Price Dispersion and International Relative Price Volatility

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ABSTRACT

A central puzzle in international economics is that international relative prices, at the aggregate and disaggregate level, are volatile and persistent. Existing models based on perfect competition, monopolistic competition with constant elasticity demand, or sticky prices are unable to generate these relative price movements nor can they explain evidence of price dispersion of identical products within countries. This paper presents a model with search frictions to explain price dispersion. Introducing price dispersion into an otherwise standard two-country, two-good flexible price model substantially increases the volatility of the terms of trade and real exchange rate. With high price dispersion and risk aversion, the model accounts for 90 percent of the volatility in the terms of trade and all of the volatility in the real exchange rate. The model also accounts for 80 percent of the persistence in international relative prices. Moreover, the model breaks the tight link between the real exchange rate and the ratio of consumption across countries common to models with complete international risk sharing.

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1. Introduction

A central puzzle in international economics is that international relative prices are volatile and persistent. These relative price fluctuations are generally attributed to nominal rigidities interacting with monetary shocks. This paper proposes an alternative explanation in which technology shocks are the driving force. The key element is a novel price setting mechanism that leads to dispersion in the price of identical products within countries. With price dispersion, the model generates international relative price fluctuations that are quite close to the data.

In an otherwise standard two country, two good flexible price model, introducing price dispersion substantially increases the volatility of international relative prices. With high risk aversion, the model accounts for over 90% of the volatility international relative prices. With persistent productivity shocks, the model accounts for 80% of the persistence of international relative prices. In contrast, for a sticky price model to generate the same amount of persistence, firms would need to set their prices for two years at a time. We achieve these results because our mechanism leading to price dispersion breaks the tight link between the ratio of consumption across countries and the real exchange rate common to models with complete international risk sharing.

There is overwhelming evidence supporting our emphasis on a price setting mechanism generating price dispersion within countries. A brief stroll around the mall or quick search on the Internet turns up similar, if not identical, goods selling for vastly different prices. Beginning with the work of Stigler (1961) and Stigler and Kindahl (1970), numerous papers document substantial and persistent price dispersion for a variety of goods\(^2\). We present additional evidence of price dispersion for a broad range of goods in the U.S. Using a sample of over 10,000 highly disaggregated goods we find on average the highest transaction price of a good is over 3 times its mean price. The theoretical literature attributes price dispersion to imperfect information of prices and costly search frictions\(^3\). We primarily focus on this source of price dispersion rather than dispersion in the cost of differentiated goods. For our purposes, we extend the price dispersion model of Burdett and Judd (1983) to a general equilibrium environment. In this model, search influences the elasticity of substitution between firms and leads to a new form of monopolistic competition with non constant elasticity of demand. It also implies, as Diamond (1971) notes, that price determination may be counter-intuitive and lead some firms to charge the consumer’s reservation price even for small

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search cost. This is an attractive feature of the model.

The two features of the model that lead to price dispersion are 1) the need for consumers to actively search to purchase consumption goods and 2) the nature of search is noisy. In regard to the first feature, search takes time so its opportunity cost is in terms of foregone labor income\(^4\). This links the highest, or reservation, price a consumer is willing to pay for a good with the local wage rate. Search is also noisy, as in Burdett and Judd (1983), in that some consumers get one price quote while other consumers get multiple price quotes. The idiosyncratic elements of search make consumers appear heterogenous to firms. From the firm’s perspective, this leads to a trade-off between charging a high price and attracting a few consumers and charging a low price and attracting more consumers. In equilibrium, this leads to a distribution of prices with a finite support in each country where the reservation price determines the upper limit of this distribution. The distribution of prices will differ across countries when the opportunity cost of search differs. Changes in productivity that shift the international relative wage shift the entire distribution of prices in the home country relative to that in the foreign country. Parsley and Wei (2001) find evidence of precisely these types of movements in international relative prices.

Introducing price dispersion alters the traditional link between quantities and prices by introducing another margin on which consumers can change their allocations. With price dispersion and search, a consumer can increase consumption either by accepting higher prices or by spending more time searching. More time searching means less time working and lower labor income. At the margin, a consumer is indifferent between accepting a good at his reservation price and further search. By engaging in additional search, the consumer gives up some labor income and expects to purchase the good at the average price in the market. If a consumer is willing to purchase a good at the reservation price, then it is this price, and not the average price, that matters at the margin. In this respect, consumers equate the marginal rate of substitution between two goods to the ratio of reservation prices for these goods, and not average prices. Similarly, the international risk sharing condition equates the ratio of the marginal utility of consumption to the ratio of reservation prices across countries and not the real exchange rate.

Most of the fluctuations in aggregate international relative prices, like the real exchange rate, can be attributed to deviations from the law of one price in traded goods across countries.\(^5\) This

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\(^4\)Search is a very specific form of household production. In this respect, the paper follows Benhabib, Rogerson and Wright (1991) and Greenwood and Hercowitz (1991) in studying the role of household production for properties of the business cycle.

\(^5\)See the work of Engel (1993 and 1999).
has lead a number of researchers (such as Betts and Devereux (2000), Bergin and Feenstra (2001), and Chari, Kehoe, and McGrattan (2001)) to concentrate on models in which deviations from the law of one price result from sticky nominal prices in local currencies and nominal exchange rate shocks\(^6\). However, there is evidence these deviations from the law of one price result from firms segmenting markets internationally and price discriminating across countries\(^7\). Krugman (1987) calls this *pricing to market*, and attributes it to firms facing different local market conditions in each market they serve. Unlike sticky price models, in my price dispersion model, firms *price to market* because they face consumers with different opportunity costs of search across countries. Over time, changes in productivity interact with the search frictions to change the opportunity cost of search in each country. At the aggregate level, this leads to markups that vary endogenously across countries and goods over time.

Many researchers have studied the properties of international relative prices in theoretical models. This literature can be divided into three strands. First, there are the flexible price models. Backus, Kehoe and Kydland (1995) and Stockman and Tesar (1995) demonstrate that models with perfect competition do not generate deviations from the law of one price and are unable to generate volatile relative prices with productivity or demand shocks. Extending these models to allow for monopolistic competition and constant markup pricing does not alter this finding. However, Lapham and Vigneault (2001) find that allowing for markups to vary exogenously over time and across countries increases the volatility of international relative prices. In our model, markups vary endogenously in response to productivity shocks. Second, there are the sticky price models. Betts and Devereux (2000), Chari, et al. (2001) and Bergin and Feenstra (2001) demonstrate that models incorporating nominal rigidities can increase the volatility of international relative prices. However, in these models international relative prices are still not persistent enough. Finally, Dumas (1992) and Obstfeld and Rogoff (2000) demonstrate that transaction costs can increase the volatility of international relative prices. In those papers, the emphasis is on iceberg shipping costs while in our paper we consider the role of imperfect information and costly search.

The next section of the paper documents some facts about international relative prices and describes evidence of price dispersion from U.S. imports. Section 3 introduces a static closed economy

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\(^6\)A notable exception is recent work by Betts and Kehoe (2002) which explores the role of non-tradeables inputs in real exchange rate movements.

version of a model in which consumers must search in order to purchase a good. In Section 4 a two
country version of the model is developed. Section 5 examines some quantitative properties of the
model. Section 6 concludes.

2. Data

In this section, some properties of international relative prices and domestic price dispersion
for the United States are reported. Statistics and figures relating to the aggregate relative prices
are based on quarterly data from the IMF’s International Financial Statistics database. Statistics
and figures relating to price dispersion are from the U.S. Census’ Import History. Census data is
from the period 1974 to 1994, although there is a major revision in the product categories between
1988 and 1989. Census data includes quantities and values of US imports by either TSUSA or
harmonized code by source country.\(^8\)

A. Relative Prices

At the aggregate level, one measure of relative prices is the real exchange rate, which is
defined as the relative price of two baskets of goods, expressed in a common currency or

\[
RER_t = \frac{P_t}{P_t^*}.
\]

Figure 1 plots two measures of the real exchange rate and the terms of trade between the U.S. and a
composite of its trading partners. The real exchange rate is measured as either the ratio of consumer
price indices or as the ratio of the wholesale price indices expressed in a common currency. Both of
these measures are volatile and persistent. Similarly, the terms of trade which is measured as the
relative price of imports to exports,

\[
TOT_t = \frac{P_t^{Imports}}{P_t^{Exports}},
\]

is also volatile and persistent. Each of these variables has an autocorrelation of approximately 0.85.
For the US, Chari, et al (2001) find that the standard deviation of the real exchange rate is nearly
four and a half times the standard deviation of output, while the terms of trade is nearly four times
as volatile as output.

At the disaggregate level, Knetter (1989 and 1993), Engel and Rogers (1996), Engel (1993

\(^8\)The Tarrif Schedule of the United States (TSUSA) was used from 1972 to 1988 and the Harmonized code was
used from 1989 to the present.
and 1999) and Parsley and Wei (2001) find that relative prices are also volatile and persistent across countries. In particular, they find that the relative price of nearly identical goods consumed in different countries are nearly as volatile and persistent as aggregate relative prices.

B. Price Dispersion

There is compelling evidence of substantial price dispersion within countries. In a study of 39 basic household goods and services Pratt, Wise and Zeckhauser (1979) find that for 22 of these products the ratio of the maximum price exceeds the minimum price by more than 50%.\(^9\) In a study of the airline industry, Borenstein and Rose (1994) find an expected absolute difference in fares between two passengers on a route of 36 percent of the airline’s average ticket price. Using plant level data from the Census of Manufactures for 13 industries, Roberts and Supina (2000) find the coefficient of variation exceeds 25% for nine of the industries. For goods sold through the Internet, Brynjolfsson and Smith (1999) find that prices differ by an average of 33% for books and 25% for CD’s. We construct a broader measure of US price dispersion by examining the price dispersion of imports into the U.S.

For goods imported from multiple countries, a measure of price dispersion is constructed using the quantity weighted variance of the log relative price by source country (see appendix). As there are many goods in each year of the sample, a distribution of price dispersion across goods is generated. Figure 2 plots the average price dispersion across goods in each year from 1972 to 1994. Over the entire sample, mean price dispersion is approximately 40%. Although there is some year to year variation, price dispersion has grown substantially over this period.

An alternative measure of price dispersion is the ratio of the maximum price to the mean price (max-mean ratio) by good. There is substantial variance in the max-mean ratio across goods in a particular year. Figure 3 plots the ratio for the median good in the distribution of max-mean ratios in each year. During this period, the max-mean ratio climbs from approximately 1.9 in 1972 to 3.0 in 1994.

It is possible that price dispersion may be a result of product heterogeneity within classifications. To account for this, price dispersion is examined for the goods that appear continuously in our sample from 1989 to 1994. This period is chosen as the product categories are most consistent over the sample period. For each good the mean level of price dispersion is calculated as a simple mean

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\(^9\)Some of the goods included: Raleigh Grand Prix 10 speed, 20 gallon aquarium, One hour horoscope reading including charting, Texas Instruments SR-50, Dennman styling brush.
of yearly price dispersion and then the variance of price dispersion over these years is calculated. Figure 4 is a scatterplot of the log of the mean and variance of price dispersion over the sample period. There is a positive correlation between the mean and variance of price dispersion for each good suggesting that goods with high price dispersion also experience large changes in price dispersion over time. This seems unlikely if price dispersion results solely from product heterogeneity.

Another way of measuring the amount price dispersion changes over time is to examine how a good’s price dispersion changes relative to other goods over time. This is done by dividing each year’s distribution of price dispersion into quintiles and then examining the transition probability across quintiles. The results are reported in the appendix. Price Dispersion is somewhat persistent as there is 60% chance that a good in the upper quintile of price dispersion will remain in the upper quintile of price dispersion in the following year. This clearly suggests that price dispersion is not random. In addition, there is a 15% chance that a good with price dispersion in the top 20% in one year will be in the bottom 60% of price dispersion the following year. In conjunction with previous work, this evidence suggests price dispersion is a common phenomenon that occurs in a broad range of goods.

The evidence of price dispersion within countries offers a new interpretation of the evidence of international deviations from the law of one price. Across countries and over time, the ratio of the average price of identical goods is quite volatile and persistent. This implies that the distribution of prices for a good in one country must be shifting relative to the distribution of prices in another country. In the remainder of the paper, we develop a model with this feature and examine its implications for the volatility of international relative prices.

3. Closed Economy Model

We develop a static closed economy general equilibrium model of price dispersion. This simplified model introduces the basic price setting mechanism. It also allows us to explore the general equilibrium properties of the model through comparative statics. These comparative statics shed light on the connection between price dispersion and price levels that are useful for our study of international relative price movements.

In our model, there is a single homogenous consumption good. It is produced by a continuum of identical firms. These firms hire labor for production and then sell their output through geographically distinct stores. These stores potentially charge different prices. The distribution of prices being charged in stores is common knowledge, but consumers do not know where to find the
store with lowest price. To find the lowest price possible, consumers must search. Because search takes time, consumers are willing to accept some prices that are higher than the lowest price in the market. In this respect, each firm has some monopoly power over consumers entering their stores. This monopoly power allows firms to charge potentially different prices, above marginal cost, and still sell positive quantities of the same good.

A. Consumer’s problem

Each consumer is modelled as a coalition, or family, of a continuum of agents. This approach eliminates any uncertainty from search and allows us to maintain a representative agent framework. There is a continuum of unit mass of identical families and without loss of generality, the ratio of firms to consumers is normalized to one. Within each family, agents are divided between searching, or shopping, for consumption goods and working. There is no disutility to either activity, although there is an opportunity cost to shopping in terms of foregone labor income. Shoppers and workers share equally in consumption at the end of each period and use the income from wages and profits to pay for shoppers’ purchases.

We choose the simplest possible reason for search: to reduce the cost of a unit of consumption. By searching consumers can shift their purchases towards firms with low prices. We assume that shoppers cannot communicate once in the market; each shopper can purchase at most one unit; and search is noisy in that with probability 1 – q a shopper receives price quotes from two firms and with probability q receives only one price quote. These assumptions imply that shoppers are given a simple reservation price rule: purchase one unit if \( p \leq \bar{p} \) and none otherwise. When a shopper receives multiple price quotes, the shopper purchases from the lowest priced firm as long as the price is less than the reservation price.

Given the distribution of prices quoted by firms, \( G(p) \), with finite support \([P, \bar{P}]\), the distribution of the lowest price drawn by a shopper as is defined as

\[
(1) \quad H(p) = qG(p) + (1 - q) \left(1 - [1 - G(p)]^2\right).
\]

Clearly, \( H(p) \) is a convex combination of the distribution of prices conditional on a single offer and

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10 An equivalent, but notationally more complex, approach would be to allow agents to trade a complete set of contingent claims over the uncertainty from searching.
11 Allowing consumers to search for a particular variety or quality of product does not influence our results.
12 Relating this assumption so that shoppers can purchase multiple goods does not alter our results. What matters is that there is an opportunity cost of searching in terms of work.
the distribution of the lowest price conditional on two price quotes.

Given a reservation price of $\tilde{p}$ a shopper will purchase with probability $H(\tilde{p})$ at an expected price of

$$P(\tilde{p}) = \int_0^{\tilde{p}} p \frac{dH(p)}{dp} dp / H(\tilde{p}).$$  \hfill (2)

A lower reservation price lowers both the expected price of a unit of consumption and probability of purchasing. With many identical shoppers, this expected price is also the average purchase price in the market. With noisy search, the average purchase price, $P(\tilde{p})$, is always lower than the average price quoted by firms because more purchases are made at the lower prices. We believe the notion of average purchase price most closely matches the price measured in the data.

Given the distribution of prices, $G(p)$, wage rate, $w$, and aggregate profits, $\Pi$, each family chooses $l$ workers, $n$ shoppers, and the reservation price $\tilde{p}$ to maximize

$$\max_{n,l,\tilde{p}} c = nH(\tilde{p}),$$  \hfill (3)

subject to

$$n + l \leq 1,$$
$$nH(\tilde{p}) P(\tilde{p}) \leq wl + \Pi.$$

If an interior solution exists, the reservation price is defined implicitly by

$$\tilde{p} = \frac{w}{H(\tilde{p})} + P(\tilde{p}).$$  \hfill (4)

This equation has a unique reservation price at which the family is indifferent between increasing consumption by sending out more shoppers or sending out shoppers with a higher reservation price rule. At the margin, a family is indifferent between paying the reservation price for a unit of consumption or sending out more shoppers. To increase consumption by one unit the family must send out $1/H(\tilde{p})$ more shoppers, each foregoing a wage of $w$, and expects to purchase a unit at a price of $P(\tilde{p})$. This equation is best interpreted as an arbitrage condition that closely ties the highest price a consumer is willing to pay with the local wage and the average price in the market.

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13Rearranging equation 4 generates the following implicit equation $\int_0^{\tilde{p}} (\tilde{p} - p) \frac{dH(p)}{dp} dp = w$. The l.h.s. of this equation is strictly increasing in $\tilde{p}$ so that if there is a solution, it is unique.
B. Firm’s problem

Each firm’s problem is similar to a problem examined in Burdett and Judd (1983). There is a large number of firms (normalized to a continuum of mass) selling a homogenous good. Each firm is identical and faces a per unit cost of production of $w/\phi$, where $\phi$ defines labor productivity.

Each firm quotes a price to $(q + 2(1-q))N$ consumers. Of these consumers, a fraction $\frac{q}{q+2(1-q)}$ only receive a quote from one firm while $\frac{2(1-q)}{q+2(1-q)}$ have a second price quote$^{14}$. The firm can not distinguish between shoppers with one or two price quotes and can only quote one price.

The price charged does not affect the number of shoppers that receive price quotes or the cost of production so that the firm’s pricing decision is summarized by the problem of maximizing profits per shopper. All shoppers are identical in their reservation price, so the maximum price a firm can charge is the consumer’s reservation price, denoted by $\bar{P}$. An uppercase distinguishes between individual and aggregate variables. While each shopper has the same reservation price, noisy search implies that shoppers may differ in their outside option. Some shoppers will have multiple price quotes and among these shoppers their second price quote may differ. For firms, this leads to trade-off between price and the probability that a shopper with two quotes makes a purchase. Based on this trade-off, the probability a shopper purchases from a firm charging $p$ is

\[
Q(p) = \begin{cases} 
0 & \text{if } p > \bar{P} \\
\frac{q}{q+2(1-q)} + \frac{2(1-q)G(p)}{q+2(1-q)} & \text{if } p \leq \bar{P} 
\end{cases}
\]

(5)

If the firm charges a price above the reservation price, it will not make a sale for certain. If the firm charges a price below the reservation price, the firm make a sale for certain if the shopper has one price quote. If the shopper has two price quotes, then the firm will only make a sale if the firm’s price is lower than the shopper’s other price quote which is drawn from $G(p)$.

Combining the demand per shopper and the number of shoppers to which a firm quotes a price generates the following demand curve per shopper of

\[
\hat{Q}(p) = q + 2(1-q)(1 - G(p))
\]

(6)

This demand curve is not constant elasticity. Instead, the elasticity of substitution between firms depends on search frictions and the distribution of prices of other firms. In this respect, the model

\[14\text{Note that the probability a customer has one price quote is less than the probability that a shopper receives one price quote. This is because each shopper expects to receive } 2-q \text{ quotes.} \]
generates interactions between firms that are quite different from standard models of monopolistic competition and constant elasticity of demand.

Given the distribution of prices in the economy, $G(p)$, consumers’ reservation price, $\tilde{P}$, the demand per shopper, $\tilde{Q}(p)$, the mass of shoppers, $N$, and the unit cost of production, $w/\phi$, a firm’s problem is to charge a price that maximizes the profits per shopper,

$$\pi = \max_p \left( p - \frac{w}{\phi} \right) \tilde{Q}(p)$$

This is a well defined problem with at least one solution. Furthermore, Burdett and Judd (1983) demonstrate that if firms have the same cost of production\(^{15}\), each firm earns the same profit $\pi$ on the support $[\underline{P}, \overline{P}]$ of the distribution of prices and that the highest price charged is equal to the reservation price ($\overline{P} = \tilde{P}$).

$$\pi = q \left( \overline{P} - \frac{w}{\phi} \right),$$

$$G(p) = \begin{cases} 0 & p < \underline{P} \\ 1 - \frac{\overline{P} - p}{\frac{w}{\phi} + \frac{q}{2(1-q)}} & p \in [\underline{P}, \overline{P}], \\ 1 & p > \overline{P} \end{cases}$$

$$\overline{P} = \frac{2(1-q)\frac{w}{\phi} + q\overline{P}}{2-q}.$$

In total, individual and aggregate firm profits are

$$\Pi = \pi N.$$

Because firms are indifferent between charging any price on the support of the distribution, they can be viewed as randomizing. With a continuum of firms, the law of large number holds and this randomizing results in a continuous distribution of prices, $G(p)$.

**C. Equilibrium**

An equilibrium is characterized by a distribution of prices, $G(p)$, wages, $w$, individual decision rules $(l, n, \bar{p})$ and aggregate decision rules $(L, N, \bar{P})$ such that:

1. Given a distribution of prices, $G(p)$, a wage rate, $w$, and profits, $\Pi$, consumers send out $n$

\(^{15}\text{Allowing firms to have different costs of production will lead to even greater dispersion in prices but does not change the model’s predictions.}\)
shoppers with a reservation price rule $\tilde{p}$ and send $l$ workers to solve equation 3.

2. Given a distribution of prices, $G(p)$, a reservation price rule, $\tilde{P}$, and a production cost, $w/\phi$, each firm chooses a price to solve equation 7 which generates $G(p)$ as defined in equations 9 and 10.

3. The resource constraints are satisfied

\begin{align}
(12) \quad NH(\tilde{P}) &= \phi L \\
(13) \quad L + N &= 1
\end{align}

4. Individual and Aggregate decisions are consistent

\begin{align}
(14) \quad n &= N, \quad l = L, \quad \tilde{p} = \tilde{P}
\end{align}

The key element in solving the model is determining the equilibrium reservation price and requires the following result.

**Proposition 1.** The highest price is equal to the shoppers’ reservation price ($\tilde{P} = \overline{P}$).

Proposition 1. implies that no consumer returns empty handed so that total consumption is equal to the mass of shoppers. The intuition for this result is straightforward. If some shoppers returned without a good ($\tilde{P} < \overline{P}$) those firms charging $\overline{P}$ can increase profits by charging $\tilde{P}$. Similarly, if shoppers are willing to pay more than the highest price in the market ($\tilde{P} > \overline{P}$), then those firms charging $\overline{P}$ can increase profits by raising their price to $\tilde{P}$.

Based on proposition 1., the reservation price is implicitly defined as the $\tilde{P}$ that satisfies

\begin{align}
(15) \quad \tilde{P} = w + P(\tilde{P}; \tilde{P} = \overline{P}),
\end{align}

where $P(\tilde{P})$ is defined in equation 2. In other words, the family is indifferent between purchasing a unit at the reservation or sending out one more shopper who will purchase at the average price in the market but must forego some labor income. This implies that the local wage matters because it affects 1) firms’ cost of production and 2) the opportunity cost of search and hence the consumer’s reservation price. This is an important result. In an international context, a firm will be able to charge a different price across countries whenever the opportunity cost of search, or wage, differs.
We can now solve for quantities and prices in the model:

\begin{align}
L &= \frac{1}{1+\phi}, \\
N &= \frac{\phi}{1+\phi}, \\
\Pi &= \frac{\phi qw}{(1-q)(1+\phi)}, \\
P(\bar{P}) &= \left(1 + \frac{\phi q}{1-q}\right) \frac{w}{\phi}, \\
\overline{P} &= \left(1 + \frac{\phi}{1-q}\right) \frac{w}{\phi}, \\
\underline{P} &= \left(1 + \frac{\phi q}{(2-q)(1-q)}\right) \frac{w}{\phi}.
\end{align}

A couple of things are worth noting. First, search only affects consumption and output through the resources spent searching. Second, prices are a simple markups over the cost of production which depend on both the search and production technologies. Third, if we just concentrate on average prices then the static model looks very much like a model of monopolistic competition with constant elasticity demand. However, unlike a constant elasticity model the markup is endogenous and will change with technology.

**D. Comparative Statics**

Having solved the model, we examine its properties with respect to a change in productivity and the noisy search parameter. These results will clarify the mechanisms at work in the two country model.

An increase in productivity increases output, consumption and the number of shoppers equally, but leads to a drop in hours worked as

\begin{align*}
\frac{\partial L}{L} &= \frac{\phi}{1+\phi}, \\
\frac{\partial N}{N} &= \frac{\partial Y}{Y} = \frac{1}{1+\phi}.
\end{align*}

With no intertemporal or international linkages, the simple labor market decision implies that an increase in productivity will lower the amount of labor supplied. This reduction in labor is necessary in order to send out more shoppers to purchase the extra output available for consumption.

An increase in productivity lowers all prices. The highest price drops by less than the average
price and the average price drops by less than lowest price. In particular,

\[
0 > \frac{\partial \bar{P}}{\partial \phi} = -\frac{1 - q}{1 - q + \phi} > \frac{\partial \bar{P}(\bar{P})}{\partial \phi} = \frac{1 - q}{1 - q + \phi q} > \frac{\partial \bar{P}_D}{\partial \phi} = -\frac{(1 - q)(2 - q)}{(1 - q)(2 - q) + \phi q}.
\]

The increase in productivity raises the opportunity cost of search and lowers the cost of production. A higher opportunity cost of search allows each firm to charge a higher markup to those consumers with a single price quote. However, the higher markup also leads firms to compete more heavily for those consumers with two price quotes. This extra competition leads both the average and lowest prices in the market to drop relative to the reservation price. This effect on prices is stronger the more competitive markets \((q \text{ close to } 0)\). The decrease in costs more than offsets the increase in the markup so that the reservation price actually falls. Consequently an increase in productivity leads to a decrease in the price level and an increase in the real wage \(w/P\).

Consider the effect of a change in productivity on the average purchase price, \(\frac{d\bar{P}}{d\phi} \), and output, \(\frac{dY}{d\phi} \). A productivity shock has a larger effect on average prices than output whenever more agents have two price quotes than one \((q < 1/2)\). As markets become more competitive \((q \to 0)\) prices respond much more than output to changes in productivity. Similarly, as economies become more productive (higher \(\phi\)’s) the effect of an increase in productivity will lead to an even larger change in prices relative to output. The logic for both of these results is the same. An increase in productivity raises the opportunity cost of search for consumers. This raises profits and leads firms to compete more heavily for additional customers, those with multiple price quotes. When more agents have two price quotes the incentive to compete for these additional customers is stronger.

In an international context a productivity shock has a different effect on the price level and output. Relative wage changes will affect price level changes and the ability to export will affect how much is produced. However, the strong effect on price levels through competition will remain important. We now develop an international version of the model.

### 4. Two Country Model

In this section our closed economy model is extended to include 2 countries, Domestic and Foreign, \(\{D,F\}\), two goods \(\{d,f\}\) and uncertainty in productivity. The structure of the shocks follow Chari, et al. (2001). Every period there is a possible state of the world \(s_t \in S\). Let \(s^t\) be the history of all states up to time \(t\) and let \(\sigma(s^t|s_0)\) represent the time 0 probability of history \(s^t\). Country \(D\) specializes in the production of good \(d\) while country \(F\) specializes in the production of
good $f$. Preferences of Domestic agents are given by the utility function $u(c)$ and Foreign preferences are given by $u(c^*)$. Consumption in each country is obtained by aggregating the quantity of each good consumed so that

$$c(s^t) = A [d(s^t), f(s^t)],$$
$$c^*(s^t) = A [f^*(s^t), d^*(s^t)].$$

where the aggregator may exhibit a bias for locally produced goods. With the exception of the search frictions, this structure is nearly identical to that considered by Backus, et al. (1995).

In each country, there are many stores specializing in the sale of either the locally producer good or the imported good. These stores require no local input and are viewed as part of the producing firm. Moreover stores selling imported goods face no cost of shipping goods internationally. With these simplifications any deviations from the law of one price across countries will be strictly due to our search frictions.

Given the mechanics of search, it is natural to allow firms to segment markets internationally and potentially price discriminate across countries. With two countries and two differentiated goods, this implies there will potentially be four price distributions. The distribution of prices charged by country $i \in \{D, F\}$ firms in Domestic will be denoted by $G_i(p)$. The distribution of prices charged by country $i$ firms in Foreign will be denoted by $G^*_i(p)$. Firms producing goods in country $D$ face a per unit cost of production of $w_D(s^t) / \phi_D(s^t)$ regardless of where the good is sold. Similarly, firms producing in country $F$ face unit cost of $w_F(s^t) / \phi_F(s^t)$.

Consumers direct their shoppers towards stores selling either good $d$ or $f$ so that a shopper sent out to purchase a particular good will only collect price quotes from firms selling that type of good. Shoppers are subject to the same noisy search and shopping technologies as before. Shoppers do not return to stores they have visited in previous period as the relationships between stores and shoppers only lasts the current period.

We assume that a complete set of one period state contingent securities are traded every period. Without loss of generality, we assume these securities are denominated in terms of the Domestic wage, so that one security costs $\chi(s^{t+1}|s^t)$ in state $s^t$ and pays one unit if and only if

\begin{footnotesize}
\begin{enumerate}
\item In a related paper, Alessandria (1999) explores the role of long-term relationships and search frictions for firm pricing across countries.
\item Because of price dispersion it is easier to choose labor in one country as the numeraire. An equivalent specification would be to set the average price of $d$ in country $D$ to be the numeraire. Alternatively we could introduce money as a unit of account.
\end{enumerate}
\end{footnotesize}
the state tomorrow is \( s^{t+1} \).

### A. Consumer’s Problem

Each household is assumed to be a coalition of agents of the same nationality. Domestic and Foreign agents are restricted from forming coalitions and it is assumed to be too costly for shoppers to be sent out of the country to shop. This approach is consistent with the type of market completeness assumed models in which firms price discriminate internationally (see Chari, et al. (2001)).

Given the distribution of prices, \( G_D \) and \( G_F \), the Domestic wage, \( w \), and the price of one period state contingent securities, \( \chi (s^{t+1}|s^t) \), each Domestic household must divide shoppers into each sector, \( \{ n_D (s^t), n_F (s^t) \} \), devise reservation price rules \( \{ \tilde{p}_D (s^t), \tilde{p}_F (s^t) \} \) and choose bond holdings \( \{ b (s^{t+1}) \} \) to solve

\[
V_0 = \max_{\{ n_D (s^t), n_F (s^t), b (s^{t+1}) \}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \sum_{s^t} \sigma (s^t|s_0) u (A [d (s^t), f (s^t)]) ,
\]

subject to \( (\forall s^t) \):

\[
\begin{align*}
d (s^t) &= n_D (s^t) H_D (\tilde{p}_D (s^t); s^t), \\
f (s^t) &= n_F (s^t) H_F (\tilde{p}_F (s^t); s^t), \\
l (s^t) + n_D (s^t) + n_F (s^t) &= 1, \\
\sum_{i \in \{D,F\}} n_i (s^t) H_i (\tilde{p}_i (s^t)|s^t) P_i (\tilde{p}_i (s^t); s^t) \\
&\quad + \sum_{s^{t+1}|s^t} \chi (s^{t+1}|s^t) b (s^{t+1}) \leq w (s^t) l (s^t) + \Pi (s^t) + b (s^t).
\end{align*}
\]

The distribution of price quotes for goods from country \( i \in \{D,F\} \) is

\[
H_i (p; s^t) = q G_i (p; s^t) + (1 - q) \left( 1 - [1 - G_i (p; s^t)]^2 \right).
\]

With a reservation price \( \tilde{p}_i \) the expected purchase price for good \( i \) is

\[
P_i (\tilde{p}_i; s^t) = \frac{\int_0^{\tilde{p}_i} \frac{\partial H_i (p, s^t)}{\partial p}}{H_i (\tilde{p}_i; s^t)}.
\]

Plugging the strict equalities into the utility function and the budget constraint and differ-
entiating leads to the following system of first order conditions

$$
\begin{align*}
   u' A_d H_D (\bar{p}_D (s^t) ; s^t) &= \mu (s^t) \left[ w + H_D (\bar{p}_D (s^t) ; s^t) P_D (\bar{p}_D (s^t) ; s^t) \right], \\
   u' A_f H_F (\bar{p}_F (s^t) ; s^t) &= \mu (s^t) \left[ w + H_F (\bar{p}_F (s^t) ; s^t) P_F (\bar{p}_F (s^t) ; s^t) \right], \\
   u' A_d n_D (s^t) &\frac{\partial H_D (\bar{p}_D (s^t) ; s^t)}{\partial \bar{p}_D} = \mu (s^t) n_D (s^t) \bar{p}_D (s^t) \frac{\partial H_D (\bar{p}_D (s^t) ; s^t)}{\partial \bar{p}_D}, \\
   u' A_f n_F (s^t) &\frac{\partial H_F (\bar{p}_F (s^t) ; s^t)}{\partial \bar{p}_F} = \mu (s^t) n_F (s^t) \bar{p}_F (s^t) \frac{\partial H_F (\bar{p}_F (s^t) ; s^t)}{\partial \bar{p}_F}, \\
   \mu (s^t) \chi (s^{t+1} | s^t) &= \mu (s^{t+1}).
\end{align*}
$$

where $A_i$ represents the partial derivative of the aggregator with respect to good $i \in \{d, f\}$ and $\mu (s^t)$ is the multiplier on the budget constraint in state $s^t$. Assuming $\bar{p}_i = \bar{P}_i$, and rearranging terms generates the following implicit equations for the reservation prices in Domestic

$$
\begin{align*}
   (22) \quad \bar{p}_D (s^t) &= w (s^t) + P_D (\bar{p}_D; s^t), \\
   (23) \quad \bar{p}_F (s^t) &= w (s^t) + P_F (\bar{p}_F; s^t),
\end{align*}
$$

and the relative price equation

$$
(24) \quad \frac{A_d (s^t)}{A_f (s^t)} = \frac{\bar{p}_D (s^t)}{\bar{p}_F (s^t)}
$$

The reservation price equations (22 and 23) are similar to the closed economy and have the same interpretation. Equation 24 states that agents equate the marginal rate of substitution between the two goods to the ratio of their reservation prices. This is a very different condition than models without price dispersion. In those models, agents equate the marginal rate of substitution between two goods to their relative price, which is also the ratio of average prices. However, with price dispersion the marginal unit of each good is purchased at the reservation price so that it is the ratio of reservation prices that matters at the margin.

Substituting out the reservation prices yields the following relationship between average prices and the marginal rate of substitution,

$$
\frac{A_d (s^t)}{A_f (s^t)} = \frac{w (s^t) + P_D (\bar{p}_D; s^t)}{w (s^t) + P_F (\bar{p}_F; s^t)}
$$

Compared to standard models, in our model a change in average prices will have a much smaller effect on quantities consumed because of the local wage/opportunity cost of search. In this respect
a given amount of volatility in quantities will require much more volatility in average prices in our price dispersion model than in a model without price dispersion. In this respect, introducing these search frictions lowers the actual elasticity of substitution between Domestic and Foreign goods.

In the foreign country, we get a similar set of equations,

\begin{align}
\tilde{p}_D^* (s^t) &= w^* (s^t) + P^*_D (\tilde{p}_D^*; s^t), \\
\tilde{p}_F^* (s^t) &= w^* (s^t) + P^*_F (\tilde{p}_F^*; s^t), \\
\frac{A_f^*(s^t)}{A_d^*(s^t)} &= \frac{\tilde{p}_D^* (s^t)}{\tilde{p}_F^* (s^t)}.
\end{align}

Additionally, the arbitrage condition in asset markets generates the following risk sharing condition\(^{18}\)

\begin{equation}
\frac{u_c A_d}{u_c A_d^*} = \frac{\tilde{p}_D}{\tilde{p}_D^*},
\end{equation}

which implies that the ratio of marginal utilities will not be equal to the ratio of the price levels across countries. This risk sharing condition should still be quite intuitive. Risk sharing leads agents to equate the ratio of marginal utilities to the relative price of consumption. In this model, the expected cost of a unit of consumption is the reservation price. Consequently, the ratio of marginal utilities is related to the ratio of reservation prices.

Returning to equations 23 and 25, it is clear that the local wage will influence the price importing firms charge through the reservation price local consumers are willing to accept. In particular, a higher local wage increases the opportunity cost of search and makes local consumer’s more willing to accept a higher price. This implies that high wage countries (or regions) will have higher reservation prices and average prices for goods than low wage countries. It also implies that shocks that affect the relative wage between countries will shift the distribution of prices being charged for the same goods in different countries and lead to deviations from the law of one price.

B. Firm’s Problem

Given our assumption that noisy search is sector specific, no consumer receives a price quote from both a firm selling \(d\) and a firm selling \(f\). Consequently, a representative firm from country \(i\) is only concerned with the distribution of prices charged by potential competitors, \(G_i (p)\). Also, with no dynamic linkages, firms will face a very similar problem to the closed economy problem. The only difference is that a type \(i\) firm can expect to quote a price to \((q + 2 (1 - q)) N_i\) agents.

\(^{18}\)This assumes that countries are ex-ante identical.
Because price setting is not influenced by the flow in of shoppers, the distribution of prices will have the same properties as in the closed economy.

As before, firms will choose their price by randomizing over the support of the distribution. In this respect, firm pricing will generally not be persistent. For example, firms charging a low-price on average over time will not have chosen a low price strategy, but rather will have repeatedly drawn a low price in the process of randomizing. In a recent study, Lach (2002) finds precisely this type of behavior across a set of stores in Israel. Similarly, some firms may draw the same, or nearly the same, price from one period to the next, even in the face of a large shock to their production costs. In this respect, some prices may appear not to adjust to large shocks to costs.

C. Equilibrium

The definition of an equilibrium is a straightforward generalization of the closed economy model. An equilibrium\(^{19}\) is characterized by the distribution of prices in each country \(\{G_D(p), G_F(p), G^*_D(p), G^*_F(p)\}\), wages \(\{w_D, w_F\}\), prices for securities \(\chi(s^{t+1}|s^t)\), decision rules \(\{n_D, n_F, \tilde{p}_D, \tilde{p}_F, b\}\) for agents in country D and decision rules \(\{n^*_D, n^*_F, \tilde{p}^*_D, \tilde{p}^*_F, b^*\}\) for agents in country F and aggregate decision rules \(N_D, N_F, \tilde{P}_D, \tilde{P}_F, N^*_D, N^*_F, \tilde{P}^*_D, \tilde{P}^*_F\) such that:

1. Given the distribution of prices, wages and profits, individual decision rules solve household’s problem in country D (country F).
2. Given the distribution of prices, reservation price rule, and wages each firm chooses a price to solve its problem.
3. The resource constraints are satisfied.

\[
N_D H_D \left( \tilde{P}_D \right) + N^*_D H^*_D \left( \tilde{P}^*_D \right) = \phi_D \left[ 1 - N_D - N_F \right] \\
N_F H_F \left( \tilde{P}_F \right) + N^*_F H^*_F \left( \tilde{P}^*_F \right) = \phi_F \left[ 1 - N^*_D - N^*_F \right] \\
b + b^* = 0
\]

\(^{19}\)We have dropped the state \(s^t\) in the definition to save space.
4. Individual and Aggregate decisions are consistent

\[ n_D = N_D \quad n_D^* = N_D^* \]
\[ n_F = N_F \quad n_F^* = N_F^* \]
\[ l = L \quad l^* = L^* \]
\[ \tilde{p}_D = \bar{p}_D = \bar{T}_D \quad \tilde{p}_D^* = \bar{p}_D^* = \bar{T}_D^* \]
\[ \tilde{p}_F = \bar{p}_F = \bar{T}_F \quad \tilde{p}_F^* = \bar{p}_F^* = \bar{T}_F^* \]

As before, firms will not charge a higher price than the consumer’s reservation price so that each shopper will return home with a good. Moreover, firms have no incentive to charge a price below the reservation price, so the highest price in the market will equal the reservation price.

5. Findings

In this section the model is evaluated qualitatively and quantitatively. To explain the mechanisms at work in the model we begin with a simple numerical example. We then compare quantitative properties of the model to the data and competing models. We find that introducing price dispersion from search increases the volatility of international relative by 300% compared to a model without search friction. With high price dispersion and risk aversion, our model can account for all of the volatility in real exchange rates and 90% of the volatility in the terms of trade. Moreover, the model generates substantial and persistent deviations from the law of one price within and across countries.

A. Calibration

The following functional forms are chosen for the utility and aggregator functions,

\[ u(c) = \frac{c^{1-\sigma}}{1-\sigma} \]
\[ A(d, f) = \left[ d^{\frac{2-\gamma}{\gamma}} + \omega f^{\frac{2-\gamma}{\gamma}} \right]^{\frac{\gamma}{2-\gamma}}. \]

The likelihood of a single price quote \((q)\) and productivity \((\phi)\) are calibrated from data on the max-mean price ratio and the share of income from profits. The profit share is set to 36%, a fairly common value, and the maximum price is set to twice the mean transaction price in the market\(^{20}\). Import

\(^{20}\)An alternative measure of price dispersion would be the ratio of the maximum price to the mean quoted price \((\bar{P}/ \int_0^\phi p dG(p))\). Since more goods are sold at lower prices, our measure may seem to overstate price dispersion. For instance, a max-mean purchase price ratio of 2 (3) generates a smaller max-mean quoted price ratio of 1.81 (2.56).
data suggests that this choice for the ratio of the maximum to mean price is low as approximately 50% of the goods imported in 1994 had ratios higher than 3. Given the elasticity of substitution, the bias parameter is chosen so imports are 15% of output. The parameters of the model are

\[ q = 0.26470588, \quad \phi = 1.5625, \quad \sigma = 2, \quad \gamma = 1.5, \]

Given the aggregator, it is possible to define the price of the marginal unit of consumption in country \( D \) and \( F \) as

\[
P = \left( \tilde{P}_D^{1-\gamma} + \omega \gamma \tilde{P}_F^{1-\gamma} \right)^{\frac{1}{1-\gamma}},
\]

\[
\bar{P} = \left( \omega \gamma \tilde{P}_D^{1-\gamma} + \tilde{P}_F^{1-\gamma} \right)^{\frac{1}{1-\gamma}}.
\]

This leads to the following risk sharing condition,

\[
\left( \frac{c}{c^*} \right)^{-\sigma} = \frac{P}{\bar{P}},
\]

which is very different than the risk sharing condition common to the literature (see for example Backus and Smith (1993), and Chari, et al. (2001)). In particular, normally the risk sharing condition implies that the ratio of marginal utilities of consumption is equal to the Real Exchange Rate (RER) and in our model the RER is defined as

\[
RER = \left( \frac{P_D^{1-\gamma} + \omega \gamma P_F^{1-\gamma}}{\omega \gamma P_D^{1-\gamma} + P_F^{1-\gamma}} \right)^{\frac{1}{1-\gamma}} \neq \frac{P}{\bar{P}},
\]

where \( P_i \) is the average price charged by country \( i \) stores in country \( D \). Thus, the model breaks the link between the ratio of consumption and the real exchange rate in a simple and intuitive way. Chari, et al. (2001) consider breaking this link to be of central importance to understanding international business cycles.

B. An example

To study the mechanism connecting price dispersion with international relative price volatility we begin with a numerical example. We consider the impact of a 10% increase in Foreign productivity on the equilibrium in the baseline Price Dispersion model. The change in quantities and prices are reported in Table 4. We first discuss how quantities change and then prices.
**Quantities**

With complete risk sharing, an increase in Foreign productivity increases consumption in both countries. Because of the simple trade-off between working and shopping, hours decrease in both countries. Domestic hours and output decrease by 0.7% while the decrease in Foreign hours is more than offset by the increase in productivity so that output increases by 4.4%. Because of the strong bias for home goods, the decrease in Domestic hours is relatively small compared to Foreign hours.

The increased availability of good $f$, and reduced availability of good $d$ leads to a shift in consumption towards good $f$ in both countries. To compensate Domestic agents, who have a bias for their own goods, for fewer of their own goods, Domestic agents consume a larger fraction of total output of each good. However, the increase in consumption of $f$ in country $F$ is relatively more valuable so that total Foreign consumption increases relatively more (3.3% vs 0.4%). From the risk sharing condition, this implies that prices in country $F$ will have to fall substantially more than in country $D$.

**Price Distributions**

Within countries the shift in consumption towards good $f$ requires a reduction in the ratio of reservation prices, $\frac{\tilde{P}_F}{\tilde{P}_D}$ and $\frac{\tilde{P}_F^*}{\tilde{P}_D^*}$, of about 3.7%. From equations 22 and 23 it is clear that the ratio of reservation prices will only decrease if the average price of Foreign goods in each market goes down. Foreign firms will lower their prices if their costs go down, so the change in the wage of their labor input can not offset the change in productivity. In fact, the Foreign relative wage decreases by almost 2.9%, lowering the opportunity cost of search for Foreign agents. The lower Foreign wage is a result of Foreign workers cutting back on hours worked to increase consumption of their local good. This implies the cost of Foreign firms drops by more than the reservation price (13% vs 3.7%), which raises the return to attracting additional consumers. In each market, this leads Foreign firms to compete more heavily, resulting in a 7.5% drop in the average price of Foreign goods relative to the average price of Domestic goods.

The lower Foreign wage reduces the opportunity cost of search of Foreign agents. This leads a decrease in reservation prices of Foreign agents and a further reduction of prices in country $F$. In fact, both $\frac{\tilde{P}_D^*}{\tilde{P}_D}$ and $\frac{\tilde{P}_F^*}{\tilde{P}_F}$ decrease by 2%. The distribution of prices will differ across countries. These changes in reservation prices do not translate fully into average prices as it reduces the gain to attracting additional consumers in country $F$. In total, average prices for the same good are only
about 1% lower in country $F$. The law of one price no longer holds even on average. With home bias, the price of a unit of consumption in country $F$ will drop substantially more than a unit of consumption in country $D$. Moreover, the real wage increases in each country, although it increases by more in country $F$.

Even though the law of one price does not hold, and disaggregate relative prices may move around over time, most of the action in international relative prices will come from changes in the relative price of country $D$ and country $F$ goods. For instance, the terms of trade, $\tau = P_F / P_D^*$, decreases by almost 6.7% while disaggregate international relative prices $r_i = P_i^* / P_i$ decrease by a little less than 1% and the real exchange rate $RER = P^* / P$ decreases by about 6.3%. In contrast, Foreign output rises by just under 4% while Domestic output decreases slightly.

### C. Quantitative Properties

To examine whether price dispersion and search can help explain the data on international relative prices, we simulate the model. Each country is subjected to a random productivity shock and the dynamic properties of the model are determined. To quantify the importance of introducing price dispersion through search, properties of the theoretical model are compared to two other models; 1) the Standard Model and 2) the Cost Dispersion Model. In the Standard Model, we eliminate the need to shop and imperfect information so there is perfect competition. The Standard model is a no-capital version of the basic model used in Backus, et al. (1995) and is detailed in Appendix 2. In the Cost Dispersion Model, detailed in Appendix 3, in each country there is a continuum of firms producing differentiated goods in each country. Firms within countries are assumed to have different costs of production. The Cost Dispersion model is a no-capital, flexible price version of the model in Chari, et al (2001). Not surprisingly, the Cost Dispersion Model is identical to the Standard Model in terms of the volatility of international relative prices.

We also examine the implications of increasing the amount of price dispersion so that the max-mean ratio is 3.0 in an experiment titled High Price Dispersion. Increasing price dispersion in this way has almost no effect on the coefficient of variation of prices. With low price dispersion the coefficient of variation is 37.2% while with high price dispersion it is 37.5%. For comparison, Roberts and Supina (1997) find the coefficient of variation for both corrugated shipping containers and ready mix concrete was approximately 37% in 1987. Finally, we consider alternative parameter values for $\sigma$. In the low sigma experiment $\sigma = 2$, which is the standard value used in flexible price models. In the high sigma experiment, we set $\sigma = 6$, which is the value that Chari, et al (2001)
require to get enough volatility in international relative prices with nominal rigidities and staggered price setting.

In terms of productivity, we follow Backus, et al. (1995) and assume that each country is subject to random technology shocks $\mathbf{z}_t = (z_{1t}, z_{2t})$ such that $(\phi_D (s^t), \phi_F (s^t)) = (e^{z_{1t} \phi}, e^{z_{2t} \phi})$. The technology shocks in the two countries follow a vector autoregressive process of the form

$$\begin{bmatrix}
\log z_{1t+1} \\
\log z_{2t+1}
\end{bmatrix} = \begin{bmatrix}
a_1 & a_2 \\
a_2 & a_1
\end{bmatrix} \begin{bmatrix}
\log z_{1t} \\
\log z_{2t}
\end{bmatrix} + \begin{bmatrix}
\varepsilon_{1t+1} \\
\varepsilon_{2t+1}
\end{bmatrix}. $$

The innovations $\varepsilon_t = (\varepsilon_{1t}, \varepsilon_{2t})$ are serially independent, multivariate normal random variables with contemporaneous covariance matrix of $V$. In terms of the covariance matrix we set $\text{var} \varepsilon_1 = \text{var} \varepsilon_2 = 0.01^2$ and the $\text{corr} (\varepsilon_1, \varepsilon_2) = 0.25$. As our focus is on the relative volatility of prices to output, the scale of the productivity shock will have no effect on these ratios. In the current experiment, there is assumed to be no spillover of shocks across countries ($a_2 = 0$). Moreover, the persistence parameter is chosen to match the literature, $a_1 = 0.95$. With complete international risk sharing, there will be no wealth effects and the persistence of international relative prices will be determined by persistence of the ratio of the productivity shocks. The properties of the models with respect to prices are reported in Table 5. These results are discussed for each parameterization.

**Low Sigma** Introducing price dispersion generates a significant improvement in relative price volatility over the competing models, increasing real exchange rate volatility by 142% and terms of trade volatility by 66%. Moreover, the Price Dispersion Model generates deviations from the law of one price while the Standard Model does not. The source of price dispersion is important as the Cost Dispersion Model is identical to the Standard Model. However, the baseline Price Dispersion model still does not generate enough relative price variability compared to the data. In fact, relative prices in the model are only about one-half as volatile as in the data.

Increasing the amount of price dispersion generates substantially more volatile prices. Now, the model can account for approximately 82% of the volatility in the terms of trade and 64% of the volatility of the real exchange rate. This increase in relative price volatility is a result of a higher opportunity cost of search. To increase the amount of price dispersion we recalibrate the model. To increase price dispersion, and maintain the same profit share, requires an increase in productivity and decrease in the probability of a single price quote. The increase in productivity increases the opportunity cost of shopping through the wage so that changes in average prices will have an even
smaller impact on quantities. The decrease in the probability of a single price quote makes markets more competitive. When markets are more competitive, changes in productivity have a much larger effect on average prices. This is precisely the mechanism that was described in the closed economy. There is evidence that as international relative prices have become more volatile, price dispersion has increased over the period 1974 to 1994.

**High Sigma** Increasing the risk aversion parameter to 6 increases the volatility of relative prices in the Price Dispersion models substantially with respect to the Standard model. This is particularly true for the volatility of the real exchange rate and disaggregate relative prices. Now, the High Price Dispersion model can account for 100% of the real exchange rate volatility, nearly 90% of the terms of trade volatility and generates disaggregate relative prices that are more volatile than output. With this parameterization the model matches up quite well with the data.

Consider how increasing the risk aversion parameter increases international relative price volatility. With higher risk aversion agents would like to smooth consumption as much as possible. This leads the ratio of consumption across countries to vary less. Compared to a low risk aversion parameter, a country with a good shock will increase consumption by less. This means more production in the country with the good shock and less production in the other country. To shift consumption towards the good from the high productivity country, there must be a larger drop in the reservation price of these goods. This occurs if firms face a very low cost of producing this good, which is only possible if the wage in the high productivity country drops substantially. This leads to increased volatility in relative wages across countries. Within countries, the larger cost differential between Domestic and Foreign firms leads to larger changes in the ratio of average prices of Domestic and Foreign. Across countries, the increased wage differential leads to larger deviations from the law of one price across countries. With respect to the terms of trade, the effect on relative prices within countries is partially offset by the effect of increased deviations from the law of one price so that the volatility of the terms of trade increases less. On the other hand, these two effects reinforce each other for the Real Exchange Rate. Hence, increasing the risk aversion parameter leads to a large increase in the volatility of the real exchange rate.

**Persistence** None of the models we consider generate international relative prices that are as persistent as the data (.68 vs .85). This is not surprising as international relative prices take on the persistence of the ratio of the productivity shocks, even in the Standard Model. In the Standard
Model, the correct amount of persistence can be achieved by allowing for capital accumulation. We believe that extending our model in this direction may increase persistence as well. Even though the price dispersion model does not generate enough persistence in relative prices, it does quite well compared to a sticky price model. In fact, the sticky price model of Chari, et al. (2001) requires almost 2 full years of price rigidity to achieve the same amount of persistence as in our price dispersion model.

**Sensitivity** Figure 6 measures the effect of changing the elasticity of substitution (gamma) and risk aversion (sigma) parameters on relative prices. The effect of increasing risk aversion parameter has already been discussed. Making goods less substitutable increases the volatility of international relative prices. This effect is strongest on the real exchange rate and disaggregate relative prices (deviations from the law of one price). When the elasticity of substitution is lowered to 2/3 and risk aversion is high, our baseline price dispersion model generates real exchange rates that are nearly 5 times as volatile as output and disaggregate relative prices that are almost twice as volatile as output.

Figure 7 depicts the effect of changing the amount of price dispersion and the share of income from profits on relative prices. Taken together, these figures demonstrate the importance of different sources of real exchange rate volatility. As previously discussed, increasing price dispersion has a strong effect on real exchange rate variability through the terms of trade because it raises the opportunity cost of search. Consequently, to shift consumption between goods from different countries there must be large terms of trade changes. On the other hand, increasing the profit ratio raises the average markup on goods so that the opportunity cost of search becomes relatively less important and smaller terms of trade changes can achieve the same change in quantities. However, the smaller terms of trade changes lead to larger changes in relative wages and international deviations from the law of one price.

**6. Conclusions**

A major puzzle in international economics is the volatility and persistence of international relative prices at the aggregate and disaggregate level. These relative price fluctuations are inconsistent with existing models based on perfect competition, monopolistic competition with constant elasticity demand, or sticky prices. These models are also at odds with evidence of price dispersion within countries. This paper presents a two country model with monopolistic competition and search frictions that generates price dispersion within countries and volatile relative prices across
countries. With a reasonable amount of price dispersion and a high risk aversion parameter, the model accounts for all of the real exchange rate volatility and most terms of trade volatility with productivity shocks alone. The model also generates international relative prices that are quite persistent.

The source of price dispersion matters for the volatility of international relative prices. When price dispersion reflects cost dispersion in varieties of differentiated goods, then international relative prices are no more volatile than in a model without price dispersion. However, when price dispersion results from search frictions then we can come close to matching the volatility of international relative prices. The key is that search, by altering the perceived elasticity of substitution between firms and across goods, leads to markups that vary endogenously across country. Moreover, by introducing another channel to determine consumption allocations, with search frictions the traditional links between quantities and prices no longer hold. In this respect, taking account of the sources of price dispersion is important for determining the relationship between average prices and quantities.

Introducing search frictions suggests a novel relationship between local wages and prices. When search takes time and is imperfect, consumers face a choice between accepting a high price for a good or spending more time searching. Search is costly as consumers must forego work, but it allows consumers to find goods at a lower price. In this respect the highest price a firm can charge is related to the local wage. Differences in wages across countries lead to differences in the opportunity cost of search and allow firms to price discriminate internationally. Over time, changes in productivity that change the international relative wage also lead to changes in the relative price of goods at the disaggregate level.

The model also matches the observation that over time the entire distribution of prices in one country is shifting relative to the distribution of prices in another country. In this model, these shifts are due to productivity shocks altering the international relative wage and the opportunity cost of search across countries. Moreover, the observation that some firms do not change their prices following a relative cost shock (like the exchange rate) can easily be interpreted in the model. Because firms set prices by randomizing over a finite support of prices, which do not change much over time, it is likely that some firms will charge the same price even after a large shock to its costs.

Finally, our main ideas have been developed in a model with no capital accumulation or money. Both of these elements are likely to play an important role in the transmission of international business cycles. In particular, we believe capital accumulation is likely to increase the persistence of international relative prices in our model in much the same way it does in other real
models. Also, the current model generates large deviations from the law of one price from productivity shocks only when international relative wages vary substantially. Introducing another reason for international relative wages to move around, perhaps through sticky wage contracts interacting with monetary shocks, represents an alternative transmission channel. We are currently working on both of these extensions.
Appendix


The US Imports History includes units ($Q_{ijt}$) and values ($V_{ijt}$) for good $i$ from country $j$ in period $t$. These are used to construct log prices for each good by source country in each period ($p_{ijt}$) and mean log prices ($p_{it}$) by good in each period:

$$p_{ijt} = \ln \frac{V_{ijt}}{Q_{ijt}}, \quad p_{it} = \ln \frac{\sum_j V_{ijt}}{\sum_j Q_{ijt}}.$$

These log prices are then converted into log relative prices and then quantity weighted to find the variance by good in each period of

$$\sigma^Q_{it} = \sqrt{\sum_j \frac{Q_{ijt}}{Q_{it}} (p_{ijt} - p_{it})^2}.$$

Over time the mean variance ($\sigma_i$) and variance of the variance ($\rho_i$) for good $i$ is computed as

$$\sigma_i = \frac{1}{T} \sum_{t=1}^{T} \sigma^Q_{it}, \quad \rho_i = \sqrt{\sum_t (\sigma_{it} - \sigma_i)^2}.$$

The max-mean ratio for good $i$ is computed as

$$MM_{it} = \exp \left( \max_j \{p_{ijt}\} - p_{it} \right).$$
A2. Standard Model

Consumers work a fixed amount each period and must divide consumption between the two goods. The country $D$ good is chosen as the numeraire. Domestic agents solve the following problem

$$V_0 = \max_{\{d,f,b\}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \sum_{s^t} \sigma (s^t|s_0) u \left( A \left[ d \left(s^t \right), f \left(s^t \right) \right] \right),$$

subject to: $d \left(s^t \right) + f \left(s^t \right) P_F \left(s^t \right) + \sum_{s^{t+1}|s^t} \chi \left(s^{t+1}|s^t \right) b \left(s^{t+1} \right) \leq w \left(s^t \right) + b \left(s^t \right).$

Foreign agents face a similar problem leading to the following first order conditions

$$\frac{A_f}{A_d} = P_F = \frac{A_f^*}{A_d^*},$$

$$\left( \frac{c [d \left(s^t \right), f \left(s^t \right)]}{c [f^* \left(s^t \right), d^* \left(s^t \right)]} \right)^{-\sigma} = \frac{P}{P^*},$$

where $P$ is the price of the Domestic basket of goods and $P^*$ is the price of Foreign basket of goods. These FOC combine with the resource constraints

$$d \left(s^t \right) + d^* \left(s^t \right) = \phi_D \left(s^t \right),$$

$$f \left(s^t \right) + f^* \left(s^t \right) = \phi_F \left(s^t \right),$$

to completely describe the model.
A3. Cost Dispersion Model

What we refer to as the Cost Dispersion Model is a simple variant of Chari, et al (2001) with heterogenous firms and no capital, money or nominal rigidities. Rather than develop the entire model, we describe some key elements of the model. In each country, consumers purchase a final good produced from differentiated Domestic and Foreign intermediates using a CES production function. Final goods producers solve the following problem:

$$\max P (s^t) - \int_{0}^{1} P_D (i, s^t) y_D (i, s^t) \, di - \int_{0}^{1} P_F (i, s^t) y_F (i, s^t) \, di,$$

subject to:

$$y (s^t) = \left[ a_1 \left( \int_{0}^{1} y_D (i, s^t)^{\theta} \right)^{\frac{1}{\theta}} + (1 - a_1) \left( \int_{0}^{1} y_F (i, s^t)^{\theta} \right)^{\frac{1}{\theta}} \right]^{\frac{1}{\theta}}.$$

In country $j$, the producer of variety $i \in [0, 1]$ produces $\phi_j (i; s^t) = \phi_j (s^t) \ast (1 + \alpha i)$ units from each unit of labor, where $\phi_j (s^t)$ is the time varying aggregate productivity shock. Because each producer is a monopolist in its variety and faces constant elasticity demand, it will set its price to be the same constant markup over cost in each country. Defining $p_j (i)$ as the price of variety $i$ of a country $j \in \{D, F\}$ good, the ratio of the highest to lowest price is

$$\frac{p_D (0)}{p_D (1)} = \frac{p_F (0)}{p_F (1)} = 1 + \alpha.$$

With shocks to $\phi_j (s^t)$ this model is identical, in terms of volatilities of the real exchange rate and terms of trade, to the Standard Model for all $\theta$. Moreover, the model converges to the Standard Model as $\theta \to 1$. 
A4. Calibration

We can determine \((q, \phi)\) based on the ratio of profit income (capital’s share) to labor income and the ratio of reservation price to the average transaction price in the closed economy.

\[
\begin{align*}
k &\equiv \frac{\Pi}{wL} = \frac{q \left( \frac{\mathcal{P}}{\frac{w}{\phi}} \right) N}{wL} \\
\mu &\equiv \frac{\mathcal{P}}{P(\mathcal{P})} = \frac{1 - q + \phi}{1 - q + \phi q} \\
\Rightarrow \quad q &= \frac{k}{\mu - 1 + \mu k} \\
\Rightarrow \quad \phi &= (\mu - 1)(1 + k)
\end{align*}
\]

An increase in the max mean ratio \((\mu)\) leads to an increase in productivity and a decrease in \(q\) (increase in competition). An increase in profit income leads to an increase in productivity and increase in \(q\) (decrease in competition).

\[
\frac{\partial q}{\partial k} = \frac{\mu - 1}{[\mu - 1 + \mu k]^2} > 0
\]
References


Table 1 - Summary Statistics on US Imports

<table>
<thead>
<tr>
<th>Year</th>
<th>Obs</th>
<th>Goods</th>
<th>Obs_good</th>
<th>1+</th>
<th>5+</th>
<th>10+</th>
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<tbody>
<tr>
<td>1974</td>
<td>61792</td>
<td>6960</td>
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<td>0.86</td>
<td>0.60</td>
<td>0.34</td>
</tr>
<tr>
<td>1975</td>
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<td>0.85</td>
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<td>0.33</td>
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<tr>
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<td>7295</td>
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<td>0.86</td>
<td>0.60</td>
<td>0.34</td>
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<td>8.85</td>
<td>0.86</td>
<td>0.59</td>
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</tr>
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<td>0.89</td>
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<td>0.39</td>
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<td>10.73</td>
<td>0.89</td>
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<tr>
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<td>97095</td>
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<td>0.89</td>
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<td>12.54</td>
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<td>0.89</td>
<td>0.67</td>
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<td>157628</td>
<td>12979</td>
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<td>12.72</td>
<td>0.94</td>
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<td>1994</td>
<td>182039</td>
<td>13671</td>
<td>13.32</td>
<td>0.95</td>
<td>0.76</td>
<td>0.51</td>
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1+ represents fraction of goods imported more than 1 country

5+ (10+) represents fraction of goods imported more than 5 (10) countries
Table 2 - Summary Statistics on US Imports Categories

<table>
<thead>
<tr>
<th>Year</th>
<th>First Yr</th>
<th>Last Yr</th>
<th>Goods</th>
<th>Plus</th>
<th>Minus</th>
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<tr>
<td>1974</td>
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<td>240</td>
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<td>5.6%</td>
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<td>517</td>
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<tr>
<td>1976</td>
<td>549</td>
<td>369</td>
<td>9258</td>
<td>6.1%</td>
<td>2.6%</td>
</tr>
<tr>
<td>1977</td>
<td>574</td>
<td>1483</td>
<td>9463</td>
<td>6.2%</td>
<td>4.0%</td>
</tr>
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<td>1978</td>
<td>3019</td>
<td>238</td>
<td>10999</td>
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<tr>
<td>1979</td>
<td>290</td>
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<td>2.2%</td>
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<tr>
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<tr>
<td>1981</td>
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<td>3.7%</td>
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<tr>
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<td>2399</td>
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<tr>
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<tr>
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<td>323</td>
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<td>393</td>
<td>369</td>
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<td>2.4%</td>
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<tr>
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<td>773</td>
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<tr>
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<tr>
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<tr>
<td>1998</td>
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<tr>
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<td>2489</td>
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<td>3.7%</td>
</tr>
<tr>
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<td>17578</td>
<td>15255</td>
<td>11.1%</td>
<td>3.7%</td>
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</tbody>
</table>
Table 3 - Transition Matrix for $RQ_{it}$ to $RQ_{it+1}$

<table>
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<tr>
<th></th>
<th>Not In Sample</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
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<td>Not In Sample</td>
<td>0.711*</td>
<td>0.109</td>
<td>0.044</td>
<td>0.043</td>
<td>0.046</td>
<td>0.047</td>
</tr>
<tr>
<td>$T + 1$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>0.101</td>
<td>0.575</td>
<td>0.185</td>
<td>0.066</td>
<td>0.041</td>
<td>0.032</td>
</tr>
<tr>
<td>2</td>
<td>0.038</td>
<td>0.179</td>
<td>0.465</td>
<td>0.203</td>
<td>0.076</td>
<td>0.039</td>
</tr>
<tr>
<td>3</td>
<td>0.036</td>
<td>0.064</td>
<td>0.199</td>
<td>0.419</td>
<td>0.208</td>
<td>0.074</td>
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<td>4</td>
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<td>0.078</td>
<td>0.203</td>
<td>0.438</td>
<td><strong>0.204</strong></td>
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<tr>
<td>5</td>
<td>0.035</td>
<td>0.031</td>
<td>0.04</td>
<td>0.076</td>
<td>0.201</td>
<td>0.616</td>
</tr>
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</table>

These matrices measure how a good's price dispersion changes over time relative to other goods. Each year a distribution of price dispersion is generated. Goods are divided into 5 equally sized bins and then the transition across bins (or out of the sample is measured). For instance, the highlighted term indicates that there is a 20.4% chance that a good with price dispersion in the highest quintile in one year has price dispersion in the second highest quintile in the next year.

* A good not in the sample in one year has a 71.1% chance of not being in the sample in the next year.
Table 4 - Impact of 10% increase in country F technology

<table>
<thead>
<tr>
<th>QUANTITIES</th>
<th>Base</th>
<th>+10%</th>
<th>Δ</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi_F/\phi_D$</td>
<td>1.000</td>
<td>1.100</td>
<td>10.0%</td>
</tr>
<tr>
<td>$Y$</td>
<td>0.609</td>
<td>0.605</td>
<td>-0.7%</td>
</tr>
<tr>
<td>$Y^*$</td>
<td>0.609</td>
<td>0.636</td>
<td>4.4%</td>
</tr>
<tr>
<td>$d$</td>
<td>0.518</td>
<td>0.516</td>
<td>-0.5%</td>
</tr>
<tr>
<td>$f$</td>
<td>0.091</td>
<td>0.096</td>
<td>5.4%</td>
</tr>
<tr>
<td>$d^*$</td>
<td>0.091</td>
<td>0.090</td>
<td>-1.7%</td>
</tr>
<tr>
<td>$f^*$</td>
<td>0.518</td>
<td>0.540</td>
<td>4.2%</td>
</tr>
<tr>
<td>$f/d$</td>
<td>0.176</td>
<td>0.187</td>
<td>5.9%</td>
</tr>
<tr>
<td>$C$</td>
<td>0.843</td>
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<td>0.4%</td>
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<tr>
<td>$C^*$</td>
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<td>0.871</td>
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</table>

<table>
<thead>
<tr>
<th>PRICES</th>
<th>Base</th>
<th>+10%</th>
<th>Δ</th>
</tr>
</thead>
<tbody>
<tr>
<td>$W^*/W$</td>
<td>1.000</td>
<td>0.971</td>
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</tr>
<tr>
<td>$P_D^*/P_D$</td>
<td>1.000</td>
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<td>-1.0%</td>
</tr>
<tr>
<td>$P_F^*/P_D$</td>
<td>1.000</td>
<td>0.989</td>
<td>-1.1%</td>
</tr>
<tr>
<td>$\tau = P_F/P_D^*$</td>
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<td>-6.5%</td>
</tr>
<tr>
<td>$P$</td>
<td>0.723</td>
<td>0.714</td>
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</tr>
<tr>
<td>$P^*$</td>
<td>0.723</td>
<td>0.669</td>
<td>-7.4%</td>
</tr>
<tr>
<td>$P_F^*/P_D$</td>
<td>1.000</td>
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<tr>
<td>$P/P^*$</td>
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<tr>
<td>$\bar{P}_F/\bar{P}_D$</td>
<td>1.000</td>
<td>0.963</td>
<td>-3.7%</td>
</tr>
<tr>
<td>$\bar{P}_F^*/\bar{P}_D$</td>
<td>1.000</td>
<td>0.962</td>
<td>-3.8%</td>
</tr>
<tr>
<td>$\bar{P}_F/\bar{P}_F^*$</td>
<td>1.000</td>
<td>1.021</td>
<td>2.1%</td>
</tr>
<tr>
<td>$\bar{P}_D/P_D^*$</td>
<td>1.000</td>
<td>1.020</td>
<td>2.0%</td>
</tr>
</tbody>
</table>
Table 5 Relative Prices in Theoretical Economies

Low Sigma Economies \((\sigma = 2)\)

<table>
<thead>
<tr>
<th></th>
<th>Volatility</th>
<th>Auto Corr.</th>
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</thead>
<tbody>
<tr>
<td></td>
<td>TOT</td>
<td>RER</td>
</tr>
<tr>
<td>DATA</td>
<td>3.68</td>
<td>4.43</td>
</tr>
<tr>
<td>Standard</td>
<td>1.21</td>
<td>0.85</td>
</tr>
<tr>
<td>Cost Dispersion</td>
<td>1.21</td>
<td>0.85</td>
</tr>
<tr>
<td>Price Dispersion</td>
<td>2.02</td>
<td>2.06</td>
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<tr>
<td>High Price Dispersion</td>
<td>3.03</td>
<td>2.85</td>
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</table>

High Sigma Economies \((\sigma = 6)\)

<table>
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<th>Volatility</th>
<th>Auto Corr.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
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<tr>
<td>Cost Dispersion</td>
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<tr>
<td>Price Dispersion</td>
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<tr>
<td>High Price Dispersion</td>
<td>3.35</td>
<td>4.52</td>
</tr>
</tbody>
</table>

All statistics are standard deviation of relative price divided by standard deviation of output. Terms of Trade statistics (TOT) are from Backus, et al. (95) and Real Exchange Rate (RER) data are from Chari, et al. (2001). All statistics are based on Hodrick-Prescott filtered data. Entries are averages over 100 simulations of length 120.
Figure 1: US Relative Prices

Figure 2: Mean Price Dispersion by Year
Figure 3: Median ratio of Max Price and Mean Price of Imports

Figure 4: Changes in Price Dispersion
Figure 5: Shifting Price Distributions
Figure 6: Relative Prices and Parameters
Figure 7: Relative Prices and Max-Mean ratio and Profit