U.S.-CANADA FTA IN THE MONOPOLISTIC COMPETITION MODEL

Set $w = 1$ as numeraire.

We know that

\[
\begin{align*}
\rho &= w = 1 \\
y_0 &= \frac{w \bar{\ell}}{2p_0} = \frac{\bar{\ell}}{2p_0} \\
y_i &= \bar{y} = \frac{\rho(n-1)\bar{\ell}}{2n^2b} \\
p_i &= \bar{p} = \frac{\bar{\ell}y^{\rho-1}}{2n\bar{y}^{\rho}} = \frac{\bar{\ell}}{2n\bar{y}^{\rho}} = \frac{bn}{\rho(n-1)}.
\end{align*}
\]

\[
n = \frac{(1-\rho)\bar{\ell} + \sqrt{(1-\rho)^2\bar{\ell}^2 + 4(\rho\bar{\ell})(2f)}}{4f}.
\]

Autarky equilibrium in Canada

\[
\begin{align*}
b &= 1, f = 2, \rho = 1/2, \bar{\ell} = 18 \\
n &= \frac{9 + \sqrt{9^2 + 4(9)(4)}}{8} \\
n &= 3 \\
\bar{y} &= 1, \bar{p} = 3 \\
p_0 &= w = 1, y_0 = 9
\end{align*}
\]

Utility:

\[
\log 9 + 2\log(3(1)^{1/2}) = 4.3944
\]

Homogenous of degree one representation of utility (a real income index):

\[
\exp[(1/2)(\log 9 + 2\log(3(1)^{1/2}))] = \exp[(1/2)(4.3944)] = 9.00.
\]
Autarky equilibrium in the United States

\[ b = 1, f = 2, \rho = 1/2, \bar{\ell} = 180 \]

\[ n = \frac{90 + \sqrt{(90)^2 + 4(90)(4)}}{8} \]

\[ n = 23.4591 \]

\[ \bar{y} = 1.8365, \bar{p} = 2.0891 \]

\[ p_0 = w = 1, y_0 = 90 \]

Utility:

\[ \log 90 + 2\log(23.4591 \times 1.8365) = 11.4182 \]

Homogenous of degree one representation of utility (a real income index):

\[ \exp[(1/2)\log(90 + 2\log(23.4591 \times 1.8365))] = \exp[(1/2)(13.2418)] = 301.60. \]

### Autarky equilibrium

<table>
<thead>
<tr>
<th></th>
<th>( \hat{n}^i )</th>
<th>( \hat{p}_0^i )</th>
<th>( \bar{p} )</th>
<th>( \hat{w}^i )</th>
<th>( \hat{c}_0^i )</th>
<th>( \bar{c}^i )</th>
<th>( \hat{y}_0^i )</th>
<th>( \bar{y}^i )</th>
<th>( \bar{l}^i )</th>
<th>( e^{\hat{w}/2} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Canada</td>
<td>3.0000</td>
<td>1.0</td>
<td>3.0000</td>
<td>1.0</td>
<td>9.0</td>
<td>1.0000</td>
<td>9.0</td>
<td>9.0</td>
<td>1.0000</td>
<td>3.0000</td>
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<tr>
<td>U.S.</td>
<td>23.4591</td>
<td>1.0</td>
<td>2.0891</td>
<td>1.0</td>
<td>90.0</td>
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<td>90.0</td>
<td>90.0</td>
<td>1.8365</td>
<td>3.8365</td>
</tr>
</tbody>
</table>

### Free trade equilibrium

First calculate the equilibrium for the North American economy with \( \bar{\ell} = \bar{\ell}^{CA} + \bar{\ell}^{US} = 198 \):

\[ n = \frac{99 + \sqrt{(99)^2 + 4(99)(4)}}{8} \]

\[ n = 25.7126 \]

\[ \bar{y} = 1.8503, \bar{p} = 2.0809 \]

\[ p_0 = w = 1, y_0 = 99 \]
To calculate consumption of each variety in each country, we just divide the world production of the variety $\bar{y}$ proportionally. In Canada, for example,

$$\bar{c}^{CA} = \frac{\bar{P}^{CA}}{\bar{P}^{CA} + \bar{Q}^{CA}} \bar{y} = \frac{18}{198} 1.8503 = 0.1682.$$  

We also divide the production and the consumption of the agricultural good proportionally.

**Free trade equilibrium**

<table>
<thead>
<tr>
<th></th>
<th>$\hat{n}_i$</th>
<th>$\hat{p}_i^0$</th>
<th>$\bar{p}$</th>
<th>$\hat{w}_i$</th>
<th>$\hat{c}_0$</th>
<th>$\bar{c}_i$</th>
<th>$\hat{y}_0^i$</th>
<th>$\tilde{y}_0^i$</th>
<th>$\bar{y}^i$</th>
<th>$\bar{e}$</th>
<th>$e^{u/2}$</th>
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</thead>
<tbody>
<tr>
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<td>2.0809</td>
<td>1.0</td>
<td>9.0</td>
<td>0.1682</td>
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<td>9.0</td>
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<td>3.8503</td>
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<tr>
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<td>1.6821</td>
<td>90.0</td>
<td>90.0</td>
<td>1.8503</td>
<td>3.8503</td>
<td>330.57</td>
</tr>
</tbody>
</table>

Notice that Canada stands to gain far more from trade than does the United States:

In Canada real income increases by 267 percent (33.06 / 9.00 = 3.67).

In the United States real income increases by 10 percent (330.57 / 301.60 = 1.10).