. Rank goods in terms of sales ’90-’92
  4. Categorize *traded* goods in 10 bins
Large increases in trade for least traded goods

Costas Arkolakis: Market Access Costs & the New Consumers Margin
Comparing Calibrated Model to Data from NAFTA Episode

- Look at growth of trade for previously traded goods for $\beta = 0, 1$

- Use calibrated parameters, consider a firm as a good

- Change variable trade costs symmetrically across goods
  - Match increase in trade in previously traded goods
    - Fixed Cost model: 12.5% decrease in variable trade costs
    - My model: 9.5% decrease in variable trade costs (e.g. $\tau'_{ij} = 0.905 \tau_{ij}$)
Calibrated Endogenous Cost model predicts increases in trade for least traded goods

US imports from Mexico for previously traded goods categorized by sales in 1990-92

Endogenous cost ($\beta=1$)

Fixed cost ($\beta=0$)
New Consumers Margin and New Trade

- Recent theory emphasizes increase in trade due to many new firms (EK02, Chaney ’06 à la Melitz ’03)

- Decompose contribution of the 3 margins to total trade
  - Intensive margin growth (total growth in sales per consumer)
  - New consumers margin (total growth in extensive margin of consumers)
  - New firms margin (total growth in extensive margin of firms)
Pareto Density and Number of Firms with Productivity $\phi$

Firms with productivity $\phi$ ($\mu(\phi)$)

$\phi^*$

Productivity
Density of exports

Exports of firms with productivity $\psi(n(\phi)x(\phi)\mu(\phi))$ vs Productivity
New Consumers Margin and new trade

Intensive margin growth

Exports of firms with productivity $\varphi (n(\varphi) x(\varphi) \mu(\varphi))$

Productivity
New Consumers Margin and new trade

Exports of firms with productivity $\varphi(n(\varphi)x(\varphi)\mu(\varphi))$

New consumers margin

Intensive margin growth

Productivity
New Consumers Margin and new trade

Exits of firms with productivity $\psi(n(\phi)x(\phi)\mu(\phi))$

New consumers margin

Intensive margin growth

New firms margin
New Consumers Margin and new trade

New consumers margin (33.3%)

Intensive margin growth (52%)

New firms margin (14.7%)

Exports of firms with productivity $\varphi$ ($n(\varphi) = \varphi \mu(\varphi)$)
New Firms Margin and the Fixed Cost model ($\beta = 0$)
New Firms Margin and new trade ($\beta = 0$)

- New firms margin (48%)
- Intensive margin growth (52%)
7. Growth theory needs to be reconsidered in the light of trade theory. In particular, a growth model that includes trade can have the opposite convergence properties from a model of closed economies.

Trade and Growth

In 2004 Mexico has income per capita of 6500 U.S. dollars. In 1935 the United States had income per capita of about 6600 U.S. dollars (real 2004 U.S. dollars).

To study what will happen in Mexico over the next 70 years, should we study what happened to the United States since 1935?

…or should we take into account that the United States was the country with the highest income in the world in 1935, while Mexico has a very large trade relation with the United States — a country with a level of income per capita approximately 6 times larger in 2004?

We study this question using the Heckscher-Ohlin model of international trade: Countries differ in their initial endowments of capital per worker.
The General Dynamic Heckscher-Ohlin Model

$n$ countries
countries differ in initial capital-labor ratios $\bar{k}_0^i$
and in size of population $L^i$.

two traded goods — a capital intensive good and a labor intensive good

\[ y_j = \phi_j(k_j, \ell_j) \]
\[ \frac{\phi_{1L}(k/\ell, 1)}{\phi_{1K}(k/\ell, 1)} < \frac{\phi_{2L}(k/\ell, 1)}{\phi_{2K}(k/\ell, 1)} \]

nontraded investment good

\[ x = f(x_1, x_2) \]

Feasibility:
\[ \sum_{i=1}^{n} L^i (c^i_{jt} + x^i_{jt}) = \sum_{i=1}^{n} L^i y^i_{jt} = \sum_{i=1}^{n} L^i \phi_j (k^i_{jt}, \ell^i_{jt}). \]

\[ k^i_{1t} + k^i_{2t} = k^i_t \]

\[ \ell^i_{1t} + \ell^i_{2t} = 1 \]

\[ k^i_{t+1} - (1 - \delta)k^i_t = x^i_t = f(x^i_{1t}, x^i_{2t}) \]
Infinitely-Lived Consumers

consumer in country $i$, $i = 1, \ldots, n$:

$$\max \sum_{t=0}^{\infty} \beta^t u(c_{1t}^i, c_{2t}^i)$$

s.t. $p_{1t} c_{1t}^i + p_{2t} c_{2t}^i + q_t^i x_t^i + b_{t+1}^i = w_t^i + (1 + r_t^{bi}) b_t^i + r_t^i k_t^i$

$$k_{t+1}^i - (1 - \delta) k_t^i = x_t^i$$

$$c_{jt}^i \geq 0, \quad x_t^i \geq 0, \quad b_t^i \geq -B$$

$$k_0^i = k_{-1}^i, \quad b_0^i = 0.$$

Notice that since $p_{1t}$ and $p_{2t}$ are equalized across countries by trade, we can set

$$q_t^i = q_t = 1.$$

The factor prices $w_t^i$ and $r_t^i$ are potentially different across countries. International borrowing and lending:
\[ \sum_{i=1}^{n} L^i b_t^i = 0, \]

No international borrowing and lending:

\[ b_t^i = 0. \]

International borrowing and lending implies that \( r_t^{bi} = r_t^b, t = 1, 2, \ldots \). No arbitrage implies that \( r_t^i = r_t = r_t^b + \delta \).
Characterization and computation of equilibrium is relatively easy when we can solve for equilibrium of an artificial world economy in which we ignore restrictions on factor mobility and then disaggregate the consumption, production, and investment decisions.

This is a guess-and-verify approach: We first solve for the integrated equilibrium of the world economy and then we see if we can disaggregate the consumption, production, and investment decisions.

Potential problem: We cannot assign each country nonnegative production plans for each of the two goods while maintaining factor prices equal to those in the world equilibrium.

Another potential problem: We cannot assign each country nonnegative investment.
If the integrated equilibrium approach does not work, it could be very difficult to calculate an equilibrium.

We would have to determine the pattern of specialization over an infinite time horizon.
$p_1 \phi_1(k, \ell) = 1$

$k_1 / \ell_1$

$p_2 \phi_2(k, \ell) = 1$

$rk + w\ell = 1$
\[ p_1 \phi_1(k, \ell) = 1 \]

\[ k_1 / \ell_1 \]

\[ (1, k^i) \]

\[ k_2 / \ell_2 \]

\[ p_2 \phi_2(k, \ell) = 1 \]

\[ rk + w\ell = 1 \]
The diagram shows a graph with axes labeled $k$ and $\ell$. The equations $p_1 \phi_1(k, \ell) = 1$, $p_2 \phi_2(k, \ell) = 1$, and $rk + w\ell = 1$ are indicated on the graph. Points and lines are marked to illustrate the relationships between these equations.
Results for General Model

International borrowing and lending implies factor price equalization in period $t = 1, 2, ...$ Production plans and international trade patterns are indeterminate.

Any steady state or sustained growth path has factor price equalization.

If there exists a steady state in which the total capital stock is positive or a sustained growth path, then there exists a continuum of such steady states or sustained growth paths, indexed by the distribution of world capital $\hat{k}_1 / \hat{k}, ..., \hat{k}_n / \hat{k}$.

International trade occurs in every steady state or sustained growth path of the model in which $\hat{k}_i / \hat{k} \neq 1$ for some $i$.

We focus on models with no international borrowing and lending.
For analysis of general model with infinitely lived consumers and comparison with model with overlapping generations, see

Ventura Model

\[ u(c_1,c_2) = v(f(c_1,c_2)) = \log(f(c_1,c_2)) \]

\[ \phi_1(k_1,\ell_1) = k_1 \]

\[ \phi_2(k_2,\ell_2) = \ell_2 \]

\[ f(x_1,x_2) = \begin{cases} 
  d \left( a_1 x_1^b + a_2 x_2^b \right)^{1/b} & \text{if } b \neq 0 \\
  d x_1^{a_1} x_2^{a_2} & \text{if } b = 0 
\end{cases} \]

Ventura (1997) examines the continuous-time version of this model.
In the Ventura model, we can solve for the equilibrium of the world economy by solving a one-sector growth model in which $c_t = f(c_{1t},c_{2t})$:

$$\max \sum_{t=0}^{\infty} \beta^t \log c_t$$

s.t. $c_t + x_t = f(k_t,1)$

$k_{t+1} = (1-\delta)k_t = x_t$

$c_t \geq 0$, $k_t \geq 0$

$k_0 = \bar{k}_0$.

If $b < 0$ and $1/\beta - 1 + \delta > da_1^{1/b}$, the equilibrium converges to $\hat{k} = 0$.

If $b > 0$ and $1/\beta - 1 + \delta < da_1^{1/b}$, the economy grows without bound, and the equilibrium converges to a sustained growth path.

In every other case, the equilibrium converges to a steady state in which $f_K(\hat{k},1) = 1/\beta - 1 + \delta$. 
The 2 sectors matter a lot for disaggregating the integrated equilibrium!

In particular, we cannot solve for the equilibrium values of the variables for one of the countries by solving an optimal growth problem for that country in isolation.

Instead, the equilibrium path for $k_t^i$ and the steady state value of $\hat{k}^i$ depends on $\overline{k}_0^i$ as well as on the path for $k_t$ and the steady state value of $\hat{k}$. 
**Proposition:** Let \( y^i_t = p_{1t}y^i_{1t} + p_{2t}y^i_{2t} = r^i_t k^i_t + w_t \). Suppose that \( x^i_t > 0 \) for all \( i \) and all \( t \). Then

\[
\frac{y^i_{t+1} - y^i_{t+1}}{y^i_{t+1}} = \frac{r^i_{t+1} c^i_t / y^i_{t+1}}{r^i_t c^i_{t-1} / y^i_t} \left( \frac{y^i_t - y^i_t}{y^i_t} \right)
\]

If \( \delta = 1 \),

\[
\frac{y^i_{t+1} - y^i_{t+1}}{y^i_{t+1}} = \frac{s^i_{t+1}}{s^i_t} \left( \frac{y^i_t - y^i_t}{y^i_t} \right)
\]

where \( s^i_t = c^i_t / y^i_t \).
Proof: The first-order conditions from the consumers’ problems are
\[
\frac{c_t^i}{c_{t-1}^i} = \frac{c_t}{c_{t-1}} = \beta(1 + r_t - \delta).
\]
The demand functions are
\[
c_t^i = (1 - \beta) \left( \sum_{s=t}^{\infty} \left( \prod_{\tau=t+1}^{s} \frac{1}{1 + r_{\tau} - \delta} \right) w_s + (1 + r_t - \delta) k_t^i \right)
\]
\[
c_t^i - c_t = (1 - \beta)(1 + r_t - \delta)(k_t^i - k_t).
\]
The budget constraint implies that
\[
c_t^i - c_t + k_t^{i+1} - k_t^{t+1} = (1 + r_t - \delta)(k_t^i - k_t).
\]
Combining these conditions, we obtain
\[
k_t^{i+1} - k_t^{t+1} = \frac{c_t}{c_{t-1}}(k_t^i - k_t).
\]
The difference between a country's income per worker and the world's income per worker can be written as

\[ y_{t+1}^i - y_{t+1} = r_{t+1} (k_{t+1}^i - k_{t+1}). \]

Using the expression for \( k_{t+1}^i - k_{t+1} \) found above and operating, we obtain:

\[ \frac{y_{t+1}^i - y_{t+1}}{y_{t+1}} = \frac{r_{t+1} c_t}{y_{t+1}} \left( \frac{y_t^i - y_t}{y_t} \right). \]

In the case \( \delta = 1 \) this becomes (using \( c_{t+1} / c_t = \beta r_{t+1} \)),

\[ \frac{y_{t+1}^i - y_{t+1}}{y_{t+1}} = \frac{s_{t+1}}{s_t} \left( \frac{y_t^i - y_t}{y_t} \right), \]

where \( s_t = c_t / y_t \). ■
\[
\frac{y_t^i - y_t}{y_t} = s_t \left( \frac{y_0^i - y_0}{y_0} \right)
\]

**Proposition.** Suppose that \( \delta = 1 \), that \( 0 < k_0 < \hat{k} \), and that \( x_t^i > 0 \) for all \( i \) and all \( t \). Then

if \( b > 0 \), differences in relative income levels decrease over time;

if \( b = 0 \), differences in relative income levels stay constant over time;

and

if \( b < 0 \), differences in relative income levels increase over time.
\[
\frac{y_t^i - y_t}{y_t} = s_t \left( \frac{y_0^i - y_0}{y_0} \right)
\]

**Proposition.** Suppose that $\delta = 1$, that $0 < k_0 < \hat{k}$, and that $x_t^i > 0$ for all $i$ and all $t$. Then

- if $b > 0$, differences in relative income levels decrease over time;
- if $b = 0$, differences in relative income levels stay constant over time;
- and
- if $b < 0$, differences in relative income levels increase over time.

**Notice contrast with convergence results for world of closed economies!**
What about corner solutions in investment?

If \( x_t^i > 0 \) for all \( i \) and all \( t \), then

\[
\frac{k_{t+1}^i - k_t^i}{k_{t+1}^i} = \frac{c_t}{k_{t+1}} \left( \frac{k_t^i - k_t^i}{k_t^i} \right) = \frac{z_{t+1}}{z_t} \left( \frac{k_t^i - k_t^i}{k_t^i} \right)
\]

where \( z_t = c_{t-1} / k_t \) and \( z_0 = c_0 / (\beta r_0 k_0) \).

The sequence \( z_t \) has the same monotonicity properties as the sequence \( s_t = c_t / y_t \).
Proposition: Suppose that the sequence $s_t = c_t / y_t$ in the equilibrium of the integrated economy is constant or strictly decreasing. There exists an equilibrium where $x_i^t > 0$ for all $i$ and all $t$. 
**Proposition:** Suppose that the sequence \( s_t = c_t / y_t \) in the equilibrium of the integrated economy is strictly increasing. Let

\[
\hat{z} = \lim_{t \to \infty} \frac{c_{t-1}}{k_t},
\]

and let \( \bar{k}_0 \leq \bar{k}_0^i, i = 1,...,n \). If

\[
\frac{\hat{z}}{z_0} \left( \frac{\bar{k}_0^{i_{\text{min}}} - \bar{k}_0}{\bar{k}_0} \right) \geq -1,
\]

then there exists an equilibrium where \( x_t^i > 0 \) for all \( i \) and all \( t \).

Otherwise, there is no equilibrium where \( x_t^i > 0 \) for all \( i \) and all \( t \). When there exists an equilibrium with no corner solutions in investment, it is the unique such equilibrium.
Numerical example 1: Two countries. \( \beta = 0.95, \delta = 1, \) and \( L^1 = L^2 = 10. \)

\[
f(x_1, x_2) = 10 \left( 0.5x_1^{-0.5} + 0.5x_2^{-0.5} \right)^{-2}.
\]

We contrast two different worlds:

In the first world, \( \bar{k}_0^1 = 5 \) and \( \bar{k}_0^2 = 3. \) Here there is an equilibrium with no corner solutions for investment.

In the second world, \( \bar{k}_0^1 = 6 \) and \( \bar{k}_0^2 = 2. \) Country 2 has \( x_t^i = k_t^i = 0 \) starting in period 3.
Example 1: Capital-labor ratios

\[ k_t^1 \quad \left( \bar{k}_0^1 = 6, \bar{k}_0^2 = 2 \right) \]

\[ k_t^1 \quad \left( \bar{k}_0^1 = 5, \bar{k}_0^2 = 3 \right) \]

\[ k_t^2 \quad \left( \bar{k}_0^1 = 5, \bar{k}_0^2 = 3 \right) \]

\[ k_t^2 \quad \left( \bar{k}_0^1 = 6, \bar{k}_0^2 = 2 \right) \]
Example 1: Relative income in country 1

\[ \bar{k}_0^1 = 6, \quad \bar{k}_0^2 = 2 \]

\[ \bar{k}_0^1 = 5, \quad \bar{k}_0^2 = 3 \]
Generalized Ventura Model

\[ u(c_1, c_2) = v(f(c_1, c_2)) = \log(f(c_1, c_2)), \text{ and } f, \phi_1, \text{ and } \phi_2 \text{ are general constant-elasticity-of-substitution functions} \]

Define

\[ F(k, \ell) = \max f(y_1, y_2) \]

\[ \text{s.t. } y_1 = \phi_1(k_1, \ell_1) \]
\[ y_2 = \phi_2(k_2, \ell_2) \]
\[ k_1 + k_2 = k \]
\[ \ell_1 + \ell_2 = \ell \]
\[ k_j \geq 0, \ell_j \geq 0. \]

In Ventura model \( F(k, \ell) = f(k, \ell) \).
C. E. S. Model

\[ y_1 = \phi_1(k_1, \ell_1) = \theta_1 \left( \alpha_1 k_1^b + (1 - \alpha_1) \ell_1^b \right)^{1/b} \]

\[ y_2 = \phi_2(k_2, \ell_2) = \theta_2 \left( \alpha_2 k_2^b + (1 - \alpha_2) \ell_2^b \right)^{1/b} \]

\[ f(y_1, y_2) = d \left( a_1 y_1^b + a_2 y_2^b \right)^{1/b} \]

(All elasticities of substitution are equal.)
In this case,

\[
F(k, \ell) = D \left( A_1 k^b + A_2 \ell^b \right)^{1/b}
\]

where

\[
A_1 = \frac{\left[ \left( a_1 \alpha_1 \theta_1^b \right)^{1/b} + \left( a_2 \alpha_2 \theta_2^b \right)^{1/b} \right]^{1-b}}{\left[ \left( a_1 \alpha_1 \theta_1^b \right)^{1/b} + \left( a_2 \alpha_2 \theta_2^b \right)^{1/b} \right]^{1-b} + \left[ \left( a_1 (1-\alpha_1) \theta_1^b \right)^{1/b} + \left( a_2 (1-\alpha_2) \theta_2^b \right)^{1/b} \right]^{1-b}}
\]

\[
A_2 = 1 - A_1
\]

\[
D = d \left\{ \left[ \left( a_1 \alpha_1 \theta_1^b \right)^{1/b} + \left( a_2 \alpha_2 \theta_2^b \right)^{1/b} \right]^{1-b} + \left[ \left( a_1 (1-\alpha_1) \theta_1^b \right)^{1/b} + \left( a_2 (1-\alpha_2) \theta_2^b \right)^{1/b} \right]^{1-b} \right\}^{1/b}
\].
The cone of diversification for the integrated economy has the form
\( \kappa_i k_t \geq k^i_t \geq \kappa_2 k_t \).

\[
\kappa_i = \left( \frac{\alpha_i}{1 - \alpha_i} \right)^{\frac{1}{1-b}} \left( \frac{a_1 (1 - \alpha_1) \theta_1^b}{\theta_1^b} \right)^{\frac{1}{1-b}} + \left( \frac{a_2 (1 - \alpha_2) \theta_2^b}{\theta_2^b} \right)^{\frac{1}{1-b}}.
\]
The cone of diversification for the integrated economy has the form 
\( \bar{\kappa}_i k_i \geq k_i^i \geq \bar{\kappa}_2 k_i \).

\[
\bar{\kappa}_i = \left( \frac{\alpha_i}{1 - \alpha_i} \right)^{1-b} \left( a_1 (1 - \alpha_1) \theta_1^b \right)^{1-b} + \left( \frac{a_2 (1 - \alpha_2) \theta_2^b}{\alpha_1 \alpha_1 \theta_1^b + a_2 \alpha_2 \theta_2^b} \right)^{1-b}.
\]

This is not the cone of diversification when factor prices are not equalized.

\[
\kappa_1(p_2 / p_1) = \left( \frac{\alpha_1}{1 - \alpha_1} \right)^{1-b} \left[ \frac{1}{\alpha_1^{1-b} \theta_1^b} - \frac{1}{\alpha_2^{1-b} (\theta_2 p_2 / p_1)^{1-b}} \right]^{b}
\]

\[
\kappa_1(p_2 / p_1) = \left[ \left( \frac{\alpha_2}{1 - \alpha_2} \right) \left( \frac{1 - \alpha_1}{\alpha_1} \right) \right]^{1-b} \kappa_2(p_2 / p_1).
\]
Cobb-Douglas Model

\[ y_1 = \phi_1 (k_1, \ell_1) = \theta_1 k_1^\alpha_1 \ell_1^{1-\alpha_1} \]

\[ y_2 = \phi_2 (k_2, l_2) = \theta_2 k_2^\alpha_2 l_2^{1-\alpha_2} \]

\[ f (y_1, y_2) = d y_1^{\alpha_1} y_2^{\alpha_2} \]

(This is the special case of the C.E.S. model where \( b = 0 \).)
In this case

\[ F(k, \ell) = Dk^{A_1} \ell^{A_2} \]

where

\[ A_1 = a_1 \alpha_1 + a_2 \alpha_2 \]

\[ A_2 = 1 - A_1 \]

\[ D = \frac{d\left[ \theta_1 a_1 \alpha_1^{\alpha_1} (1 - \alpha_1)^{1-\alpha_1} \right]^{a_1}}{A_1^{A_1} A_2^{A_2}} \frac{d\left[ \theta_2 a_2 \alpha_2^{\alpha_2} (1 - \alpha_2)^{1-\alpha_2} \right]^{a_2}}{A_1^{A_1} A_2^{A_2}} \]

\[ \bar{\kappa}_i = \left( \frac{\alpha_i}{1 - \alpha_i} \right) \frac{A_2}{A_1}. \]
**Proposition:** In the Cobb-Douglas model with $\delta = 1$, suppose that factor price equalization occurs at period $T$. Then factor price equalization occurs at all $t \geq T$. Furthermore, the equilibrium capital stocks can be solved for as

\[ k_t^i = \gamma^i k_t \]

where $\gamma^i = k_T^i / k_T$ and $k_{t+1} = \beta A_1 D k_t^{A_1}$ for $t \geq T$. 
Proposition: In the C.E.S. model with $\delta = 1$, suppose that the sequence $s_t = c_t / y_t$ in the equilibrium of the integrated economy is weakly decreasing. Suppose that factor price equalization occurs in period $T$. Then there exists an equilibrium in which factor price equalization occurs at all $t \geq T$. Furthermore, this equilibrium is the only such equilibrium.
**Proposition:** In the C.E.S. model with $\delta = 1$, suppose that the sequence $s_t = c_t / y_t$ in the equilibrium of the integrated economy is strictly increasing. Again let $z_t = c_{t-1} / k_t$, $z_0 = c_0 / (\beta r_0 k_0)$, and $\hat{z} = \lim_{t \to \infty} c_{t-1} / k_t$.

Let $k_0^{i_{min}} \leq k_0^{i} \leq k_0^{i_{max}}, i = 1,...,n$. If

$$\frac{\hat{z}}{z_0} \left( \frac{k_0^{i_{min}} - k_0}{k_0} \right) \geq \kappa_2 - 1, \quad \frac{\hat{z}}{z_0} \left( \frac{k_0^{i_{max}} - k_0}{k_0} \right) \leq \kappa_1 - 1,$$

then there exists an equilibrium with factor price equalization in every period. If, however, either of these conditions is violated, there is no equilibrium with factor price equalization in every period. When there exists an equilibrium with factor price equalization in every period, it is the unique such equilibrium.
Numerical example 2: Two countries. $\beta = 0.95$, $\delta = 1$, and $L^1 = L^2 = 10$.

$$\phi_1(k, \ell) = 10k^{0.6}\ell^{0.4}$$

$$\phi_2(k, \ell) = 10k^{0.4}\ell^{0.6}$$

$$f(x_1, x_2) = x_1^{0.5}x_2^{0.5}$$

$\bar{k}_0^1 = 4, \bar{k}_0^2 = 0.1$. 
Example 2: Capital-labor ratios

\[ \kappa_1 \left( \frac{p_{2t}}{p_{1t}} \right) \]

\[ \kappa_2 \left( \frac{p_{2t}}{p_{1t}} \right) \]

\[ \bar{\kappa}_1 k_t \]

\[ \bar{\kappa}_2 k_t \]

\[ k^1_t \]

\[ k^2_t \]
Example 2: Relative income in country 1
Numerical example 3: Two countries. $\beta = 0.95$, $\delta = 1$, and $L^1 = L^2 = 10$.

\[
\begin{align*}
\phi_1(k, \ell) &= 10 \left( 0.8k^{-0.5} + 0.2\ell^{-0.5} \right)^2 \\
\phi_2(k, \ell) &= 10 \left( 0.2k^{-0.5} + 0.8\ell^{-0.5} \right)^2 \\
f(x_1, x_2) &= \left( 0.5x_1^{-0.5} + 0.5x_2^{-0.5} \right)^2 \\
\bar{k}_0^1 &= 5, \bar{k}_0^2 = 2.
\end{align*}
\]

Contrast with the Ventura model with the same integrated equilibrium:

\[
f(x_1, x_2) = 5.7328 \left( 0.5x_1^{-0.5} + 0.5x_2^{-0.5} \right)^2.
\]
Example 3: Capital labor ratios

\begin{align*}
\kappa_1(k_{2t}/k_{1t}) \\
k^1_t \text{ (Ventura model)} \\
k^1_t \text{ (C.E.S. model)} \\
k^2_t \text{ (C.E.S. model)} \\
k^2_t \text{ (Ventura model)}
\end{align*}
Example 3: Capital labor ratios (detail)

\[ k_t^1 \]

\[ \kappa_2 \left( \frac{p_{2t}}{p_{1t}} \right) \]

\[ k_t^2 \quad \text{(C.E.S. model)} \]

\[ k_t^2 \quad \text{(Ventura model)} \]
Example 3: Relative income in country 1

![Graph showing deviation from average income over periods for Ventura model and C.E.S. model.](image-url)
8. Favorable changes in the terms of trade and/or reductions in tariffs make it easier to import intermediate goods. Although this is often interpreted as an increase in productivity, it does not show up as such in productivity measures that use real GDP as a measure of output.

A deterioration in the terms of trade makes it expensive for an economy to import intermediate goods.

We can think of international trade as part of the production technology. Exports are inputs, imports are outputs. A deterioration in the terms of trade corresponds to a negative technology shock.

Can this negative “technology shock” account for the drop in TFP during the crisis?
A deterioration in the terms of trade makes it expensive for an economy to import intermediate goods.

We can think of international trade as part of the production technology. Exports are inputs, imports are outputs. A deterioration in the terms of trade corresponds to a negative technology shock.

Can this negative “technology shock” account for the drop in TFP during the crisis?

No!

Standard national income accounting (SNA, NIPA) implies that terms of trade shocks have no first-order effects on real output (GDP, GNP)
A simple model with intermediate goods

Labor
\[ \ell_t = \bar{\ell} \]

Final good
\[ y_t = f(\bar{\ell}, m_t) \]

Intermediate good
\[ m_t = \frac{x_t}{a_t} \]

Feasibility
\[ c_t + x_t = y_t \]

Real GDP
\[ c_t = y_t - x_t = f(\bar{\ell}, m_t) - a_t m_t \]
Competitive economy solves

$$\max_{m_t} f(\ell, m_t) - a m_t$$

$$f_m(\ell, m(a_t)) \equiv a_t$$

$$f_{mm}(\ell, m(a_t)) m'(a_t) = 1$$

$$m'(a_t) = \frac{1}{f_{mm}(\ell, m(a_t))} < 0$$

How does real GDP change with an increase in $a$ — a negative shock to the intermediate goods producing technology?

$$Y(a_t) \equiv f(\ell, m(a_t)) - a_t m(a_t)$$

$$Y'(a_t) = f_m(\ell, m(a_t)) m'(a_t) - a_t m'(a_t) - m(a_t) = -m(a_t) < 0$$
A model with international trade

Suppose now that

\( m \) is imported intermediate inputs,
\( x \) is exports,
\( p = a \) is terms of trade (real exchange rate)

Balanced trade

\[ p_t m_t = x_t \]

Real GDP

\[ c_t + x_t - p_0 m_t = y_t - p_0 m_t = f(\bar{l}, m_t) - p_0 m_t \]

where \( p_0 \) is price of imports in the base year.
Competitive economy continues to solve

\[
\max_{m_t} f(\bar{\ell}, m_t) - p_t m_t
\]

\[
f_m(\bar{\ell}, m(p_t)) \equiv p_t
\]

\[
m'(p_t) = \frac{1}{f_{mm}(\bar{\ell}, m(p_t))} < 0
\]

How does real GDP change with an increase in \( p_t \) — a deterioration in the terms of trade (depreciation in the real exchange rate)?

\[
Y(p_t) \equiv f(\bar{\ell}, m(p_t)) - p_0 m(p_t)
\]

\[
Y'(p_t) = f_m(\bar{\ell}, m(p_t)) m'(p_t) - p_0 m'(p_t) = (p_t - p_0) m'(p_t)
\]

\[
p_0 \approx p_t \Rightarrow Y'(p_t) \approx 0
\]
Alternative accounting concepts

- Diewert and Morrison (1974, 1986)
- U.S. Bureau of Economic Analysis (Command Basis GDP)
- United Nations Statistics Division (Gross National Income)
- GNP, GDP (SNA, NIPA) do not.
Alternative accounting concepts

- Diewert and Morrison (1974, 1986)
- U.S. Bureau of Economic Analysis (Command Basis GDP)
- United Nations Statistics Division (Gross National Income)
- GNP, GDP (SNA, NIPA) do not.

Terms of trade shocks are worse than you think!
Extensions

Chain weighted price indices

Changes in tariffs

Endogenous labor
9. In models with heterogeneous firms (for example, Melitz, Chaney), trade liberalization can cause resources to shift from less productive firms to more productive firms. Although this is often interpreted as an increase in productivity, it does not show up as such in productivity measures that use real GDP as a measure of output.

Some countries experience aggregate productivity increases following trade liberalization

What is the economic mechanism through which this occurs?

Does trade liberalization increase aggregate productivity through reallocation toward more productive firms or through productivity increases at individual firms?
Reallocation mechanism

Technology of each firm is fixed

Trade liberalization results in a reallocation of resources:

The least efficient firms exit

Resources are moved toward more efficient firms, particularly exporters
Main findings

Reallocation following trade liberalization has no first-order effect on productivity, but it matters for welfare.

Productivity gains must primarily come from firm-level productivity increases.

Gibson studies a technology adoption mechanism in which firms can upgrade to a better technology, but it is costly to do so. Trade liberalization encourages technology adoption.
Model

$I$ symmetric countries, each with an *ad valorem* tariff on imports

Monopolistically competitive firms that are heterogeneous in technological efficiency

Sunk cost of entering export markets — only the most efficient firms export

Fixed cost of production — not all firms choose to operate

No aggregate uncertainty
Consumer’s problem

\[
\max \sum_{t=0}^{\infty} \beta^t \log \left( \int_{z \in Z_t} c_t(z)^\rho \, dz \right)^{1/\rho}
\]

s.t. \[
\int_{z \in Z_t^d} p_t(z) c_t(z) \, dz + (1 + \tau_t) \int_{z \in Z_t^x} p_t(z) c_t(z) \, dz = \bar{N} + \Pi_t + T_t
\]
Aggregation

Ideal real income index:

\[ C_t = \left( \int_{z \in Z_t} c_t(z)^\rho \, dz \right)^{1/\rho} \]

Ideal price index:

\[ P_t = \left( \int_{z \in Z_t^d} p_t(z)^{-\rho} \, dz + (1 + \tau_t)^{-\rho} \int_{z \in Z_t^x} p_t(z)^{-\rho} \, dz \right)^{-{(1-\rho)/\rho}} \]

Budget constraint again:

\[ P_t C_t = \bar{N} + \Pi_t + \Gamma_t \]
Demand functions

Firms take the consumer’s demand functions as given

Demand for domestically produced goods:

\[ \tilde{c}_t^d(p) = \left( \frac{P_t}{p} \right)^{\frac{1}{1-\rho}} C_t \]

Demand for imported goods:

\[ \tilde{c}_t^x(p) = \left( \frac{P_t}{(1 + \tau_t)p} \right)^{\frac{1}{1-\rho}} C_t \]
Firms: Timing within a period

Entrants learn their efficiencies

Each firm decides whether to operate or exit — producing requires paying a fixed cost of $f^p$ units of labor

Non-exporters decide whether to pay the sunk cost of entering export markets, $f^x$ units of labor

After producing, each firm faces exogenous probability of death $\delta$
Technologies

A firm of type $a$ has the increasing-returns technology

$$y(n; a) = \max\left[ a(n - f^p), 0 \right]$$

$a \in [1, \infty)$ is the firm’s technology draw from Pareto distribution $F(a) = 1 - a^{-n}$

$f^p$ is the fixed cost, in units of labor, of producing
Firm’s static problem: Maximize period profits

Non-exporters:

\[ \pi^d_t(a) = \max_{p,n} p \tilde{c}^d_t(p) - n \]

s.t. \[ a(n - f^p) = \tilde{c}^d_t(p) \]

Exporters:

\[ \pi^x_t(a) = \max_{p,n} p(\tilde{c}^d_t(p) + (I-1)\tilde{c}^x_t(p)) - n \]

s.t. \[ a(n - f^p) = \tilde{c}^d_t(p) + (I-1)\tilde{c}^x_t(p) \]
Prices

The profit-maximizing price is a constant markup over marginal cost:

\[ p(a) = \frac{1}{\rho a} \]

The price of a good is inversely related to the efficiency with which it is produced.
Exporter’s dynamic problem

\[
\nu_t^x(a) = \max \left[ 0, \ \pi_t^x(a) + \frac{1-\delta}{1+r_{t+1}} \nu_{t+1}^x(a) \right]
\]

Non-exporter’s dynamic problem

\[
\nu_t^d(a) = \max \left[ 0, \ \pi_t^d(a) + \frac{1-\delta}{1+r_{t+1}} \max \left[ \nu_{t+1}^d(a), \nu_{t+1}^x(a) - \frac{1+r_{t+1}}{1-\delta} f^x \right] \right]
\]

Outer maximization: Whether to operate

Inner maximization: Whether to devote \( f^x \) units of labor to enter export markets
**Firm entry**

There is free entry of firms, and firms enter as non-exporters.

The cost of a technology draw from probability distribution $F$ is $f^e$ units of labor.

The measure of draws taken, $e_t$, is determined endogenously through a free-entry condition:

$$
\frac{1}{1 + r_{t+1}} \int v^d_{t+1}(a) F(da) - f^e \leq 0, \quad = 0 \text{ if } e_t > 0
$$

The inequality reflects the constraint that $e_t \geq 0$. 
Distributions of firms by efficiency

Suppose that at the beginning of period $t$ the distribution of non-exporters is $m_t^d$ and the distribution of exporters is $m_t^x$.

To obtain the distributions of firms that choose to operate, apply the decision rules:

$$
\mu_t^x(a) = \int_1^a \chi_t^x(\alpha)m_t^x(d\alpha)
$$

$$
\mu_t^d(a) = \int_1^a \chi_t^d(\alpha)m_t^d(d\alpha)
$$

Distributions evolve in response to firm entry, $e_t$ and changes in export status, $\chi^e_t$. 
Labor market clearing

The supply of labor is fixed at $\bar{N}$ and is allocated among 3 activities: production, entering export markets, and entering the domestic market

$$\sum_s \int \left( n_t^d (a) \mu_t^d (da) + n_t^x (a) \mu_t^x (da) + f^x \chi_t^e (a) \mu_t^d (da) \right) + f^e e_t = \bar{N}. $$

Measuring productivity

Labor productivity in the data is a measure of real value added per worker or per hour

Standard way of calculating real value added is to use base-period prices
Measuring real value added per worker

Value added at current prices:

\[ y_t = \int_{z \in Z_t^d} p_t(z) y_t(z) \, dz \]

Value added at base-period (period-0) prices:

\[ Y_t = \int_{z \in Z_t^d} p_0(z) y_t(z) \, dz \]

Real value added per worker is \( Y_t / \bar{N} \)
What if a good was not produced in the base period?

This is an issue in the data as well

The standard recommendation for obtaining a proxy for the base-period price is to deflate the current price by the price index for a basket of goods that were produced in both periods, say $\tilde{Z}$:

$$\tilde{P}_t = \frac{\int_{\tilde{Z}} p_t(z) y_0(z) \, dz}{\int_{\tilde{Z}} p_0(z) y_0(z) \, dz}$$

Proxy for the period-0 price of a good not produced in period 0:

$$p_0(z) = \frac{p_t(z)}{\tilde{P}_t}$$
Measuring social welfare

Ideal real income index:

\[
\frac{\bar{N} + \Pi_t + \Gamma_t}{P_t} = C_t = \left(\int_{z \in Z_t} c_t(z)^\rho \; dz\right)^{1/\rho}
\]

The ideal price index \( P_t \) takes into account changes in variety and the consumer’s elasticity of substitution — in contrast to price indices in the data.
To what extent can reallocation following trade liberalization account for long-term productivity gains?

To determine the long-term effects of trade liberalization, we compare stationary equilibria of the model

two versions of the model:

Static version with $\beta \rightarrow 1$ (similar to Melitz (2003)): analytical result

Dynamic version with $0 < \beta < 1$: illustrative numerical example
Static model: An analytical finding

Proposition: In a stationary equilibrium with $\beta \to 1$, real value added per worker does not depend on the level of the tariff

To see why:

With $\beta \to 1$, $\Pi = 0$, so the budget constraint gives

$$ \int_{z \in Z_i^d} p_t(z) c_t(z) \, dz + \int_{z \in Z_i^x} p_t(z) c_t(z) \, dz = \bar{N} $$

The balanced trade condition is

$$ \int_{z \in Z_i^d} p_t(z)(y_t(z) - c_t(z)) \, dz + \int_{z \in Z_i^x} p_t(z) c_t(z) \, dz = \int_{z \in Z_i^x} p_t(z) c_t(z) $$
Add them together to get

\[ \int_{z \in Z_t^d} p_t(z) y_t(z) \, dz = \bar{N} \]

So value added at current prices is constant, does not depend on \( \tau \)

What about base-period prices? Without technology adoption, the price of each good in the economy is constant: \( p(z; a) = 1/(\rho a) \)

So base-period prices are equal to current prices and the prices of new goods do not get deflated

Result:

\[ Y_t = \int_{z \in Z_t^d} p_0(z) y_t(z) \, dz = \int_{z \in Z_t^d} p_t(z) y_t(z) \, dz = \bar{N} \]
Intuition for the result

Reallocation following trade liberalization has no long-term effect on measured productivity

Why? Two factors:

Prices — they are inversely related to the efficiency with which a good is produced

General equilibrium effects — changes in the real wage (partial equilibrium analysis would predict a substantial increase in measured productivity)
Parameterization for illustrative numerical experiment

$\bar{N} = 1$ \hspace{1em} Normalization

$\rho = 0.5$ \hspace{1em} Elasticity of substitution of 2 (Ruhl 2003)

$\eta = 1.5$

$\delta = 0.05$

$f^e = 1$

$f^x$ \hspace{1em} 20 percent of firms export initially

$f^p$ \hspace{1em} Efficiency cutoff for operating is 1 initially
Illustrative numerical experiment in the static model

$\beta \rightarrow 1$

Policy experiment: Eliminate a 10 percent tariff between 2 countries

Compare stationary equilibria to assess long-term effects of trade liberalization:

- Percent change in measured productivity $0.0$
- Percent change in welfare $0.5$
A note on the welfare increase

The increase in welfare following trade liberalization is not due to an increase in variety — the measure of varieties available to the consumer decreases.

Reallocation toward more efficient firms drives the welfare increase.

This is in sharp contrast to trade models with homogeneous firms, in which the increase in welfare is driven by an increase in variety.

Main point: Reallocation matters for welfare but not for measured productivity.
Illustrative numerical experiment in the dynamic model

To what extent can the fully dynamic model account for measured productivity gains?

\[ \beta = 0.96 \] \hspace{1cm} \text{Real interest rate of 4 percent}

Same numerical experiment:

- Percent change in measured productivity \quad 0.7
- Percent change in welfare \quad 1.8
1. Consider a two-sector growth model in which the representative consumer has the utility function

\[ \sum_{t=0}^{\infty} \beta^t \log(a_1 c_{1t}^{b_1} + a_2 c_{2t}^{b_2})^{1/b}. \]

The investment good is produced according to

\[ k_{t+1} = d(a_1 x_{1t}^{b_1} + a_2 x_{2t}^{b_2})^{1/b}. \]

In particular, the depreciation rate is \( \delta = 1 \). Feasible consumption/investment plans satisfy the feasibility constraints

\[ c_{1t} + x_{1t} = \phi_1(k_{1t}, \ell_{1t}) = k_{1t} \]
\[ c_{2t} + x_{2t} = \phi_2(k_{2t}, \ell_{2t}) = \ell_{2t}. \]

where

\[ k_{1t} + k_{2t} = k_i \]
\[ \ell_{1t} + \ell_{2t} = \ell_i. \]

The initial value of \( k_i \) is \( \bar{k}_0 \). \( \ell_i \) is normalized to 1.

a) Define an equilibrium for this economy.

b) Explain how you can reduce the equilibrium conditions of part a to two difference equations in \( k_i \) and \( c_i \) and a transversality condition. Here \( c_i = d(a_1 c_{1t}^{b_1} + a_2 c_{2t}^{b_2})^{1/b} \) is aggregate consumption. (You do not need to go through all of the algebra, but you need to explain all of the logical steps carefully.)

c) Suppose now that there is a world made up of two different countries, each with the same technologies and preferences, but with different constant populations, \( L^j_i = \bar{L}_i \), and with different initial capital-labor ratios \( \bar{k}_0^j \). Suppose that goods 1 and 2 can be freely traded across countries, but that the investment good cannot be traded. Suppose too that there is no international borrowing. Define an equilibrium for the world economy.

d) State and prove versions of the factor price equalization theorem, the Stolper-Samuelson theorem, the Rybczynski theorem, and the Heckscher-Ohlin theorem for this particular world economy.
e) Let \( s_t = c_t / y_t \) where \( y_t = p_{t1}k_{t1} + p_{t2} = r_tk_t + w_t = d(a_tk_t^b + a_2)^{1/b} \) is income per capita. Transform the two difference equation in part b into two difference equations in \( k_t \) and \( s_t \). Prove that

\[
\frac{y_t^i - y_t}{y_t} = \frac{s_{t+1}}{s_t} \left( \frac{y_{t+1}^i - y_{t+1}}{y_{t+1}} \right) = \frac{s_{t+1}}{s_t} \left( \frac{y_t^i - y_0}{y_0} \right),
\]

where \( y_t^i = p_{t1}k_t^i + p_{t2} = r_tk_t^i + w_t = d(a_tk_t^ib + a_2)^{1/b} \) is income per capita in country \( i \).

f) Discuss the economic significance of the result in part e.

2. Consider an economy where the consumers have Dixit-Stiglitz utility functions and solve the problem

\[
\max \ (1- \alpha) \log c_0 + (\alpha / \rho) \log \int_0^m c(v)^\rho \, dv
\]

s.t. \( p_0c_0 + \int_0^m p(v)c(v)dv = w\bar{t} + \pi \)

\( c(v) \geq 0. \)

Here \( \pi \) are profits of the firms, which are owned by the consumers.

a) Suppose that the producer of good \( v \) takes the price function \( p(v) \) as given. Suppose too that this producer has the production function

\( y(v) = \max \left[ z(v)(\ell(v)-f), 0 \right]. \)

Solve the consumer’s profit maximization problem to derive and optimal pricing rule.

b) Suppose that there is a measure \( \mu \) of potential firms. Firm productivities are distributed on the interval \( z \geq 1 \) according to the Pareto distribution with distribution function

\( F(z) = 1 - z^{-\gamma}. \)

Define an equilibrium for this economy.

c) Suppose that \( \mu \) is large enough so that not all firms can earn nonnegative profits in equilibrium. Find an expression for the cutoff productivity level \( \bar{z} \) such that firms with productivity \( \bar{z} \) earn zero profits. Find an expression that relates the measure of goods consumed \( m \) to the measure of potential firms \( \mu \) and the cutoff level \( \bar{z} \). Find an expression for profits \( \pi \).
d) Suppose now that there are two countries that engage in free trade. Each country $i$, $i = 1, 2$, has a population of $\ell_i$ and a measure of potential firms of $\mu_i$. Firms’ productivities are again distributed according to the Pareto distribution, $F(z) = 1 - z^{-\gamma}$. A firm in country $i$ faces a fixed cost of exporting to country $j$, $j \neq i$, of $f_x$ where $f_x > f_d = f$ and an iceberg transportation cost of $\tau - 1 \geq 0$. Define an equilibrium for this economy.

e) Suppose now that the two countries in part d are symmetric in the sense that $\ell_1 = \ell_2 = \ell$ and $\mu_1 = \mu_2 = \mu$. Suppose too that $\mu$ is large enough so that not all firms can earn nonnegative profits in equilibrium. Explain now why there are two relevant cutoff levels of firm productivity, $\overline{z}_t$ and $\underline{z}_t$. Find expressions for these cutoff productivity levels. Find an expression for profits $\pi$.

f) Discuss the strengths and weaknesses of this model. In particular: What economic phenomena can this sort of model help to account for? What sort of phenomena can it not account for? How can we modify the model to account for these phenomena?

3. Find data to calculate the bilateral real exchange rate between two countries who have a bilateral trade relation that is important to at least one of the countries. Find data on the prices of traded goods in these two countries. Calculate a decomposition of the bilateral real exchange rate of the form

$$rer_i = rer_i^T + rer_i^N,$$

where $rer_i$ is the natural logarithm of the bilateral real exchange rate and $rer_i^T$ is the logarithm of the bilateral real exchange rate for traded goods. Calculate the correlation between $rer_i^T$ and $rer_i^N$ in levels, in 1 year differences, and in 4 year differences.

Calculate ratio of the standard deviations of $rer_i$ and $rer_i^N$ in levels, in 1 year differences, and in 4 year differences. Calculate a variance decomposition of $rer_i$ in terms of $rer_i^T$ and $rer_i^N$ in levels, in 1 year differences, and in 4 year differences.