LEARNING BY DOING AND THE DYNAMIC EFFECTS OF INTERNATIONAL TRADE*

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Using an endogenous growth model in which learning by doing, although bounded in each good, exhibits spillovers across goods, this paper investigates the dynamic effects of international trade. Examining the interaction of an LDC and a DC, the latter distinguished by a higher initial level of knowledge, I find that under free trade the LDC (DC) experiences rates of technical progress and GDP growth less than or equal (greater than or equal) to those enjoyed under autarky. Since both countries enjoy the usual static gains from trade, free trade may, nevertheless, improve the welfare of LDC consumers.

I. INTRODUCTION

In the theory of international trade the static gains from trade and losses from protection have been thoroughly established. Trade theory, however, provides little indication as to what are the dynamic effects of international trade on growth, technical progress, and welfare. The postwar empirical experience of export-oriented economies such as Hong Kong, Singapore, South Korea, and Taiwan, when contrasted with that of protectionist economies such as Argentina and Ghana, has led many to argue that outward-oriented policies will induce rates of growth and technical progress much greater than those achievable with inward-oriented policies.

Unfortunately, the theoretical arguments put forward by proponents of this viewpoint appear to suffer from an inability to distinguish between what Solow [1956] allowed us to separate as growth versus level effects. Thus, for example, the World Bank's *World Development Report 1987* argues that the adoption of an outward-oriented policy will raise savings and investment rates, eliminate DUP rent seeking, increase X-efficiency, and "correctly" value the exchange rate, all of which are obviously level effects.[1] Further, the current focus on the postwar relative superiority of

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1. When finally referring to some growth effects, such as the impact of trade policy on productivity growth and rates of innovation, the report, despite offering some favorable evidence, becomes more equivocal, acknowledging that "little is known about technological innovation in relation to trade policy" [*World Bank*, 1987, p. 91].

outward-oriented policies is empirically myopic, ignoring the fact that historically many capitalist economies, such as the United States and Japan, experienced rapid growth under early conditions of protectionism.[2]

In defense of the more rigorous proponents of the positive growth effects associated with free trade,[3] the neoclassical growth model, in which the principal engine of growth is exogenous technical change, while providing many valuable lessons, did not provide a very good framework for analyzing long-run growth. With the development during the 1980s of models endogenizing long-run growth (e.g., Romer [1986, 1987], Lucas [1988], Prescott and Boyd [1987]), economists now have the formal techniques with which to explore the relationship between trade policy and long-run growth. Thus, in a series of recent papers Grossman and Helpman [1988, 1989a,b], using models in which endogenous growth is generated either by the development of new varieties of intermediate or final goods or by the improvement of an existing set of goods, relate the dynamic effects of various economic policies to their impact on the R&D efforts of two trading economies.[4] In this paper I develop a model in which endogenous growth is generated by learning by doing which, although bounded in each good, exhibits spillovers across goods, and use this model to examine the impact of the movement from autarky to free trade on the growth rates, rates of technical progress, and intertemporal consumer welfare of two economies, one of which (the LDC) is initially less technically advanced than the other (the DC).

II. A Brief Description of the Model and its Relation to Previous Literature

The learning-by-doing model in this paper incorporates two important characteristics suggested by industry level analyses of technical progress. First, there appear to be substantial spillover

2. In addition, blanket references to the modern export-led growth successes of the East Asian economies mask remarkable variations in policy among them, from Hong Kong's laissez-faire environment, to Singapore's forced domestic saving and encouragement of foreign investment, to South Korea's huge government-backed conglomerates.

3. E.g., Krueger and Harberger.

4. In their "Quality Ladders" [1989b] paper Grossman and Helpman do not emphasize the dynamic effects of different policies, focusing instead on deriving the autarky and free trade equilibria, but their model clearly contains policy implications, as illustrated by Grossman's [1989] further analysis.

effects in the development of knowledge across industries, with technical innovations originating in particular industries finding important applications, as well as instigating further technical change, in other economic sectors. At the anectodal level these spillovers have been documented extensively by Rosenberg [1982]. In a more formal econometric analysis Jaffe [1986] found that the number of patents a firm received was increasing not only in its own R&D efforts, but also in the R&D efforts of its technical neighbors. Although no formal analysis of cross-industry spillover effects in learning by doing has been made, it seems reasonable to assume that many of the technical and managerial advances brought about by experience in the production of certain products have applications elsewhere. Thus, in my model productivity increases in each industry are not only a function of productive activity in that industry, but also the result of spillovers from learning by doing in other industries. The concept of spillover effects in learning by doing has previously been incorporated in models by Boldrin and Scheinkman [1988], Stokey [1988], and Succar [1987].

The second empirical regularity of relevance to this paper is the existence of strong diminishing returns in the learning-by-doing process. Most of the empirical studies of learning by doing have focused on the log-linear model characterized by the equation, $C_n = AE_n^{-b}$, where C_n is the cost of production of the nth unit, E_n is cumulative output up to and including the nth unit, A is the cost of the first unit, and b is the progress elasticity.[5] Whereas this formulation implies that as cumulative experience increases the marginal contribution to cost reduction of an additional unit of output goes to zero; it also implies that a 1 percent increase in cumulative experience will always yield a b percent reduction in costs; i.e., that the productivity gains from learning by doing are essentially unbounded. In contrast, a number of researchers have argued that although the learning curve is initially approximately log-linear, a plateauing phase is ultimately reached in which additional proportional increments to experience yield absolutely

5. See, for examples, Wright [1936], Hirsch [1956], Alchian [1963], Boston Consulting Group [1968], and Lieberman [1984]. In some formulations C_n is the average cost of all n units or the price of the nth unit. Sheshinski [1967] and Levhari and Sheshinski [1973] assume forms that are log-linear in output: $Q = E^bF(K, L)$, with Q, K, L, E, and b denoting output, capital, labor, cumulative experience, and the experience elasticity of output, respectively.

no additional gains in productivity.[6] Unfortunately, most of the empirical work on learning by doing is remarkably sloppy, with little or no control for ongoing investment,[7] R&D, and product design changes, rare use of significance tests (curves are frequently drawn through the data and eyeballed to determine whether the fit is good), and no rigorous attempts to test rival hypotheses on the nature of learning by doing (e.g., perpetual log-linear versus plateauing).[8] Thus, it is difficult to draw decisive conclusions on the nature of learning by doing on the basis of the empirical work to date. Nevertheless, sufficient evidence of plateauing exists to justify its consideration as a model of learning by doing.

Perhaps learning by doing can be conceived of as the exploration and actualization of the productive potential of new technologies; if you will, a series of minor technical innovations that are learned from a major technical breakthrough. Thus, the development of new productive technologies, perhaps as the result of R&D efforts, and their use in the production of existing or new goods initially leads to rapid learning by doing. After some time, however, the productive capability of these new technologies is exhausted, and learning by doing slows and perhaps ultimately stops. In the absence of the introduction of new technical processes, it is likely that learning by doing cannot be sustained.[9] This would explain why, despite considerable economic activity, learning by doing did not lead to sustained economic growth prior to the modern era. The unique aspect of the learning-by-doing model in this paper is that this concept is incorporated explicitly in the imposition of a bound on the cumulative productivity gains from learning by doing in the production of any particular good.

Turning to a description of the formal structure of the model,

6. See Carr [1946], Asher [1956], Conway and Schultz [1959], and Baloff [1966].

7. Which might introduce scale effects or embody technology developed from R&D efforts.

8. Some of the more egregiously poor examples are Garg and Milliman [1961], Boston Consulting Group [1968], and Goel and Becknell [1972]. Lieberman's [1984] study of the chemical industry and Levhari and Sheshinski's [1973] analysis of the Israeli diamond industry are among the most careful and illuminating studies. Although Levhari and Sheshinski did not explicitly seek to test rival hypotheses on the nature of learning by doing, they did find that a formulation in which the elasticity of output with respect to experience was a quadratic and concave function of the level of experience provided as good a fit as the log-linear model.

9. It is interesting to note that in his seminal paper Arrow [1962] used cumulative investment, as opposed to cumulative output, as his index of experience precisely because he believed that, in the absence of substantial changes in the productive environment, learning by doing is bounded.

in order to focus on the effects of learning by doing, this paper will abstract from the development of new goods and productive processes and take as a datum the existence of a continuum of goods, indexed along the real line, any of which could conceivably be produced at any given time (although in practice only a small subset will be produced at any given time). Furthermore, the bound on learning by doing in each of these goods will also be taken as exogenous. It is assumed that learning by doing exhibits spillovers across goods and that the knowledge so generated is in the public domain, i.e., is nonappropriable by the firms engaged in production. With labor as the sole factor of production and preferences that are symmetric and separable in the goods, the equilibrium output of perfectly competitive firms will endogenously determine the evolution through time of a function describing the unit output labor requirements of each of the goods. Thus, fundamentally, the model examines the endogenous exploration of the productive potential of a series of technologies, whose development, for the sake of simplicity, is taken as a given.

At any given time, learning by doing will have been exhausted in a subset of goods, but will continue in the remainder. Some fairly weak assumptions on the nature of preferences and the learning-by-doing technology are then sufficient to ensure unbounded growth. Over time, growth will involve the production of a changing basket of goods, with both the quantity and variety of goods consumed increasing, which accords well with the empirical experience of most economies. It is notable that although production takes place under conditions of perfect competition, there will be gains from increasing variety, which have previously been confined exclusively to monopolistically competitive models.[10]

Assuming that there is no international diffusion of knowledge, the effect of trade on technical progress and growth will depend upon whether static comparative advantage leads an economy to specialize in goods in which it has mostly exhausted learning by doing or in goods in which learning by doing still proceeds apace. Trade will obviously improve the intertemporal welfare of consumers in economies in which it accelerates technical progress and growth. However, even in an economy in which trade slows technical progress, consumers might experience an improve-

10. In both static (e.g., Dixit and Stiglitz [1977], Lancaster [1980]) and dynamic ([Romer, 1987; Grossman and Helpman, 1989a]) settings.

ment in intertemporal welfare, since under free trade technical progress abroad can improve real consumption income at home.

Having outlined the model of this paper, we can now relate it to the previous literature on endogenous growth and the effect of trade on growth and technical progress.[11] A model with an infinite continuum of producible goods, of which only a finite number are produced at any given time, was first proposed by Wan [1975]. Ordering goods according to their level of technical sophistication, Wan assumed that technical progress in each good was a function of the levels of technical capability in all lower numbered goods, and, allowing as well for the international diffusion of knowledge, examined the conditions under which the technological levels of different trading economies would converge. Stokey [1988] considered a similar infinite continuum of goods, with consumer preferences specified over goods characteristics and goods ranked according to the number of characteristics they contain. With endogenous technical change based upon learning by doing, Stokey derived a set of assumptions, not all of which are intuitively compelling,[12] sufficient to guarantee the introduction of new goods and the disappearance of old goods over time. The model of this paper is a melding of these two approaches. I rank goods hierarchically by their level of technical sophistication and use fairly intuitive assumptions, i.e., that learning by doing is bounded in each good and that the ultimate productivity of labor is rising in the technical sophistication of goods, to generate a learning-by-doing "movement to the right," with old goods eventually being discarded in favor of new and more advanced goods.

Ranging from Bardhan's [1970] and Clemhout and Wan's [1970] analyses of the optimal subsidy for an infant industry in a small open economy, to Lucas's recent [1988] exposition of a global

11. I shall not discuss the neoclassical (e.g., Oniki and Uzawa [1965]) and North-South (e.g., Findlay [1982, 1984]) analyses of the interaction of trading economies. These models rely upon exogenous technical change to generate growth and, hence, although illuminating in many ways, tends to beg the question of the dynamic effects of international trade on growth and technical progress. For surveys of this literature see Findlay [1984] and Smith [1984].

12. Specifically, Stokey [1988] assumed that at any given time the function describing the prices of goods (ordered on the real line) was weakly concave on $[0,m)$ and strictly convex on (m,∞). Since higher numbered goods contain additional characteristics, this implies that the marginal cost of additional characteristics is diminishing along $[0,m)$, which ensures that no goods below m will be consumed. At any time the consumer's optimal plan involves consuming goods in $[A, B]$, where $A = \max\{s \geq 0 | p(s) - s\,dp(s)/ds = 0\}$ is the good that provides the lowest average cost per characteristic across the widest range of characteristics. Stokey assumed that aggregate economywide learning by doing results in a greater proportional reduction in the cost of higher numbered goods (ensuring a rise in B) and also raises A.

learning-by-doing equilibrium, learning by doing has long been used in the formal analysis of the effects of trade on endogenous growth. In the main, these models have assumed the existence of two goods, one of which experiences more rapid learning by doing than the other, which, as in Bardhan's framework, might experience no learning whatsoever. Since at any time the model in this paper will contain two sets of goods, one in which learning by doing has ended and one in which learning by doing continues, in its static structure, with its asymmetric learning effects, it is similar to this literature. In its dynamic structure, however, it differs in that it endogenizes the movement of goods out of the learning-by-doing (infant industry) sector into the (mature) sector in which learning by doing no longer occurs. Furthermore, as ably demonstrated in a static framework by Krugman [1987] and in a more dynamic framework by Lucas [1988] and Boldrin and Scheinkman [1988], the usual learning-by-doing function will tend to enhance existing patterns of comparative advantage, leading to fairly static trade patterns. In contrast, the bounded learning by doing with spillover effects of the model in this paper leads to an evolving trade structure, which agrees with the empirical experience of most economies.

Finally, with respect to the recent papers by Grossman and Helpman investigating the relationship between trade, economic policies, and endogenous R&D induced growth, this paper, in essence, takes an opposite approach. While Grossman and Helpman examine the impact of R&D, assuming that the full productive potential of a technology is actualized at the moment of invention, I take as given the existence of technical processes produced by path-breaking R&D and explore the actualization of their productive potential. Empirically there is evidence that the actualization of the productive potential of existing technologies may lead to productivity increases several orders of magnitude greater than those associated with the original technical innovations. Thus, for example, Enos [1958] found that whereas during the initial introduction of new petroleum refining processes cost reductions of 1.5 percent per annum were achieved, subsequent improvements of these same processes led to cost reductions of 4.5 percent per annum.[13] At the same time, as postulated in this paper, learning by

13. Similarly, Mak and Walton [1972] have shown that although the initial introduction of the steamboat, between 1815 and 1820, to western inland rivers led to a significant decline in freight costs, subsequent improvements in the steamboat,

doing cannot be sustained without R&D and path-breaking innovation. Both effects are important, and a more complete and complicated model would combine R&D into new productive processes/inputs and the learning-by-doing exploitation of the ultimate productive potential of that R&D.

In Section III below I present the general model of this paper, illustrating how in an autarkic economy bounded learning by doing with spillovers across goods can result in unbounded growth. In section IV I use a specific functional form of this model to examine the effect of the movement from autarky to free trade on the growth rates, rates of technical progress, and intertemporal welfare of two economies, one of which is, initially, technically more advanced than the other. Section IV concludes with a summary of its principal results. Section V presents some final thoughts.

III. A General Model of Unbounded Growth with Bounded Learning by Doing

Assume an economy populated by a large number of consumers and perfectly competitive firms. Goods are indexed by s along $[B,\infty)$[14] and are ordered according to the sophistication of the technical processes used in their production, with the production of higher numbered goods involving more advanced technologies. Labor is the sole factor of production and for each good s there exists a lower bound on potential unit labor requirements, $\bar{a}(s)$, which is continuous and nonincreasing in s, the latter assumption reflecting a belief that the ultimate productivity of labor is nondecreasing (and probably increasing) in the technical sophistication of the production processes involved. At any time t, however, actual unit labor requirements are $a(s,t)$.[15] I assume that $a(s,t)$ is continuous in s and that $\lim_{s\to\infty} a(s,t) = \infty$.[16] The latter assumption reflects the notion that although the blueprints of all future technologies are available at each time t, an economy must pass through a

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principally changes in hull design, led to much greater declines in shipping costs between 1820 and 1860.

14. An extension to the real line of the Ricardian model with a continuum of goods, indexed on the line segment [0,1], developed by Dornbusch, Fischer, and Samuelson [1977].

15. Throughout this paper all time subscripts denote an implicit (rather than explicit) dependence on time, due to the impact of ongoing production experience.

16. Given the learning-by-doing assumptions presented farther below, if $a(s,0)$ is continuous in s and $\lim_{s\to\infty} a(s,0) = \infty$, then this is sufficient to guarantee the same for $a(s,t)$ for all t.

certain amount of production experience before the costs of production of advanced goods fall to acceptable levels.[17] Together, these two assumptions imply that at each time t there exists a good s^*_t, which it will be convenient to refer to later on, such that for all s $a(s,t) \geq a(s^*_t,t)$.

This economy exhibits bounded learning by doing with spillovers across goods. Thus, if $a(s,t) = \bar{a}(s)$, then $\partial a(s,t)/\partial t = 0$ regardless of the output of good s or of any other good. However, if $a(s,t) > \bar{a}(s)$, then

$$\frac{\partial a(s,t)/\partial t}{a(s,t)} = -\int_B^\infty B(s,v,a(v,t)/\bar{a}(v))L(v,t)dv, \quad (1)$$

where $L(v,t)$ is the amount of labor devoted to the production of good v at time t and the $B(s,v,a(v,t)/\bar{a}(v))$ are the learning by doing coefficients. Some reasonable assumptions might be that, for any industry s: (i) $B(s,v,a(v,t)/\bar{a}(v)) \geq 0$ for all v, i.e., the output of other industries cannot reduce productivity in industry s; (ii) $B(s,v,1) = 0$, i.e., economic activity in industries in which learning by doing has been exhausted (in which there is nothing left to learn) cannot contribute to productivity increases in other industries; (iii) $B(s,v,a(v,t)/\bar{a}(s)) > 0$ for all $v \in (s - \alpha_s, s + \alpha_s)$ such that $a(v,t) > \bar{a}(v)$ (where $\alpha_s > 0$ for all s), i.e., economic activity in neighboring industries (in which learning continues) contributes positively to productivity improvements in industry s; (iv) $\sup_s \sup_v B(s,v,a(v,t)/\bar{a}(v)) < \infty$, i.e., the learning-by-doing spillovers are of bounded measure; and (v) $B(s,v,a(v,t)/\bar{a}(v))$ is continuous in s, i.e., the magnitude of the learning-by-doing spillovers is a continuous function of the technological sophistication of the industry receiving the spillover.[18] As I assume that learning by doing in this economy is nonappropriable by firms, production takes place under conditions of perfect competition.

Turning to consumer preferences, these are additively separable across time, with the instantaneous utility at each time t given

17. Thus, for example, although Leonardo da Vinci developed the blueprints for a bat's wing flying machine, it is likely that, given the production knowledge of the Italian economy at the time, it would have taken an enormous amount of labor to produce one such unit. Some 400 years later, after the world had considerable experience in producing bicycles and other simple machines, the French engineer Clement Ader produced and flew a bat's wing flying machine patterned after Leonardo's original design.

18. Assumptions (iv) and (v) ensure that if $a(s,0)$ is continuous in s and $\lim_{s\to\infty} a(s,0) = \infty$, then the same is true for $a(s,t)$ for all t.

by

$$(2)\quad V(t) = \int_B^{\infty} U(C(s,t))\,ds \quad \text{with} \quad U'(0) < \infty, \quad U(0) = 0,$$

where $C(s,t)$ denotes consumption of good s at time t and where $U(\cdot)$ is strictly concave and continuously differentiable, which, together with the restriction on $U'(0)$, indicates a strong, but not unbounded, preference for variety.[19] As there is no storage technology and all consumers are identical, intertemporal consumption smoothing is not possible and, so, the (implicit) interest rate[20] will adjust to ensure that at each time t consumers, who inelastically supply one unit of labor for wage W, maximize (2) subject to the budget constraint:

$$(3)\qquad W \geq \int_B^{\infty} P(s,t)C(s,t)\,ds.$$

Or, since under perfect competition prices $P(s,t) = Wa(s,t)$:

$$(3)'\qquad 1 \geq \int_B^{\infty} a(s,t)C(s,t)\,ds.$$

Although the consumer's optimal consumption plan at each time t can be derived formally, an intuitive analysis is more illuminating.[21] Substitution between goods that are consumed in positive quantities leads the consumer to follow the usual first-order condition:

(4)

$$\frac{U'(C(s,t))}{a(s,t)} = \frac{U'(C(v,t))}{a(v,t)} \ \forall\ s,v \text{ such that } C(s,t) > 0, C(v,t) > 0.$$

Since all goods enter symmetrically into the utility function, the consumer should obviously consume the cheapest of all goods, i.e., good s_t^*. Symmetrical preferences also imply that if the consumer consumes any good x, then she also consumes all goods s such that $a(s,t) \leq a(x,t)$. For finite quantities of good s_t^* consumed, it follows from equation (4) that there exists some good M such that as the

19. Since all goods enter symmetrically into utility, the assumption that $\bar{a}(s)$ is nonincreasing in s is equivalent to maintaining that the upper bound on the utility derived per unit of labor is nondecreasing in the technical sophistication of the industry. One could, equivalently, scale units so that $\bar{a}(s)$ is increasing in s, but with higher numbered goods provided greater utility per unit. As a practical example, consider the replacement of the phonograph by the compact disc player. Although compact disc players are more expensive than phonographs, they provide greater utility per unit cost, and hence are replacing the older system.

20. On bonds in zero net supply.

21. The formal analysis is available upon request from the author.

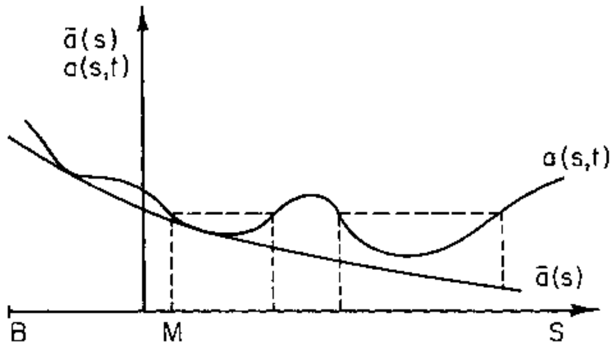


FIGURE I
Equilibrium in the General Model

prices of goods approach $a(M,t)$ from below, consumption goes to zero. Good M is the limit good. For all goods x such that $a(x,t) \geq a(M,t)$, consumption is zero as, given the symmetrical nature of goods in utility, these goods are too expensive to warrant positive consumption. For all goods s whose price is less than $a(M,t)$, consumption quantities are determined by

$$(4)' \qquad U'(C(s,t)) = a(s,t)U'(0)/a(M,t).$$

M itself is determined by the requirement that the consumer exhaust her budget constraint. The equilibrium is illustrated in Figure I. It is apparent that the continuity of $a(s,t)$ and the fact that $\bar{a}(s)$ is nonincreasing in s ensure that the consumer consumes some goods s such that $a(s,t) > \bar{a}(s)$, i.e., some goods in which learning by doing has not yet been exhausted.

Turning to the issue of growth, as a measure of growth I suggest that conventional national income accounting measure of the increase in real GDP per capita, i.e., the proportional change in the value of output (at constant prices) minus the rate of population growth:[22]

$$(5) \qquad g(t) = \frac{\int_B^\infty a(s,t)\partial X(s,t)/\partial t \, ds}{\int_B^\infty a(s,t)X(s,t) \, ds} - \frac{dL(t)/dt}{L(t)},$$

where $L(t)$ equals the population at time t and $X(s,t)$ equals aggregate output of good s at time t. Since $\int_B^\infty a(s,t)X(s,t)\, ds = L(t)$

22. Clearly, my measure differs from the national income accounts in that the base year for prices is constantly updated.

at all times t, it follows, using Leibnitz's rule, that

$$(5)' \qquad g(t) = \frac{\int_B^\infty - \partial a(s,t)/\partial t X(s,t)\, ds}{L(t)}.$$

Allowing S_L to denote the set of goods s such that $C(s,t) > 0$ and $a(s,t) > \bar{a}(s)$ and exploiting the fact that in autarky $X(s,t) = C(s,t)L(t)$,

$$(5)'' \qquad g(t) = \int_{s \in S_L} a(s,t)C(s,t) \left(\int_B^\infty B(s,v,a(v,t)/\bar{a}(v))L(v,t)dv \right) ds,$$

which is always strictly positive.[23] It is easily verified that if $\lim_{s\to\infty}\bar{a}(s) = 0$, then $\lim_{t\to\infty}V(t) = \infty$.[24] Thus, the general model is one in which learning by doing, although bounded in any particular good, can lead to unbounded growth.

IV. The Dynamic Effects of International Trade

In this section I use a specific functional form of the general model of Section III to investigate the dynamic effects of international trade. To provide a benchmark, I begin by deriving the endogenous rates of technical progress and growth of an autarkic economy, before turning to an examination of the dynamic evolution of two economies, one more technically advanced than the other, engaged in free trade. I conclude with a summary of the principal results.

A. Autarky Equilibrium

Preferences and Technology

Consider an economy population at each time t by $L(t)$ representative consumers, each of whom inelastically supplies one unit of labor at all times and seeks to maximize the intertemporal

23. From (4) it is apparent that the continuity of $a(s,t)$ ensures the continuity of $C(s,t)$, implying that $L(v,t) > 0$ for all v in some neighborhood of each s in S_L. Assumption (iii) on $B(s,v,a(v,t)/\bar{a}(v))$ earlier above combined with the continuity of $a(s,t)$ ensures that $B(s,v,a(v,t)/\bar{a}(v)) > 0$ for all v in some neighborhood of each s in S_L. It follows that $\int_B^\infty B(s,v,a(v,t)/\bar{a}(v))L(v,t)\, dv$ is strictly positive for each s in S_L, from which it follows that (5)″ is strictly positive as well.

24. Say that $\lim_{s\to\infty}\bar{a}(s) = \bar{a}$. As t goes to infinity, $a(s)$ becomes $\bar{a}(s)$, i.e., a downward sloping curve which asymptotes to $\bar{a}$. The consumer then divides her budget uniformly over a set of goods (of infinite measure) whose prices are all negligibly different from $\bar{a}$, deriving total instantaneous utility $U'(0)/\bar{a}$. The statement in the text then follows.

utility function:

$$\mathscr{P} = \int_t^\infty V(x)e^{-\rho(x-t)}\,dx, \tag{6}$$

where $V(x)$ denotes instantaneous utility at time x, which is given by

$$V(x) = \int_B^\infty \log\left(C(s,x) + 1\right) ds, \tag{7}$$

where $C(s,x)$ denotes consumption of good s at time x and B, a large negative number,[25] denotes the first good ever produced by mankind. Goods are produced using labor as the sole factor of production, with the lower bound on potential unit labor requirements given by $\bar{a}(s) = \bar{a}e^{-s}$. Let $B(s,v,a(v,t)/\bar{a}(v)) = 2$ for all s and v such that $a(v,t)/\bar{a}(v) > 1$.[26] Then, it is easily verified that if, at some arbitrary time 0, unit labor requirements are given by

$$a(s,0) = \bar{a}e^{-s}\ \forall\ s \le T(0);\ a(s,0) = \bar{a}e^{-T(0)}e^{s-T(0)}\ \forall\ s \ge T(0), \tag{8}$$

then, at each point in time t they will be given by

$$a(s,t) = \bar{a}e^{-s}\ \forall\ s \le T(t);\ a(s,t) = \bar{a}e^{-T(t)}e^{s-T(t)}\ \forall\ s \ge T(t). \tag{8$'$}$$

With $T(t)$ evolving according to the learning-by-doing equation,

$$\frac{dT(t)}{dt} = \int_{T(t)}^\infty L(s,t)\,ds, \tag{9}$$

where $L(s,t)$ denotes the labor allocated to the production of good s at time t. Thus, if unit labor requirements are initially symmetric around $T(0)$, they will remain symmetric around an ever increasing $T(t)$, which summarizes the current level of knowledge. To simplify the notation, in my analysis below, I shall frequently suppress the time subscript of the variables, all of which depend implicitly, rather than explicitly, on time.

It is easily verified that the economy set out in equations (6)–(9) is a particular example of the general model presented in Section III above. The key simplifying assumption is that all goods in which learning by doing has not yet been exhausted contribute symmetrically to economywide learning by doing, which ensures

25. Large in the sense that in both the autarky and free trade perfectly competitive equilibria (discussed below) good B is no longer produced. This assumption makes the equilibria easier to analyze and does not substantially alter the results.

26. And equal zero for all v such that $a(v,t)/\bar{a}(v) = 1$.

that unit labor requirements remain symmetric around an ever increasing $T(t)$, with learning by doing exhausted sequentially in goods. At any given moment this economy has two sets of goods, those in which there is no more learning by doing to be had ($s \leq T(t)$), and those in which learning by doing continues ($s > T(t)$). In this respect, as mentioned earlier above, it is analogous to learning-by-doing models which arbitrarily assume that some goods experience more rapid learning by doing than others. The difference lies in the fact that this model endogenizes which goods belong to which category. Thus, while retaining a simple static structure, it embodies rich dynamics.

General Equilibrium

As the preferences and production technology of this economy are identical in form to that of the economy discussed in Section III, the equilibrium has an identical structure. With no storage technology and identical consumers, the (implicit) interest rate will adjust so as to ensure that each consumer maximizes $V(t)$ subject to her instantaneous wage income W. Under perfect competition, prices $P(s) = Wa(s)$, so that the real price of goods, in units of labor, is simply $a(s)$. At each time t there exists a good M such that for all goods s such that $a(s) < a(M)$, consumption is positive; while for all goods x such that $a(x) \geq a(M)$, consumption is zero. For goods consumed in positive quantities, the usual first-order condition holds:

$$(10) \qquad 1/[(C(s) + 1)a(s)] = 1/[(C(v) + 1)a(v)].$$

As $a(s)$ goes to $a(M)$ from below, consumption of good s goes to zero. It follows, using (10), that for all goods s which are consumed in positive quantities:

$$(11) \qquad a(s)C(s) = a(M) - a(s).$$

M is then determined by the requirement that the consumer exhaust her budget constraint.

The autarky equilibrium is illustrated in Figure II. As shown, the difference between the dashed horizontal line of height $a(M)$ and the unit labor requirements curve $a(s)$ is equal to the consumer's expenditure on each good s. As $a(s)$ is symmetric around T, to each M there corresponds an N such that $a(N) = a(M)$, with N and M located equidistant from T. The consumer consumes all goods in (M,N). The shaded area denotes total expenditure and, as the consumer's budget is exhausted, equals one.

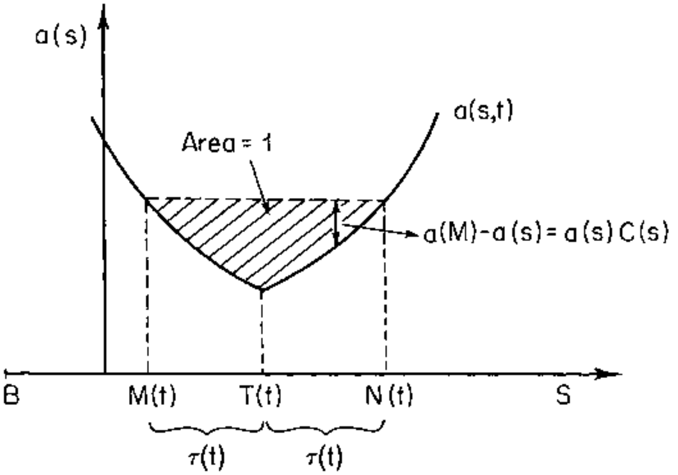


FIGURE II
Autarky Equilibrium

If we let τ denote $T - M = N - T$, i.e., the range of goods on each side of T that are consumed, then, using the binding budget constraint, we have

$$(12) \qquad 1 = \int_M^N a(s)C(s)ds = (2\tau - 2)\bar{a}e^{-M} + 2\bar{a}e^{-T},$$

or

$$(12)' \qquad e^T = (2\tau - 2)\bar{a}e^{\tau} + 2\bar{a}.$$

As $1 > d\tau/dT > 0$, it follows that as T increases, the range of goods consumed increases and moves to the right,[27] dropping off some of the lower end goods. Since

$$C(T) = e^{\tau} - 1 \qquad \text{and} \qquad C(T + \Delta) = e^{\tau}e^{-\Delta} - 1$$

as T increases, the consumption of the cheapest good, and any good located a distance Δ from that good, increases. Hence, although as T increases some goods are no longer consumed, the consumption of symmetrical substitutes rises. As knowledge improves, utility rises as cheaper production allows an increase in both the variety and the quantity of goods consumed.

27. $dM/dT = 1 - d\tau/dT > 0$.

Autarky Growth Rate and Rate of Technical Progress

Let $X(s,t)$ denote aggregate output of good s at time t. Then

$$\frac{dT(t)}{dt} = \int_{T(t)}^{\infty} L(s,t)ds = \int_{T(t)}^{N(t)} a(s,t)X(s,t)ds = \frac{L(t)}{2}. \tag{13}$$

Thus, the rate of technical progress always equals one half of the labor force. This follows naturally from the symmetrical nature of demand around $T(t)$, which leads to one half of the labor force being allocated to goods in which learning by doing has yet to be exhausted.

We know from Section III that the instantaneous growth rate of GDP per capita, at constant prices, equals

$$\begin{aligned} g(t) &= \frac{-\int_{B}^{\infty} \partial a(s,t)/\partial t X(s,t)\, ds}{L(t)} \\ &= \frac{-\int_{T(t)}^{N(t)} \frac{\partial a(s,t)/\partial t}{a(s,t)} a(s,t)X(s,t)\, ds}{L(t)} \\ &= \frac{2dT(t)/dt \int_{T(t)}^{N(t} a(s,t)X(s,t)\, ds}{L(t)} \\ &= \frac{2(dT(t)/dt)^2}{L(t)} = \frac{L(t)}{2}. \end{aligned} \tag{14}$$

Thus, in autarky the growth rate of real GDP per capita, like the rate of technical progress, is equal to $L(t)/2$. With these benchmarks in mind, we can move to an analysis of the trade equilibrium.

B. Trade Equilibrium

Preferences and Technology

Consider two economies, an LDC and a DC, with the latter denoted by a star. The two economies have constant populations L and L^* ($L >= < L^*$) with identical preferences according to equations (6) and (7) earlier above. In both economies labor is the sole factor of production with unit labor requirements $a(s)$ as in equation (8)′. The key element that distinguishes the DC from the LDC is that when we initially observe these two economies T^* is greater than T. Thus, $a^*(s) \leq a(s)$, with strict inequality for all $s > T$. Learning in each economy proceeds independently according to the learning-by-doing equation (9), above, with no spillovers of knowledge between the two economies, and there is no international borrowing or lending.

Let $X = T^* - T$ be the difference in the levels of technical knowledge. Further, let W be the numeraire and

$$\omega = W^*/W, \qquad p(s) = P(s)/W. \tag{15}$$

Thus, ω is the relative wage and $p(s)$ the goods price in terms of units of LDC labor. Under perfect competition, if the good is produced in the LDC, then $p(s) = a(s)$, whereas if it is produced in the DC, then $p(s) = \omega a^*(s)$. Obviously, each good s will be produced by the least cost producer or, in the case of equal costs, by either or both economies.

General Equilibrium

Under free trade the consumption problem of consumers in the DC and the LDC is analogous to that in autarky with $a(s)$ replaced by $p(s)$. In the absence of international lending, the (implicit) interest rate in each economy will adjust to ensure that consumers maximize their instantaneous utility subject to their instantaneous labor income. At each time t consumers in both the LDC and the DC only consume goods priced at less than $p(M)$ and $p(M^*)$, respectively, with the quantities so consumed given by

$$p(s)C(s) = p(M) - p(s) \qquad p(s)C^*(s) = p(M^*) - p(s). \tag{16}$$

Allowing GDC and $GLDC$ to denote the sets of goods produced in the DC and the LDC, respectively,[28] M is determined by the budget constraint of individuals in the LDC:

$$1 = \int_{s \in GDC} p(s)C(s)ds + \int_{s \in GLDC} p(s)C(s)ds; \tag{17}$$

M^* by the budget constraint of individuals in the DC:

$$\omega = \int_{s \in GDC} p(s)C^*(s)ds + \int_{s \in GLDC} p(s)C^*(s)ds; \tag{18}$$

and the relative wage ω by the trade balance condition:

$$\int_{s \in GDC} Lp(s)C(s)ds = \int_{s \in GLDC} L^*p(s)C^*(s)ds. \tag{19}$$

28. As will later be seen, the sets of goods produced by both the DC and the LDC can reasonably be restricted to zero measure.

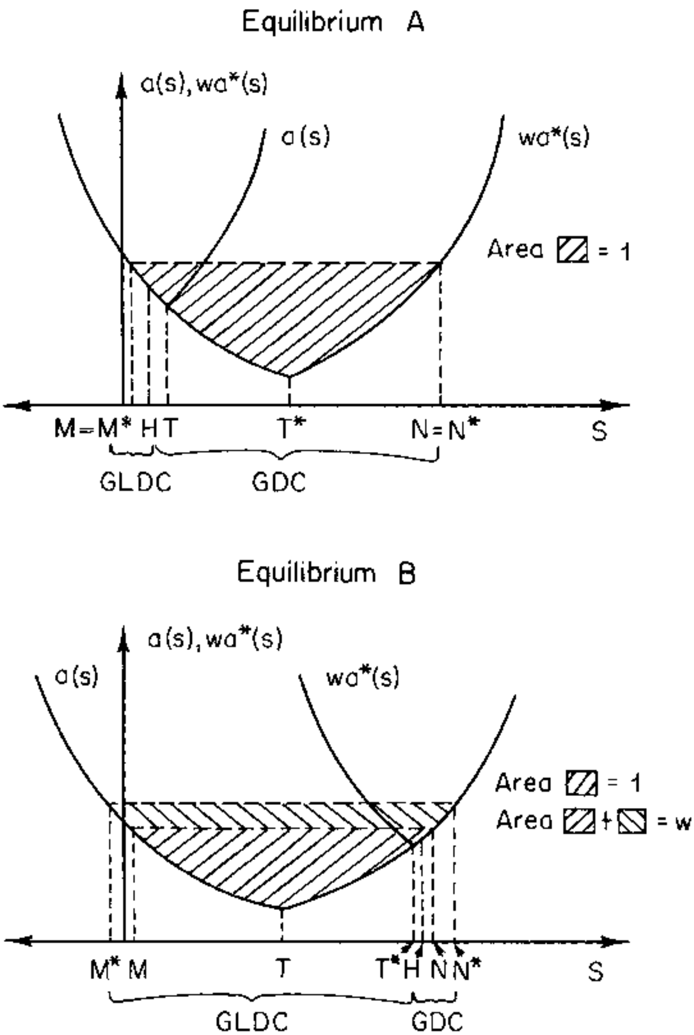
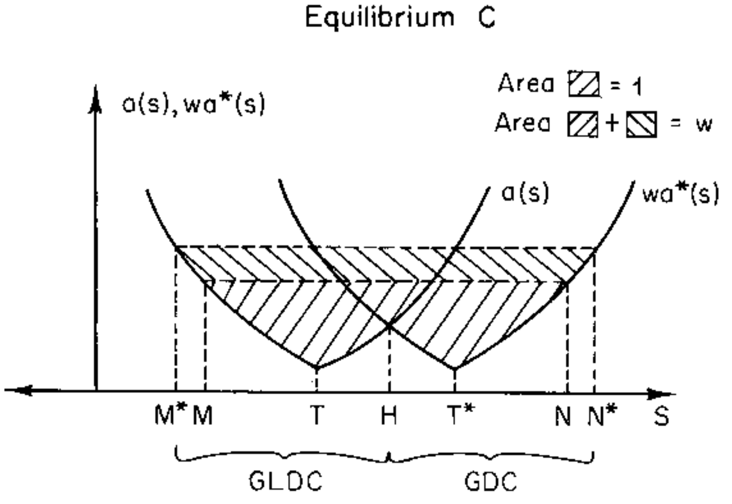


FIGURE III
Typology of Free Trade Equilibria

As Figure III illustrates, under free trade five different types of equilibria may emerge. In equilibrium type A, $\omega = 1$. For $X > 0$, this is the lowest relative wage possible, as otherwise all goods would be cheaper to produce in the DC. In this equilibrium, as $a^*(s) < a(s)$ for all $s > T$, it follows that the LDC produces no goods above T. Goods less than or equal to T might be produced by either



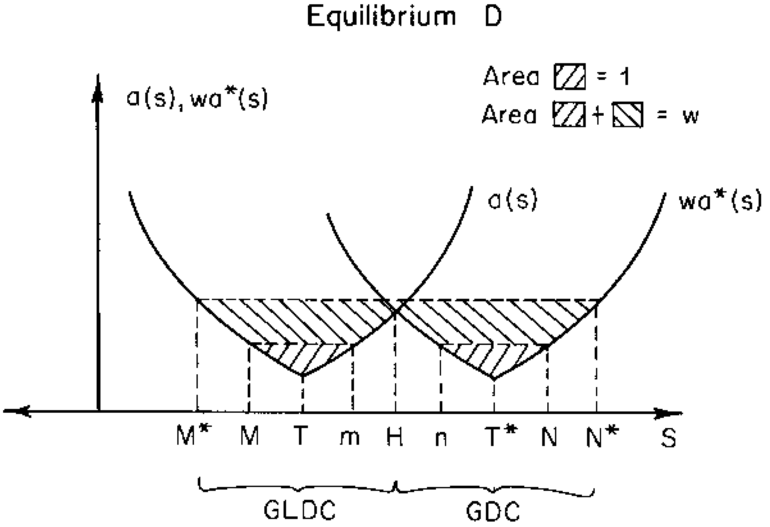


FIGURE III—CONTINUED

or both countries. Without altering any of the important variables in the analysis,[29] we can assume that all consumed goods $s > H$ ($H \leq T$) are produced by the DC and all consumed goods $s \leq H$ by

29. I.e., the utility of consumers in either economy, the relative wage rate, or the rates of technical progress.

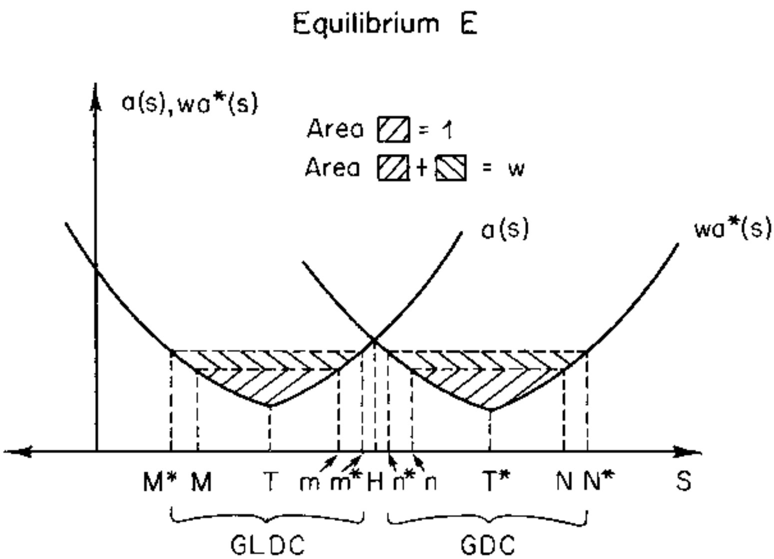


FIGURE III—CONTINUED

the LDC.[30] Given that $a^*(s)$ is monotonically increasing for $s > T^*$, in this equilibrium (as in all of the others in this model) to each $M(M^*)$ there corresponds an $N(N^*)$, which is the upper limit on the technical sophistication of the goods purchased by consumers in the LDC (DC). As the relative wage in equilibrium A is one, the budget constraints of consumers in both countries are identical, and hence $M = M^*$ and $N = N^*$, as illustrated in Figure III.

In equilibrium type B, $\omega = e^{2X}$. This is the highest possible relative wage, as for higher ω all goods are produced more cheaply in the LDC. In equilibrium B the price of producing any goods $s < T^*$ is strictly less in the LDC, and hence no such good is produced in the DC. For goods greater than or equal to T^*, the cost of production is the same in both countries. We assume that all consumed goods $s \leq H$ $(H \geq T^*)$ are produced by the LDC and all consumed goods $s > H$ by the DC. The greater income of individuals in the DC allows them to consume both a greater variety $(N^* > N, M^* < M)$ and a greater quantity[31] of goods.

In equilibria C, D, and E, $e^{2X} > \omega > 1$. The LDC produces all consumed goods $s \leq H$, $(T < H < T^*)$, and the DC produces all

30. This assumption simply eases the discussion later in the paper.
31. $C^*(s) = p(M^*)/p(s) - 1 > p(M)/p(s) - 1 = C(s)$.

consumed goods $s > H$. As $\omega > 1$, in each of these equilibria the greater personal income of individuals in the DC allows them to consume both a greater variety and a greater quantity of goods, as illustrated in the figure. Although in equilibrium C consumers in both economies consume all goods between M and N(M^* and N^*), in equilibrium D, LDC consumers only consume goods in (M,m) and (n,N), with $p(M) = p(N) = p(m) = p(n)$ and $n \geq m$. Goods in $[m,n]$ are too expensive to warrant positive consumption, given the limited income of LDC consumers and the availability of cheaper symmetrical substitutes. In equilibrium E individuals in both economies consume two sets of goods, $\{(M,m),(n,N)\}$ and $\{(M^*,m^*),(n^*,N^*)\}$. For consumers in both economies, the intermediate goods are too expensive to consume given the availability of cheaper symmetric substitutes and the limits on consumption imposed by their budget constraints.

The conditions under which the various types of equilibria apply are illustrated in Figure IV.[32] As shown, the types of equilibria attained depend upon the relative populations of the two economies. In the case where $L^* > L$, for low values of X the free trade equilibrium is equilibrium A. Under these circumstances, given the relatively large population of the DC and its relatively small technical lead over the LDC, the demand for DC products is not sufficient to raise ω above one.[33] For small values of X the highest numbered good produced by the LDC, H, is strictly less than T. As X increases, holding T constant,the world demand for DC products numbered above T increases, leading the DC to transfer labor from goods below T to those above it. Hence, H rises, but ω remains equal to one. When H reaches T, we are at the upper boundary of equilibrium A. Further increases in X, raise the demand for DC products above T and, with no more labor to be transferred from goods below T, raise the DC's relative wage above one. Hence, above the boundary of equilibrium A we move into equilibrium C. If one continues to raise X, for given T, the wage will continue to rise, and one will gradually pass into, first, equilibrium D and, then, equilibrium E. Equilibrium D is reached first since as X increases, with per capita real LDC income (in units of LDC labor) limited to one, consumers in the LDC find themselves unable

32. A formal proof of the statements that follow is available upon request from the author.

33. This result (as in equilibrium B discussed below) is analogous to the case in the standard Ricardian model when the terms of trade are determined by the cost structure of the larger economy.

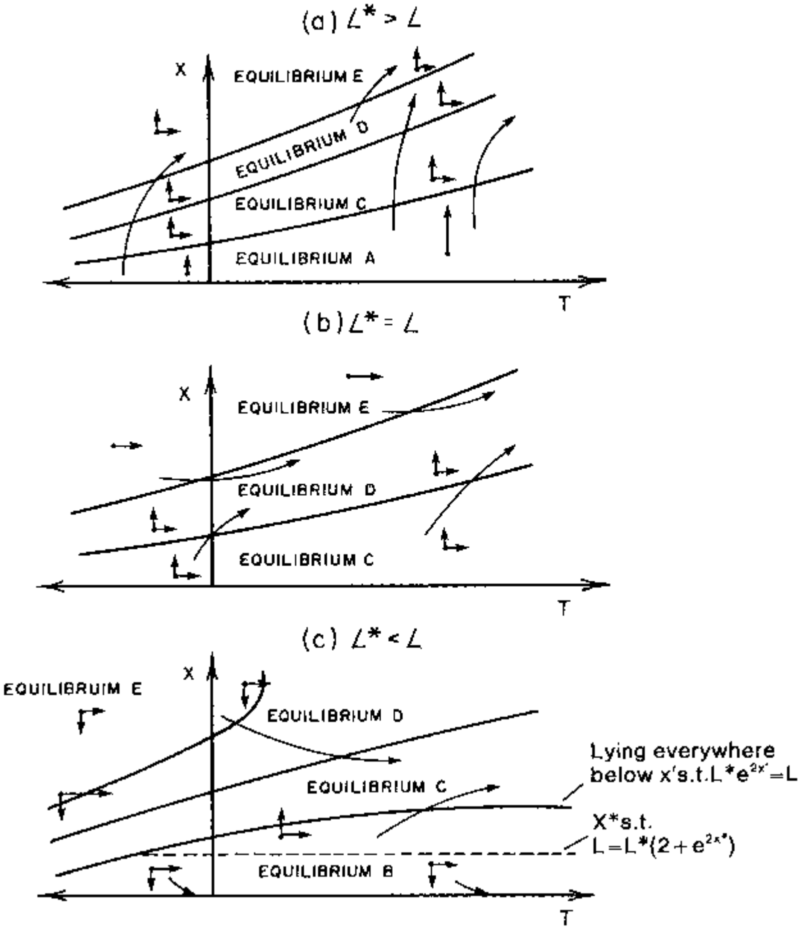


FIGURE IV
Conditions Under Which the Different Equilibria Apply

to consume all of the goods between T and T^*. Further increases in X then lead to equilibrium E as consumers in the DC, despite the increases in their real income due to the rise in ω, ultimately find themselves unable to afford all of the wide variety of goods between T and T^*. The boundaries of equilibria A, C, D, and E are all upward sloping because an increase in T (for fixed X) implies an increase in the goods purchasing power of consumers in both economies, allowing them to consume a wider variety of goods.[34]

34. Recall the effect of an increase in T in the autarky equilibrium.

For the case of $L = L^*$, for any small value of X above T the free trade equilibrium is equilibrium C, with $e^{2X} > \omega > 1$. Given the identical preferences and size of the two populations, even a small technical lead for the DC leads to an immediate rise in its relative wage. Further increases in X, for given T, lead to continued rises in ω and, for the same reasons as in the preceding paragraph, a movement into equilibria D and E. Once again, the boundaries of the different equilibria are upward sloping because an increase in T increases the real income (in units of symmetric goods) of consumers in both economies.

For the case where $L > L^*$, for low values of X the free trade equilibrium is equilibrium B. Under these circumstances, given the relatively small population of the DC the demand for its products, even for a small technical lead, is sufficient to raise its relative wage to the maximum possible. For small X, H is strictly greater than T^*. As, for given T, X increases (implying a rise in T^*), the world demand for LDC products below T^* increases, leading the LDC to transfer labor from goods above T^* to those below T^*. H falls,[35] but the relative wage, equal to e^{2X}, continues to rise. When H reaches T, we are at the upper boundary of equilibrium B. Further increases in X raise the demand for LDC products below T^* and, with no more labor, to be transferred from goods above T^*, lead to a shortage of LDC labor at the relative wage $\omega = e^{2X}$. Hence, although the relative wage continues to rise, it falls below the maximum of e^{2X}, and we move into equilibrium C. If one continues to raise X, for given T, the wage will continue to rise (but remain below e^{2X}), and we then move into equilibrium D. Given the impact of changes in T (for given X) on goods purchasing power, one can guarantee that, moving left from the lower boundary of C and equilibrium D, for some small enough T one reaches the boundary of equilibrium E, with consumers in the DC no longer able to afford all goods between T^* and T. It is not possible, however, to guarantee that moving upwards (increasing X) from the lower boundary of C and equilibrium D, one will reach equilibrium E. Given the relatively small population of the DC ($L^* < L$), increases in the level of technical knowledge of DC workers lead to fairly large rises in their relative wage, creating the possibility that they will be able to consume all of the ever increasing

35. H actually falls, as opposed to H–T^* decreasing with H constant or rising, because the greater DC income leads to a rise in DC demand for all LDC goods below the pre-dX T^* (including some low-end goods below T for which demand was previously zero).

range of products between T and T^*. Thus, I am not able to rule out the possibility that the lower boundary of equilibrium E is asymptotic to a vertical line or is backward bending, as drawn in the figure.

The intuition as to why equilibrium A holds only when $L^* > L$ and equilibrium B only when $L < L^*$ is simple. In equilibrium A the DC produces all consumed goods above T^* and some consumed goods below T, implying, given the symmetry of world demand around T^*, that more than half of world income (equal to $L^* + L$) is spent on DC products. But, as the income of DC consumers equals L^*, this in turn implies that $L^* > (L^* + L)/2$; i.e., that the DC has more than half of the world's population. In equilibrium B the LDC produces all consumed goods below T and some consumed goods above T, implying, given the symmetry of world demand around T, that more than half of world income (equal to $\omega L^* + L$) is spent on LDC products. But, as the income of LDC consumers equals L, it follows that $L > (\omega L^* + L)/2$; i.e., that the LDC has more than half of the world's population.

It should be noted that whereas it is always the case in equilibria C, D, and E that an increase in X (i.e., an increase in T^* for given T) raises the relative wage of DC workers, the effect of an increase in T (holding X constant) is more ambiguous.[36] As mentioned earlier, an increase in T (for given X) amounts to an increase in the goods purchasing power of consumers in both economies and, in this model, will raise the relative wage of workers in the economy in which less than half of world income is spent.[37] Thus, if $L^* \geq L$, it is always the case that an increase in T (for given X) lowers the relative wage of the DC. If $L^* < L$, it can be shown that there exists a horizontal line, at the point where $L^* e^X = L$, above which (below which) more (less) than half of world income is spent on DC products.[38] Above (below) this line, an increase in T (for given X) lowers (raises) the relative wage of DC workers.

Pausing to examine the realism of these results, we see that the DC always produces a higher numbered set of goods than the LDC, which accords well with empirical experience; i.e., more

36. An increase in T (holding T^* constant but allowing X to vary) always lowers the DC's relative wage.

37. This is because the income elasticity of demand for each good is inversely related to current expenditure on that good (note how the richer DC consumers consume a wider variety of goods than the LDC consumers do).

38. Along that line $\omega = e^X$. The above result then follows from the fact that $\partial\omega(X, T)/\partial X > 0$.

technically advanced economies produce more technically sophisticated goods. The greater personal income of individuals in the DC, in cases where $\omega > 1$, allows them to consume both more of each good and a greater variety of goods. Consumers in the DC enjoy frontier DC goods, e.g., personal computers and advanced medical services, which are too expensive for LDC consumers. In equilibria D and E, DC consumers also consume low-end DC goods and high-end LDC goods which are too expensive for the average LDC consumer, e.g., Italian and Brazilian shoes. Finally, DC consumers enjoy labor-intensive primitive LDC products which are no longer consumed by the LDC workers themselves. While this is something of an artifact of the model, examples, such as handicraft goods and labor-intensive tourist services, do come to mind.

With the different types of equilibria in mind, we now turn to an analysis of the evolution of T and T^*, thereby establishing the dynamics of X and T in Figure IV. Using this information subsequent sections will examine the effect of trade on the growth rates of the two economies and the intertemporal utility of their respective populations.

Technical Progress and Equilibrium Dynamics

As specified earlier, learning by doing in both economies follows a process identical to that discussed under our autarky example:

$$\frac{dT}{dt} = \int_{T}^{\infty} L(s)ds \qquad \frac{dT^*}{dt} = \int_{T^*}^{\infty} L^*(s)ds, \tag{20}$$

where $L(s)$ and $L^*(s)$ are the labor allocated to the production of good s in the LDC and the DC, respectively. In equilibrium A, given that the LDC produces no goods above T, the relative wage equals one, and world demand is symmetric around T^* (recall Figure III earlier above), it follows that

$$\frac{dT}{dt}(A) = 0 \qquad \frac{dT^*}{dt}(A) = \frac{L^* + L}{2}. \tag{21}$$

In equilibrium B, as the DC produces no goods below T^* and world demand is symmetric around T, it follows that[39]

39. The fact that the amount spent on goods below T in this equilibrium $(L/2 + \omega L^*/2)$ is less than total LDC income (L) ensures that $(L - \omega L^*)/2 > 0$.

$$\frac{dT}{dt}(B) = \frac{L - \omega L^*}{2} \qquad \frac{dT^*}{dt}(B) = L^*. \tag{22}$$

In the case of equilibrium E, as expenditure on each country's goods is distributed symmetrically around its current level of technical sophistication:

$$\frac{dT}{dt}(E) = \frac{L}{2} \qquad \frac{dT^*}{dt}(E) = \frac{L^*}{2}. \tag{23}$$

Turning to equilibria C and D, examining Figure III, it is readily apparent that, since both countries produce sets of learning-by-doing goods of positive measure, both dT/dt and dT^*/dt are strictly greater than zero. As more than one half of total expenditure on LDC goods falls on goods in which learning by doing has been exhausted, while more than one half of total expenditure on DC goods falls on goods that are still experiencing learning by doing, it follows that

$$\frac{L}{2} > \frac{dT}{dt}(C,D) > 0 \qquad \frac{dT^*}{dt}(C,D) > \frac{L^*}{2}. \tag{24}$$

To summarize our results, under free trade $L/2 \geq dT/dt \geq 0$, while $dT^*/dt \geq L^*/2$. Thus, in terms of technical progess, the DC experiences dynamic gains from trade, while the LDC experiences dynamic losses. Under free trade the keenest area of competition between the LDC and the DC is in the most advanced LDC goods (those in which it is still experiencing learning by doing) and the least advanced DC goods (those in which it has exhausted learning by doing). This competitive interaction has an asymmetric effect, driving DC labor out of the DC's low numbered industries into industries in which it is still experiencing learning by doing and, simultaneously, forcing LDC labor out of high numbered LDC industries into industries in which it has already exhausted learning by doing. Thus, the LDC's comparative advantage, while statically optimal, has detrimental effects on its rate of technical progress. Under free trade the DC experiences faster technical progress, at the expense of the LDC.

Using the above information, we can examine the dynamics of X, the difference between the levels of technical knowledge of the two economies. If $L^* > L$, since $dX/dt = dT^*/dt - dT/dt$, while $dT/dt \leq L/2$ and $dT^*/dt \geq L^*/2$, it follows that dX/dt is strictly positive in all of the equilibria. This is illustrated in Figure IVa. The larger population of the DC, coupled with its initial technical superiority, leads to a growing technical gap between the two economies.[40] In the case of $L^* = L$, as $dT/dt < L/2$ and $dT^*/dt > L^*/2$ everywhere below equilibrium E, it follows that in those regions dX/dt is always strictly positive. Within equilibrium E, $dX/dt = 0$. However, since the boundary of equilibrium E is upward sloping and within equilibrium E $dT/dt > 0$, it follows that the economies will follow paths similar to those outlined in Figure IVb, with the technical difference between the two economies growing larger and larger. Whenever the growth of X slows, the continued growth of T forces down the relative wage, accelerating technical progress in the DC and reducing learning by doing in the LDC, leading to further rises in X. Thus, if two economies are identical in all respects, but one, say as the result of temporary government subsidies, acquires a miniscule technical advantage, then it will pull farther and farther away from the other economy. The costs of a temporary shortfall can be permanent.

Tuning to the case where $L > L^*$, Figure IVc, in equilibrium E $dX/dt = L^*/2 - L/2 < 0$. Within equilibrium B, $dX/dt = L^*(2 + \omega)/2 - L/2$. As $\omega = e^{2X}$, it follows that if the LDC's population is several orders of magnitude greater than that of the DC, then there exists a horizontal line in equilibrium B above (below) which dX/dt is greater than (less than) zero.[41] Thus, if the initial difference between the two economies is small enough and the LDC's population is large enough, the LDC can draw back the DC and overtake it, as illustrated in Figure IVc. However, if the initial technical difference between the two economies is sufficiently large, while (in cases where the initial equilibrium lies somewhat above equilibrium B) the LDC might be able to reduce the technical gap between itself and the DC, it will not be able to eliminate it altogether (no matter how much larger its population).

Growth of GDP per Capita

Given their constant populations in this example, the growth rates (at constant prices) of LDC and DC GDP per capita, $g(t)$ and

40. The dynamic paths drawn in Figure IVa are intended to be suggestive. In general, whether or not any particular path crosses the boundaries between equilibria C, D, and E will depend upon the parameter values. What is certain, however, is that everywhere in equilibria C, D, and E, $dX/dt > 0$ and $dT/dt > 0$. A similar disclaimer applies to the dynamic paths shown crossing the boundary between equilibria C and D in Figures IVb and IVc.

41. Otherwise, everywhere in equilibrium B $dX/dt > 0$.

$g^*(t)$, respectively, are

$$(25)\quad g(t) = \frac{\int_B^\infty p(s,t)\partial X(s,t)/\partial t\, ds}{\int_B^\infty p(s,t)X(s,t)\, ds} \quad g^*(t) = \frac{\int_B^\infty p(s,t)\partial X^*(s,t)/\partial t\, ds}{\int_B^\infty p(s,t)X^*(s,t)\, ds},$$

where $X(s,t)$ and $X^*(s,t)$ are aggregate output of good s at time t in the LDC and the DC, respectively. As $\int_B^\infty p(s,t)X(s,t) = L$ and $\int_B^\infty p(s,t)X^*(s,t) = \omega(t)L^*$ at all times t, it is easily verified that

$$(26)\qquad g(t) = \frac{2(dT/dt)^2}{L} \qquad g^*(t) = \frac{2(dT^*/dt)^2}{L^*}.$$

It follows, using our results in the previous section, that $g(t) \leq L/2$ and $g^*(t) \geq L^*/2$, with strict equality everywhere outside of equilibrium E. Thus, relative to autarky, free trade increases the growth rate of the DC and lowers that of the LDC.

Intertemporal Welfare

In this subsection I examine the effect of international trade on the intertemporal utility enjoyed by consumers in the LDC and the DC over the time horizon $[0,\infty)$. To that end, let $V_{FT}(T_{FT}(t), T^*_{FT}(t))$, and $V_A(T_A(t))$ denote the instantaneous utility enjoyed at time t by LDC consumers under free trade and in autarky, with $V^*_{FT}(T_{FT}(t),T^*_{FT}(t))$ and $V^*_A(T^*_A(t))$ connoting the same for DC consumers, and the subscripts FT and A highlighting the fact that the time paths of T and T^* depend upon the trade regime followed. I wish to determine under which conditions, on $T(0),T^*(0)$, L^*,L and ρ, will consumers in each country enjoy higher intertemporal utility under free trade, and under which conditions will they be better off in autarky.

Although, given the static gains from trade, it is always the case that for any T and T^*;

$$(27)\qquad V_{FT}(T,T^*) \geq V_A(T) \qquad V^*_{FT}(T,T^*) \geq V^*_A(T^*),$$

more general analytical comparisons between V_{FT} and V_A (V^*_{FT} and V^*_A) are impossible. The utilities enjoyed by consumers under free trade and in autarky are functions of different variables which are themselves different implicit functions of the parameters T and T^*.[42] Therefore, although the instantaneous free trade and autarky

42. Thus, $V_A(T) = \tau_A^2$, where τ_A is the range of goods on each side of T that are consumed under autarky and is determined implicitly by equation (12)′ earlier

utilities are well defined, and could be computed for specified values of the various parameters, in general, for differing values of T_{FT}, T_A, T^*_{FT} and T^*_A it is impossible to make meaningful analytical comparisons of their relative magnitudes. Nevertheless, it is still possible to analyze the effect of trade on the intertemporal utility of consumers in the two economies using (27), as well as the following, easily proved results:[43]

$$\text{(28)} \quad \frac{\partial V_{FT}(T,T^*)}{\partial T} \geq 0, \quad \left.\frac{\partial V_{FT}(T,T^*)}{\partial T^*}\right|_{L^* \geq L} > 0, \quad \frac{dV_A}{dT} > 0, \quad \frac{\partial V^*_{FT}(T,T^*)}{\partial T^*} > 0, \quad \frac{dV^*_A}{dT^*} > 0\,,$$

i.e., the utility of consumers in both economies under free trade and in autarky in increasing in their own level of technical progress and, provided that $L^* \geq L$, the utility of LDC consumers under free trade is strictly increasing in the DC's level of technical progress.[44]

Since the DC's rate of technical progress under free trade is greater than or equal to that experienced under autarky, we know that as long as it remains the DC, i.e., as long as it is not overtaken by the LDC, $T^*_{FT}(t) \geq T^*_A(t)$. Under these circumstances, $V^*_{FT}(T_{FT}(t), T^*_{FT}(t)) \geq V^*_{FT}(T_{FT}(t), T^*_A(t)) \geq V^*_A(T^*_A(t))$ (with at least one inequality being strict for all $t > 0$),[45] i.e., the instantaneous utility enjoyed by the DC at each time t under free trade is greater than that enjoyed at the same time along the comparable autarky path. It follows that the intertemporal welfare of DC consumers is unambiguously improved. This is not surprising, since free trade increases the rate of technical progress in the DC and at the same time allows DC consumers to enjoy the usual static gains from

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above. In contrast, in equilibria C, D, and E $V_{FT}(T, T^*) = (X + \tau_{FT} - \ln(\omega))^2 + \tau^2_{FT} - (\tau_{FT} - \eta_{FT})^2$, where $X = T^* - T$, ω is the DC's relative wage, and τ_{FT} and η_{FT} are the distance from T to the lowest and highest LDC goods purchased by LDC consumers under free trade, respectively, with ω, τ_{FT} and η_{FT} all determined implicitly by T and T^* using equations (17)–(19) earlier above. The same equations, with τ^*_A, τ^*_{FT} and η^*_{FT}, describe V^*_A and V^*_{FT}.

43. $\partial V_{FT}/\partial T$ equals zero only in equilibrium A.

44. It is notable that $\partial V^*_{FT}(T, T^*)/\partial T$ may be negative, as is the case in equilibrium B. If $L > L^*$, although it can be shown that in equilibrium E, as well as above the horizontal line given by $L^*e^X = L$ (see the discussion in the "General Equilibrium" section earlier above), $\partial V_{FT}/\partial T^* > 0$, below that horizontal line, and outside equilibrium E $\partial V_{FT}/\partial T^*$ may be less than or equal to zero, as is the case in equilibrium B.

45. The first inequality is strict outside of equilibrium E and the second outside of equilibrium A.

trade. As discussed earlier above, in equilibrium B it is possible that the LDC may overtake the DC, in which case the DC would become the LDC in equilibrium A, moving subsequently into equilibrium C. We shall consider this case after examining the intertemporal utility enjoyed by LDC consumers.

If the free trade path begins in either equilibrium A or E, it is easily proved that as long as the world economy remains in those equilibria LDC consumers will, at each moment in time, enjoy higher instantaneous utilities along the free trade path.[46] It can also be shown that whenever the free trade path is in equilibrium B, LDC consumers will, at each moment, experience higher instantaneous utilities along the autarky path.[47] For the case of equilibria C and D, as well as equilibrium E entered after time 0, the analysis is complicated by the fact that while $T_{FT}(t) \leq T_A(t)$, which tends to reduce LDC utility, it is also the case that $T^*_{FT}(t) \geq T^*_A(t)$, which tends to raise LDC utility. Thus, for particular values of $T(0)$, $T^*(0)$, L^*, L, and ρ, it is not possible to establish definitively whether over the entire time horizon $[0, \infty)$ LDC consumers enjoy higher utility under free trade or in autarky.

Nevertheless, it is possible to establish some general propositions on the conditions under which free trade makes LDC consumers better or worse off. First, consider the case where $L^* > L$. Since $T^*_{FT}(t) \geq T^*(0) + tL^*/2$ and $T_{FT}(t) \geq T(0)$, it follows that $V_{FT}(T_{FT}(t), T^*_{FT}(t)) \geq V_{FT}(T(0), T^*(0) + tL^*/2)$. Thus, $V_{FT}(T(0), T^*(0) + tL^*/2)$ constitutes a lower bound on the utility enjoyed at time t along the free trade path. An increase in L^* will increase the value of this lower bound (at each time t) for two reasons. First, for given T and T^* an increase in L^* lowers ω,[48] the DC's relative wage, which increases the utility enjoyed under free trade by LDC consumers. Second, an increase in L^* raises the value of $T^*(0) + tL^*/2$ at each time t, which also enhances welfare in the LDC. Since these potential increases in $V_{FT}(T(0), T^*(0) + tL^*/2)$ are unbounded in L^*, it follows that, for any initial value of $T(0)$ and

46. In the case of equilibrium A since $dT^*_{FT(A)}/dt = L^*/2 + L/2$, it is apparent that $T^*_{FT}(t) > T_A(t)$ for as long as the world economy remains in that equilibrium. A cursory examination of Figure III confirms that $V_{FT(A)}(T_{FT}(t), T^*_{FT}(t)) = V_A(T^*_{FT}(t)) > V_A(T_A(t))$. As for when the free trade path begins in equilibrium E, as long as the two economies remain in that equilibrium, $T_{FT}(t) = T_A(t)$ and $T^*_{FT}(t) = T^*_A(t)$. It follows that $V_{FT(E)}(T_{FT}(t), T^*_{FT}) = V_{FT(E)}(T_A(t), T^*_A(t)) > V_A(T_A(t))$.

47. Regardless of whether the world economy is initially (at time 0) in equilibrium B or enters it subsequently (from equilibrium C), we know that while in this equilibrium $T_{FT}(t) < T_A(t)$ (for all $t > 0$). It follows that $V_{FT(B)}(T_{FT}(t), T^*_{FT}(t)) = V_A(T_{FT}(t)) < V_A(T_A(t))$.

48. Unless ω already equals its minimum value of one.

$T^*(0)$, one can always find some sufficiently large L^* such that the intertemporal utility enjoyed under free trade dominates that experienced in autarky. The basic intuition behind this result is that, with L^* much greater than L, under free trade the LDC enjoys the benefits of latching on to a gigantic engine of technical progress.

Second, consider the case where $L > L^*$, but the LDC is unable to overtake the DC. In this case, the LDC will find itself participating in a world economy in which it trails a leader (the DC) whose fastest conceivable rate of technical progress is bounded from above by L^*. Now, whenever $\omega = 1$, the instantaneous utility enjoyed by LDC consumers under free trade equals the utility they would enjoy in autarky if $T_A(t) = T^*_{FT}(t)$.[49] For the case we are considering, $L > L^*$, ω is greater than one and, since the utility of LDC consumers is decreasing in ω, it follows that $V_{FT}(T_{FT}(t), T^*_{FT}(t)) < V_A(T^*_{FT}(t)) \leq V_A(T^*(0) + tL^*)$. Hence, the instantaneous utility enjoyed by LDC consumers under free trade is bounded from above by $V_A(T^*(0) + tL^*)$. In contrast, under the autarky path at each time t, LDC consumers enjoy $V_A(T(0) + tL/2)$. Although, due to the static gains from trade, LDC consumers may initially enjoy higher instantaneous utility along the free trade path,[50] if L is more than twice as large as L^*, there will come a time t' when $T(0) + t'L/2 = T^*(0) + t'L^*$, after which the utility experienced under autarky will be greater than the upper bound on the utility enjoyed under free trade. As L–L^* increases, t' becomes smaller and the gap (at any time t after t') between $V_A(T(0) + tL/2)$ and $V_A(T^*(0) + tL^*)$ grows without bound. It is not hard to see that, given $L > 2L^*$, for any set of initial values of $T(0)$ and $T^*(0)$, there exists some large enough L (relative to L^*) or small enough rate of time discount ρ such that the LDC does better under autarky, losing the static gains from trade, but enjoying a more rapid rate of technical progress.

Finally, consider the case in which the LDC's population is much greater than the DC's, the world economy begins in equilibrium B, and the LDC is able to catch up with the DC. Recall from Figure IVc that, for this overtaking to occur, the initial difference between the two economies, $X(0)$, must be less than X^*, where X^* is such that $L = 2L^* + L^*e^{2X^*}$. It is easily shown that as $X(0)$ goes to X^*, the time spent in the catch-up path goes to infinity

49. Recall Figure III, equilibrium A.
50. As is the case outside of equilibrium B.

and the value of ω at each point in time along that path rises.[51] Since $V_{FT(B)} = V_A(T_{FT}(t)) = V_A(T(0) + tL/2 - \int_0^t \omega(s)L^*/2ds)$, while $V_A(T_A(t)) = V_A(T(0) + tL/2)$, it follows that both the duration and the magnitude of the losses experienced by LDC consumers along the catch-up path are rising in $X(0)$. After the LDC overtakes the DC and enters equilibrium A (as the new DC), it will begin to enjoy a more rapid rate of technical progress and may, eventually, enjoy higher instantaneous utilities than along the autarky path. Nevertheless, it is clear that for large enough ρ or $X(0)$, free trade will unambiguously lower the intertemporal utility of consumers who reside in the economy which at time 0 is the LDC in equilibrium B. If the initial equilibrium is C, D, or E, and the world economy evolves into equilibrium B, with the LDC overtaking the DC, the analysis is complicated by the fact that the LDC initially enjoys higher instantaneous utilities along the free trade path (due to the static gains from trade). In this case, for low enough ρ and a long enough time spent along the catch-up path, free trade will unambiguously make LDC consumers worse off.

As for the welfare of DC consumers in the case where the LDC overtakes the DC, in both equilibrium B and equilibrium A (in their new role as LDC consumers) the (former) DC consumers will enjoy higher instantaneous utilities along the free trade path, with the difference between V^*_{FT} and V^*_A increasing with the time spent in these equilibria. Thus, although the utility experienced by the ex-DC consumers as the world economy evolves into equilibrium C is difficult to analyze, it is possible to assert that for large enough ρ or $X(0)$, free trade will unambiguously improve the intertemporal utility of consumers who reside in the economy which at time 0 is the DC.

To summarize, free trade does the following: (1) it unambiguously improves the intertemporal utility of DC consumers if the DC is never overtaken by the LDC; (2) it will improve the intertemporal utility of DC consumers even if the DC is overtaken by the LDC,

51. Recall that in equilibrium B $\omega = e^{2X}$ and $dX/dt = L^*(2 + \omega)/2 - L/2$. From this it follows that

$$\omega = e^{2X} = \frac{(L - 2L^*)e^{(2L^* - L)t}}{L^* e^{(2L^* - L)t} + (L - 2L^*)e^{-2X(0)} - L^*},$$

and the time S when $X(S) = 0$, is given by

$$e^{(L - 2L^*)S} = \frac{L - 3L^*}{(L - 2L^*)e^{-2X(0)} - L^*}.$$

provided that either ρ or $X(0)$, and hence the time spent along the catch-up path, is sufficiently large; (3) it will unambiguously improve the intertemporal utility of LDC consumers if the LDC's population is sufficiently small relative to that of the DC; (4) it will unambiguously reduce the intertemporal utility of LDC consumers if the LDC's population is sufficiently large relative to that of the DC and it is unable to overtake the latter economy; and (5) for the case where the LDC is able to overtake the DC, it will (a) unambiguously reduce the utility of LDC consumers if either their rate of time preference or $X(0)$ is sufficiently large and the initial equilibrium is equilibrium B and (b) unambiguously reduce the utility of LDC consumers if their rate of time preference is sufficiently small, the initial equilibrium is C, D, or E, and the time spent along the catch-up path is sufficiently large.

C. Summary

This section has used a specific functional form of the model presented in Section III to explore the dynamic effects of international trade. The key simplifying assumption chosen is that all goods in which learning by doing has yet to be exhausted contribute symmetrically to productivity increases in all other such goods, which, given a monotonically increasing cost curve for these same goods, ensures that learning by doing is exhausted sequentially in goods. The results indicate that under free trade the LDC (DC) experiences rates of technical progress less than or equal (greater than or equal) to those enjoyed under autarky. If the DC's working population is greater than or equal to that of the LDC, the technical gap between the two economies will increase without bound. Furthermore, even if the LDC has a population several orders of magnitude greater than that of the DC, if the initial technical gap between the two economies is large enough the LDC will never catch up with its trading partner. Thus, to a certain degree the model is characterized by knife-edge results. If an economy, say as the result of short-term government subsidies, acquires a slight competitive edge over an equally large trading partner, any subsequent free trade interaction will give it a permanent and increasing absolute technical advantage. Although these results are no doubt somewhat dependent upon the particular functional form chosen for our analysis, intuitively it would seem likely that they would tend to be preserved in any functional form in which the learning-by-doing spillover effects are not heavily biased in favor of any particular set of goods, and learning by doing tends to be exhausted sequentially in goods.

With respect to growth rates the model of this paper suggests that free trade will tend to raise the rate of GDP growth of the DC and lower that of the LDC. To the extent that bounded learning by doing with spillovers across goods, but not across nations, is an important source of growth, it would seem that trade does not accelerate the growth of national income in all trading economies, as the proponents of the positive growth gains from trade tend to assert. Naturally, different results might emerge from an analysis of the impact of trade on factors such as R&D, the acquisition of human capital, and international spillovers of knowledge.

Finally, with respect to intertemporal welfare the results of this model are somewhat ambiguous. Since the DC enjoys both the static gains from trade and an accelerated rate of technical progress, as long as the DC economy maintains its technical lead, i.e., continues to play the role of the DC, its consumers will enjoy an unambiguous increase in intertemporal utility. As for the LDC, although it suffers a decrease in its rate of technical progress, its consumers benefit from the traditional static gains from trade, which increases as the DC experiences technical progress. As these two effects work in opposite directions, the effect of trade on the intertemporal utility of LDC consumers is ambiguous. In general, the LDC is more likely to experience an improvement (reduction) in intertemporal welfare if its population is much smaller (larger) than that of the DC.

V. Some Final Thoughts

The stylized model of this paper clearly contains many simplifying assumptions and consequent weaknesses. Not the least of these is that it takes as given the development of new goods and processes whose productive potential is then exploited by learning by doing. A more sophisticated model would capture the effect of trade on both research and development and learning by doing, perhaps modeling the interaction between the two effects. The impact of trade on technical progress, growth rates, and by extension, intertemporal welfare might also emerge from a number of other directions, such as via the effect of trade on the international diffusion of knowledge. Furthermore, whereas this paper has focused only on the dichotomy between autarky and free trade, the analysis of more common deviations from free trade, such as tariffs and quotas, including the endogenous effects of variables such as rent and tariff seeking on technical progress, should be quite interesting. This paper has provided an analysis of the impact of the movement from autarky to trade on one particular factor of importance to growth. The relationship between trade policies and growth rates obviously remains an open, exciting, and empirically relevant question.

The basic structure of the results of the model in this paper, i.e., that the LDC (DC) always experiences rates of technical progress and growth less than or equal (greater than or equal) to those enjoyed under autarky, suggests an interesting extension to a three-country world composed of an LDC, a DC, and a NIC (located technically between the LDC and the DC). One possible result, suggested, but not confirmed, by this paper, would be that if a set of countries with small populations (the NICs) enter the free trade era (say post-World War II) with a slight technical lead over the LDCs they will be driven into concentrating all of their production in goods in which they experience rapid learning by doing, while the LDCs wait in industries in which they have already exhausted learning by doing. The NICs experience extreme product concentration and rapid structural change as they progress up the product ladder. The LDCs have to wait until the NICs move out of lower industries to enter them and experience more rapid technical progress and structural change. As the NICs close in on the DCs, their rates of technical progress and GDP growth would tend to slow. To a certain degree, this simple story characterizes the postwar empirical experience of the East Asian NICs and the more recent accelerated growth of manufacturing in countries such as Thailand and the Philippines, which only occurred after the NICs had begun to move out of the low-end light industrial products. Learning by doing is most certainly not the only source of growth. Nevertheless, the model in this paper, and its multicountry extension, provides a simple means of modeling the hierarchy and movement of countries in the international ladder of production.

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