Model of a self-fulfilling debt crisis

Motivation: Mexican crisis

- Crisis occurred with “sound” fundamentals.
  - why now, and not earlier?
  - if in Mexico, why not elsewhere?

Model:

- Crisis occurrence depends upon extrinsic uncertainty.
- Crises zone: conditions for crisis depend on fundamentals:
  - debt vs. output
  - term structure of debt.
- In crisis zone probability of a crisis is arbitrary.
- Examine optimal government policy if a crisis can occur in equilibrium:
  - motivates fleeing zone by lowering debt.
- Examine role of debt maturity in preventing a crisis.
Prior literature

- Diamond and Dybvig (1993)
- Calvo (1988)
  - multiple equilibria with different default levels
  - investors expectations of default change interest rate
  - change in interest rate induces different default levels
  - (our model shows that government cannot peg interest rate)
  - a simple model of lending crisis
  - with 2 period as opposed to 1 period debt possibility of crisis reduced
  - (our model shows that once crisis has started there is nothing government can do)
- Chari and P. Kehoe (1996)
  - information cascade story for crises
  - opposite information assumption: no communication
Model

- infinite horizon, discrete time model with three types of actors and one good in each period

- actors:
  - Government:
    - cannot commit to its policies or repayment of debts
    - sequentially chooses spending and borrowing levels
    - only source of revenue is a flat rate income tax
    - borrows by issuing pure discount bonds
    - benevolent concern for consumers

\[ g_t + z_t B_t \leq \theta a_t f(k_t) + q_t B_{t+1} \]
- **(International) Bankers:**
  - large number of risk neutral lenders with discount factor $\beta$
  - price of government’s one period bonds is $q$
  - $q$ depends on amount of debt issued
    \[ x_t + q_t b_{t+1} \leq \bar{x} + z_t b_t \]

- **Consumers:** (risk neutral in consumption for simplicity)
  choose $\{c_t, k_{t+1}\}$ and have
  \[ E \sum_{t=0}^{\infty} \beta^t [c_t + v(g_t)] \]
  s.t. $c_t + k_{t+1} \leq (1 - \theta) a_t f(k_t)$
Default penalty $a_t$ falls from 1 to $\alpha < 1$ forever, and government is excluded from credit market

- They argue for direct penalties enforcing repayment.
- Cole and P. Kehoe show spillovers can motivate.
- Both provide explanations for $a_t$. 
Exogenous sunspot variable: $\zeta_t$ is i.i.d. and uniformly distributed on $[0, 1]$

Timing within a period:

1. $\zeta_t$ is realized, and the aggregate state is $s_t = (B_t, K_t, a_{t-1}, \zeta_t)$;
2. the government chooses $B_{t+1}$;
3. each banker chooses $b_{t+1}$, which along with $z_t$ determines $x_t$;
4. the government chooses $z_t$ and $g_t$;
5. each consumer chooses $k_{t+1}$ and $c_t$. 
Recursive equilibrium

- The aggregate state is $s_t = (B_t, K_t, a_{t-1}, \zeta_t)$
- A collection of value functions and policy functions:
  - For consumers, $V_c(k, s, B', g, z)$ and $c(k, s, B', g, z)$, $k'(k, s, B', g, z)$
  - For bankers, $V_b(b, s, B')$
  - For the government, $V_g(s)$ and $B'(s)$, and $g(s', B', q)$, $z(s', B', q)$
- An equation of motion for the aggregate capital stock $K'(s, B', g, z)$.
  - Consistency of consumer’s behavior $K'$:
    $$K'(s, B', g, z) = k'(K, s, B', g, z).$$
- Banker’s problem: price function
  $$q(s, B') = \beta Ez(s', B', q(s', B'(s')))$$
Consumer’s problem:

\[
V_c(k, s, B', g, z) = \max_{c, k'} c + v(g) + \beta EV_c(k', s', B'', g', z')
\]

subject to

\[
c + k' \leq (1 - \theta)a(s, z)f(k)
\]

\[
c, k' \geq 0
\]

Use the government’s policy functions, \(B'(\cdot), z(\cdot), g(\cdot),\) along with \(q(\cdot)\) and \(K'(\cdot)\) to determine \(s', B', g'\) and \(z'.\)
Government problems:

- **problem one** pick $B'$

\[ V_g(s) = \max_{B'} c(K, s, B', g, z) + v(g) + \beta EV_g(s') \]

Use the government’s policy functions, $z(\cdot), g(\cdot)$, along with $q(\cdot)$ and $K'(\cdot)$ to determine $g, z$, and $s'$.

- **problem two** pick $z$ and $g$

\[ \max_{g,z} c(K, s, B', g, z) + v(g) + \beta EV_g(s') \]

subject to

\[ g + zB \leq \theta a(s, z)f(K) + qB' \]

\[ z = 0 \text{ or } z = 1 \]

\[ g \geq 0. \]

Use $K'(\cdot)$ to determine $s'$. 
Agenda

- zero probability of crisis equilibrium
  - conditions for no-lending continuation equilibrium
- positive probability of crisis equilibrium
  - changes behavior before crisis
  - changes interest rate
- maturity of debt
  - little role if no sunspot
  - important role if sunspot
  - nothing government can do once crisis has started
- examine Mexican crisis.
What happens in default?

- Productivity falls from $a_t = 1$ to $a_t = \alpha < 1$.
- Government loses all access to credit markets after a default.
- Equilibrium price of government debt is $q = 0$.
- **Consumers:** invest $k^d$ and eat the remainder:
  \[
  k^d : (1 - \theta)\alpha \beta f^d (k^d) = 1 \\
  c^d(k) = (1 - \theta)\alpha f(k) - k^d
  \]
- **Bankers:** buy none of the government debt since they believe that $z = 0$.
- **Government:** eats everything it raises in revenue, issues no new claims and sets $z = 0$; its post-default payoff is given by
  \[
  c^d(K) + v(\theta \alpha f(K)) + \\
  \beta [c^d(k^d) + v(\theta \alpha f(k^d))] / (1 - \beta).
  \]
No crisis equilibrium with lending

- State is \((B, K, a_{-1})\) (ignore \(\zeta\)).

- Equilibrium price function:
  - \(q(s, B') = \beta\) if government has no incentive to default
  - \(q(s, B') = 0\) if it does.

- Consumers:
  - if prior default, follow default continuation equilibrium
  - if default next period, set \(k' = k^d\) and \(c = (1 - \theta)f(k) - k^d\)
  - otherwise invest \(k^n\) and eat \(c^n(k)\)
    \[
    k^n : (1 - \theta)\beta f'(k^n) = 1
    \]
    \[
    c^n(k) = (1 - \theta)f(k) - k^n.
    \]

- Bankers:
  - buy any amount of debt at the price \(\beta\) if they believe government will not default
  - buy none if they believe that the government will default.
Government:

- Payoff from defaulting today:

\[
V_g^{d}(s, B', q) = c^d(K) + v(\theta \alpha f(K) + qB') \\
+ \beta [c^d(k^d) + v(\theta \alpha f(k^d))]/(1 - \beta)
\]

- Payoff from not defaulting today, given \(a_{-1} = 1\) and others do not believe will default:

\[
V_g^{n}(s, B', q) = c^n(K) + v(\theta f(K) - B + qB') \\
+ \beta V_g(s')
\]

- Optimal default rule: choose the maximum of two payoffs,

\[
Z(s, B', q) = \begin{cases} 
1 & \text{if } V_g^{n}(s, B', \beta) \geq V_g^{d}(s, B', \beta) \\
0 & \text{otherwise}
\end{cases}
\]
Equilibrium:

- if initial debt low enough, get commitment outcome and payoff
  \[ V^n_g(s, B, \beta) \]
  - government follows stationary policy: \( B' = B \)

- if initial debt too high for stationary policy, then government runs down debt to reduce incentive to default and then goes stationary
  - recursively construct the no defaults sets of states and the government’s payoff from defaulting and show that this occurs in no more than two recursions

- if too high for this, then no lending/default only equilibrium with payoff
  \[ V^d_g(s, 0, 0) \]
No-lending equilibrium

belief that the government will default can be self-fulfilling if it induces the government to default

- no-lending continuation condition

\[ V_{gd}(s, 0, 0) > V_{gn}(s, 0, 0) \]

- payoff in no-lending continuation equilibrium:

\[ V_{gd}(s, 0, 0) \]

- in continuation equilibrium \( z(s) = 0 \) and \( q(s, B') = 0 \)

- government sets \( B'(s) = 0 \), and consumers set \( k' = k^d \).
Crisis equilibrium?

- Lending equilibrium participation constraint:

\[ V^n_g(s, B', \beta) \geq V^d_g(s, B', \beta) \]

  - Define \( \bar{B} \) : largest \( B \) such that there exists a \( B' \geq 0 \) for which participation constraint is satisfied.

- No-lending continuation condition:

\[ V^d_g(s, B', 0) = V^d_g(s, 0, 0) > V^n_g(s, 0, 0) = V^n_g(s, B', 0) \]

  - Define \( \bar{b}(K) \):

\[ V^d_g((\bar{b}(K), K, 1), 0, 0) = V^n_g((\bar{b}(K), K, 1), 0, 0) \]

  (notice that \( \bar{b}'(K) > 0 \))

- Crisis Zone exists if \( \bar{b}(k^n) < \bar{B} \).
Self-fulfilling crisis equilibrium

- equilibrium description:
  - if $\zeta < \pi$ and $B > \bar{b}(K)$, then a crisis occurs
  - if $\zeta \geq \pi$ or $B \leq \bar{b}(K)$, then a crisis cannot occur today

- equilibrium price of government debt:
  - $\beta$ if a crisis cannot occur next period
  - $\beta(1 - \pi)$ if it can occur
  - 0 if the government does not weakly prefer to repay

- policy function of the consumer:
  - $k' = k^n$ if the probability a default next period is zero
  - $k' = k^\pi$ if there could be a crisis next period

\[
k^\pi : [(1 - \pi) + \pi \alpha] (1 - \theta) \beta f'(k^\pi) = 1
\]

  - $k' = k^d$ if default has either already occurred or is believed will occur next period.
Equilibrium outcomes:

- If debt below $\bar{b}(K)$ then
  - government not in crisis zone
  - optimal policy is stationary $g$ and $B$.

- If debt slightly above $\bar{b}(K)$ then
  - government is in crisis zone
  - interest rate on debt discretely higher
  - capital stock discretely lower
  - optimal to run debt down to $\bar{b}(K)$ in one step
  - capital stock jumps up and interest rate jumps down when leave the crisis zone.

- Yet higher debt,
  - government in crisis zone
  - optimal to run down debt in several steps
  - if sufficiently high and $\pi$ sufficiently small may go stationary.
- Debt higher still,
  - participation constraint binds
  - jump debt down to secure new borrowing.

- Debt too high, default only outcome.
TRAJECTORIES

\[ q = 0, k = k^d \]

\[ q = (1-\pi) \beta \]

\[ k = k^n \]

Time
Three possible payoffs to the government:

- if a crisis cannot occur because $B' \leq \bar{b}(k^n)$, then the payoff to government is $V^g_n(s, B', \beta)$
- if the government prefers to default, then its payoff is $V^d_g(s, 0, 0)$
- if a crisis can occur,
  - let $V^T_g(s)$ denote the payoff to the government if it reduces its debt level in $T$ periods to $\bar{b}(k^n)$, at which point its payoff is $V^m_g(\bar{b}(k^n), k^n, 1, \bar{b}(k^n), \beta)$
  - note that the capital level next period is $k^\pi$, and it will continue at this level until $T$ periods hence when it rises to $k^n$
  - $g$ is constant between now and period $T - 1$ (may need to adjust in initial period to satisfy participation constraint)
  - as $T \to \infty$, $V^T_g(s)$ converges to the payoff from a policy in which the government’s debt always exceeds $\bar{b}(k^n)$, $V^\infty_g(s)$.
  - exists a best $T$ if we include $\infty$. 

Maturity of the debt and debt crises

- present value of the debt is $B$ and maximum maturity of the debt is $N$
- assume that we are in a no crisis equilibrium and hence $q = \beta$
- policy of maintaining a flat maturity structure: the value of the payments coming due in each period is constant
  - if $B_N$ is the amount coming due, then
    \[
    B_N = B/(1 + \beta + \ldots + \beta^{N-1}) = \frac{1 - \beta}{1 - \beta^N}B
    \]
    to maintain this debt structure the government issues $B_N$ units of $N$ period discount bonds
  - its net payments are $B_N(1 - \beta^N)$
- participation constraint converges to no-lending continuation condition because
  \[
  \lim_{N \to \infty} \beta^N B_N = 0
  \]
- without possibility of crisis there is little role for maturity, but with possibility of crisis there is a big role.
1994-1995 Mexican Crisis

- in 1994
  - political crisis in Mexico
  - international reserves fell sharply in March and April, then stabilized
  - Mexican central bank sterilized
  - large fraction of public debt converted to *tesobonos* and maturity shrank
  - November another run on reserves occurred
  - December-January markets refused to roll over debt coming due.

- puzzle: fundamentals sound even after devaluation
  - debt/GDP had been falling
  - maturity structure of debt did shorten.
interpretation of events using model
  - with dollar-indexed debt, default discrete event with discrete penalty
  - shortened maturity put Mexico in the CRISIS ZONE
  - political turmoil helped to stir the caldron
  - crisis only ended with offer of Clinton’s 31 January loan package.

results of model
  - models with debt roll over have crises equilibria
  - crisis can be avoided only by keeping debt down and maturity long.

given the model, surprise is that we do not see more crises.
## Debt/GDP for Selected Countries (Percent)

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Mexico</td>
<td>55.2</td>
<td>45.8</td>
<td>35.1</td>
<td>35.0</td>
<td>37.4</td>
</tr>
<tr>
<td>Belgium</td>
<td>130.7</td>
<td>132.6</td>
<td>134.4</td>
<td>141.3</td>
<td>140.1</td>
</tr>
<tr>
<td>France</td>
<td>40.4</td>
<td>41.1</td>
<td>45.6</td>
<td>52.9</td>
<td>56.8</td>
</tr>
<tr>
<td>Germany</td>
<td>43.4</td>
<td>42.7</td>
<td>47.3</td>
<td>51.8</td>
<td>54.6</td>
</tr>
<tr>
<td>Greece</td>
<td>77.7</td>
<td>81.7</td>
<td>88.6</td>
<td>117.1</td>
<td>119.8</td>
</tr>
<tr>
<td>Italy</td>
<td>100.5</td>
<td>103.9</td>
<td>111.4</td>
<td>120.2</td>
<td>122.6</td>
</tr>
<tr>
<td>Spain</td>
<td>48.7</td>
<td>49.9</td>
<td>53.0</td>
<td>59.4</td>
<td>63.5</td>
</tr>
</tbody>
</table>

## Calendar of Maturing Debt 1995
### Tesobonos and Cetes
(millions USD)

<table>
<thead>
<tr>
<th></th>
<th>Cetes</th>
<th>Tesobonos</th>
<th>Cetes plus Tesobonos</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st quarter</td>
<td>3,015.00</td>
<td>9,873.94</td>
<td>12,888.94</td>
</tr>
<tr>
<td>2nd quarter</td>
<td>1,563.47</td>
<td>6,429.26</td>
<td>7,992.72</td>
</tr>
<tr>
<td>3rd quarter</td>
<td>1,042.66</td>
<td>8,425.70</td>
<td>9,468.36</td>
</tr>
<tr>
<td>4th quarter</td>
<td>943.13</td>
<td>3,927.83</td>
<td>4,870.97</td>
</tr>
</tbody>
</table>
## Tesobonos Auctions

<table>
<thead>
<tr>
<th>Date</th>
<th>Yield (percent)</th>
<th>Amount Sold (million dollars)</th>
<th>Amount Offered (millions dollars)</th>
</tr>
</thead>
<tbody>
<tr>
<td>6 Dec 1994</td>
<td>8.39</td>
<td>420</td>
<td>420</td>
</tr>
<tr>
<td>13 Dec 1994</td>
<td>8.23</td>
<td>375</td>
<td>375</td>
</tr>
<tr>
<td>20 Dec 1994</td>
<td>8.61</td>
<td>416</td>
<td>600</td>
</tr>
<tr>
<td>27 Dec 1994</td>
<td>10.23</td>
<td>28</td>
<td>600</td>
</tr>
<tr>
<td>3 Jan 1995</td>
<td>12.31</td>
<td>52</td>
<td>500</td>
</tr>
<tr>
<td>10 Jan 1995</td>
<td>19.63</td>
<td>63</td>
<td>400</td>
</tr>
<tr>
<td>17 Jan 1995</td>
<td>19.75</td>
<td>400</td>
<td>300</td>
</tr>
<tr>
<td>24 Jan 1995</td>
<td>21.40</td>
<td>50</td>
<td>50</td>
</tr>
<tr>
<td>31 Jan 1995</td>
<td>24.98</td>
<td>155</td>
<td>150</td>
</tr>
</tbody>
</table>

Numerical Example

- Period: 2/3 year
- Utility: $E \sum_{t=0}^{\infty} 0.97^t(c_t + \log(g_t))$
- Possibility of default: $\pi = 0.02$
  - $\beta = 0.97$ implies yearly discount factor 0.955, which implies a yearly yield of 0.047 on risk free bonds
  - $\pi = 0.02$ implies a yearly yield of 0.079 on Mexican government bonds
- Feasibility constraint:
  \[ c + g + k' - 0.95k + zB \leq 2k^{0.4} + qB' \]
- $\delta = 0.05$ corresponds to a yearly discount rate of 0.074
- Capital Stock:
  \[ (1 - \theta)[(0.98 + 0.02\alpha)0.8(k^\pi)^{-0.6} - 0.05] = 0.97^{-1} - 1 \]
- Tax rate: $\theta = 0.20$
- Default Penalty: 0.05 ( $\alpha = 0.95$ )
  - $k^\pi = 39.04$
  - GDP$= (3/2)2(k^\pi)^{0.4} = 12.99$
  - capital/output ratio = 3.00
  - investment/GDP ratio = 0.23
  - tax revenues/GDP ratio = 0.15
- Initial debt: $B_0 = 2.67$
  - debt/GDP = 0.20
GOVERNMENT DEBT POLICY FUNCTION
WITH GOVERNMENT DISCOUNT RATE 0.93

New Debt Level Relative to GDP vs. Initial Debt Level Relative to GDP

45 degree line
Policy Function