After financial openings, like that in Spain and Mexico in the late 1980s, large capital inflows have been accompanied by substantial appreciations in the real exchange rate.

Previous work has shown that, to capture the timing of capital inflows and the changes in the relative prices of nontraded goods, frictions in factor markets are important.

The model here stresses the role of the imperfect substitutability between domestic and foreign traded goods in determining the terms of trade, whose movements are major components of real exchange rate fluctuations.
This is preliminary work based on

"Capital Flows and Real Exchange Rate Fluctuations Following Spain’s Entry into the European Community,” with Gonzalo Fernandez de Cordoba.

"Tradability of Goods and Real Exchange Rate Fluctuations” with Caroline M. Betts
REAL EXCHANGE RATE

\[ RER = NER \times \frac{P_{ger}}{P_{esp}} \]

units: \frac{\text{pesetas}}{\text{deutsche marks}} \times \frac{\text{deutsche marks/German basket}}{\text{pesetas/Spanish basket}}

= \frac{\text{Spanish basket}}{\text{German basket}}
Suppose $P_{esp}^T = NER \times P_{ger}^T$ (law of one price)

\[
RER^N = \frac{P_{esp}^T}{P_{ger}^T} \times \frac{P_{ger}}{P_{esp}} = \frac{(P_{ger}/P_{ger}^T)}{(P_{esp}/P_{esp}^T)}
\]

$RER^N$ is the part of the real exchange rate explained by the relative price of nontraded goods.
What is left over in $RER$ is the part explained by the terms of trade.

$$RER^T = \frac{NER \times P_{ger}^T}{P_{esp}^T}$$

Notice that

$$RER = RER^T \times RER^N$$

**TRADED**
Agriculture and Industry

**NONTRADED**
Construction and Services
MODELING CAPITAL FLOWS INTO SPAIN

\[ Y_j = AN_j^{1-\alpha} K_j^\alpha \]

\[ y_j = A k_j^\alpha \]

\( (r_j = \alpha A k_j^{\alpha-1} - \delta) \)

\[ y_{\text{esp}} = 21,875 \quad \text{(1986)} \]

\[ y_{\text{ger}} = 27,879 \]

\[ k_{\text{esp}} = 45,528 \]

\[ k_{\text{ger}} = 73,618 \]

\[ \frac{y_{\text{esp}}}{y_{\text{ger}}} = \left( \frac{k_{\text{esp}}}{k_{\text{ger}}} \right)^\alpha \]
Let $\alpha = 0.3020$

\[
\frac{y_{esp}}{y_{ger}} = 0.8649
\]

in data

\[
\frac{y_{esp}}{y_{ger}} = 0.7847
\]

Differences in capital per worker explain 63 percent of differences in output per worker between Spain and Germany.
HOW LARGE SHOULD CAPITAL FLOWS BE?

Calibrate

\[ A_{esp} = \frac{y_{esp}}{k_{esp}} = 857.3298 \]

\[ A_{ger} = \frac{y_{ger}}{k_{ger}} = 945.0353 \]

Equate marginal products

\[ \alpha A_{esp} k_{esp}^{\alpha - 1} = \alpha A_{ger} k_{ger}^{\alpha - 1} \]

\[ k_{ger} = 73,618 \quad \text{implies} \quad k_{esp} = 64,030 \]

Spanish capital stock would have to increase by 18,502, which is 85 percent of Spanish GDP, 41 percent of Spanish capital stock.

\[ (r_{ger} = 0.057 \quad \text{implies} \quad r_{esp} = 0.088) \]
Simple Model

Consumers

\[
\max \sum_{t=0}^{\infty} \beta^t (\epsilon c_{Tt}^\rho + (1 - \epsilon) c_{Nt}^\rho) / \rho
\]

subject to

\[
c_{Tt} + p_t c_{Nt} + a_{t+1} = w_t \bar{\ell} + (1 + r_t) a_t
\]

\[
a_t \geq -A
\]

where

\[
a_t = q_{t-1} k_t + b_t.
\]
Feasibility conditions

\[ c_{Nt} + z_{Nt} = A_N k_{Nt}^{\alpha_N} \ell_{Nt}^{1-\alpha_1} \]

\[ k_{Tt} + k_{Nt} = k_t \]

\[ \ell_{Tt} + \ell_{Nt} = \bar{\ell} \]

\[ k_{t+1} - (1 - \delta)k_t = G_{Zt}^{\gamma}Z_{Tt}^{1-\gamma} \]

\[ c_{Tt} + z_{TNt} + b_{t+1} = A_T k_{Tt}^{\alpha_T} \ell_{Tt}^{1-\alpha_T} + (1 + r_t)b_t \]
Calibration

\[ y_N = 1.0481k_N^{0.2869} \ell_N^{0.7131} \]
\[ y_T = 1.0214k_T^{0.3109} \ell_T^{0.6891} \]
\[ x = 1.9434z_T^{0.3802} z_N^{0.6198} \]
\[ \delta = (\delta k/y)/(k/y) = 0.0576 \]
\[ \epsilon = \frac{(c_N/c_T)^{1-\rho}}{1 + (c_N/c_T)^{1-\rho}} = \frac{0.5830^{1-\rho}}{1 + 0.5830^{1-\rho}} \]
\[ \beta = 1/(1 + r^*) = 0.9463 \]
\[ (r^* = \alpha A_{ale} k_{ale}^{\alpha-1} - \delta) \]
Labor adjustment frictions

\[ \ell_{Nt+1} \leq \lambda \ell_{Nt} \]

\[ \ell_{Tt+1} \leq \lambda \ell_{Tt} \]

\[ \lambda > 1 \]

(In the simulations, \( \lambda = 1.01 \).)
Capital adjustment frictions

\[ x_{Nt+1} + x_{Tt+1} \leq G z_N^{\gamma} z_T^{1-\gamma} \]

\[ k_{Nt+1} \leq \phi(x_{Nt+1}/k_T)k_{Nt} + (1 - \delta)k_{Nt} \]

\[ k_{Tt+1} \leq \phi(x_{Tt+1}/k_T)k_{Tt} + (1 - \delta)k_{Tt} \]

\[ \phi'(x/k) > 0, \phi''(x/k) < 0, \phi(\delta) = \delta, \phi'(\delta) = 1 \]

\[ (\phi(x/k) = (\delta^{1-\eta}(x/k)^\eta - (1 - \eta)\delta)/\eta, 0 < \eta \leq 1) \]

(In the simulaciones \( \eta = 0.9 \).)
**MODEL WITH ARMINGTON AGGREGATOR**

**Consumers**

$$\max \sum_{t=0}^{\infty} \beta^t (\epsilon c_T^T + (1 - \epsilon) c_N^T - 1) / \rho$$

subject to

$$p_T c_T + p_N c_N + a_{t+1} = w_t \bar{\ell} + (1 + r_t) a_t + T_t$$

$$a_t \geq -A$$

where

$$a_t = q_{t-1} k_t + b_t$$
Feasibility-Equilibrium Conditions

Domestically produced traded good

\[ x_{Dt} + x_{Ft} = A_D k_D^{\alpha_D} \ell_D^{1-\alpha_D} \]

Armington aggregator

\[ c_{Tt} + z_{Tt} = M(\mu x_{Dt}^\zeta + (1 - \mu)m_t^\zeta)^{1/\zeta} \]

Nontraded good

\[ c_{Nt} + z_{Nt} = A_N k_N^{\alpha_N} \ell_N^{1-\alpha_N} \]

Balance of payments

\[ m_t + b_{t+1} = p_{Dt}x_{Ft} + (1 + r_t)b_t \]

Investment

\[ k_{t+1} - (1 - \delta)k_t = Gz_T^\gamma z_{Nt}^{1-\gamma} \]
Foreign demand

\[ x_{Ft} = D((1 + \tau_{Ft})p_{Dt})^{\frac{-1}{1-\xi}} \]

Factor markets

\[ k_{Dt} + k_{Nt} = k_t, \quad \ell_{Dt} + \ell_{Nt} = \bar{\ell} \]

Transfer of tariff revenue

\[ T_t = \tau_{Dt}m_t \]
Profit maximization

\[ w_t = p_{Dt}A_D(1 - \alpha_D)(k_{Dt}/\ell_{Dt})^{\alpha_D} \]
\[ = p_{Nt}A_N(1 - \alpha_N)(k_{Nt}/\ell_{Nt})^{\alpha_N} \]

\[ 1 + r_t = (p_{Dt}A_D\alpha_D(\ell_{Dt}/k_{Dt})^{1-\alpha_D} + (1 - \delta)q_t)/q_{t-1} \]
\[ = (p_{Nt}A_N\alpha_N(\ell_{Nt}/k_{Nt})^{1-\alpha_N} + (1 - \delta)q_t)q_{t-1} \]

\[ p_{Tt} = q_t\gamma G(z_{Nt}/z_{Tt})^{1-\gamma} \]
\[ p_{Nt} = q_t(1 - \gamma)G(z_{Tt}/z_{Nt})^{\gamma} \]
\[
x_{Dt} = \mu \frac{1}{1-\xi} M^{\frac{\xi}{1-\xi}} \left( \frac{p_{Tt}}{p_{Dt}} \right) \frac{1}{1-\xi} \left( c_{Tt} + z_{Tt} \right)
\]

\[
m_t = (1 - \mu) \frac{1}{1-\xi} M^{\frac{\xi}{1-\xi}} \left( \frac{p_{Tt}}{1 + \tau_{Dt}} \right) \frac{1}{1-\xi} \left( c_{Tt} + z_{Tt} \right)
\]

where

\[
p_{Tt} = \frac{1}{M} \left[ \mu^{\frac{1}{1-\xi}} p_{Dt}^{\frac{-\xi}{1-\xi}} + (1 - \mu) \frac{1}{1-\xi} \left( 1 + \tau_{Dt} \right)^{\frac{-\xi}{1-\xi}} \right]^{\frac{-(1-\xi)}{\xi}}
\]
LABOR ADJUSTMENT FRICTIONS

\[ \ell_{D_{t+1}} \leq \lambda \ell_{D_t} \]
\[ \ell_{N_{t+1}} \leq \lambda \ell_{N_t} \]
\[ \lambda > 1 \]

If constraint binds, labor in traded goods sector receives a different wage, \( w_{D_t} \), than the wage of labor in the nontraded goods sector, \( w_{N_t} \).

(In simulations, \( \lambda = 1.01 \).)
**CAPITAL ADJUSTMENT FRICTIONS**

\[
\begin{align*}
i_{Dt+1} + i_{Nt+1} & \leq Gz_T^\gamma z_{Nt}^{1-\gamma} \\
k_{Dt+1} & \leq \phi(i_{Dt+1}/k_{Dt})k_{Dt} + (1 - \delta)k_{Dt} \\
k_{Nt+1} & \leq \phi(i_{Nt+1}/k_{Nt})k_{Nt} + (1 - \delta)k_{Nt}
\end{align*}
\]

\[\phi'(i/k) > 0, \quad \phi''(i/k) < 0, \quad \phi(\delta) = \delta, \quad \phi'(\delta) = 1\]

\[
(\phi(i/k) = (\delta^{1-\eta}(i/k)^\eta - (1 - \eta)\delta)/\eta, \quad 0 < \eta \leq 1)
\]

Adjusting the sector specific capital stock rapidly is costly. (In simulations \(\eta = 0.9\).)
REAL EXCHANGE RATE

Bilateral real exchange rate between the United States and country $i$:

$$RER_{i,us} = NER_{i,us} \frac{P_{us}}{P_i}$$

$NER_{i,us}$: nominal exchange rate — country $i$ currency units per U.S. dollar

$P_j$: price deflator or index for the basket of goods consumed or produced in country $j$, $j = us, i$. 
Decompose

\[ RER_{i,us} = \left( NER_{i,us} \frac{P^T_{us}}{P^T_i} \right) \left( \frac{P^T_i}{P_i} / \frac{P^T_{us}}{P_{us}} \right) RER_{i,us} = RER^T_{i,us} \times RER^N_{i,us} \]

where

\[ RER^T_{i,us} = NER_{i,us} \frac{P^T_{us}}{P^T_i} \]

is the real exchange rate of traded goods — the component that measures deviations from the law of one price and

\[ RER^N_{i,us} = \left( \frac{P^T_i}{P_i(P^T_i, P_i^N)} \right) / \left( \frac{P^T_{us}}{P_{us}(P^T_{us}, P_{us}^N)} \right) \]

is the (bilateral) relative price of nontraded (to traded) goods.
TRADED
Agriculture, Mining and Petroleum, and Manufacturing

NONTRADED
Construction and Services
MODELING CAPITAL FLOWS INTO MEXICO

\[ Y_j = AK_j^\alpha N_j^{1-\alpha} \]

\[ y_j = Ak_j^\alpha \]

\[ y_{\text{mex}} = 16,373, \quad y_{\text{us}} = 36,859 \quad (1989) \]

\[ r_j = \alpha Ak_j^{\alpha-1} - \delta \]

\[ r_{\text{mex}} = 0.13, \quad r_{\text{us}} = 0.05 (1989-1990) \]

\[ \frac{y_{\text{mex}}}{y_{\text{us}}} = \left( \frac{k_{\text{mex}}}{k_{\text{us}}} \right)^\alpha = \left( \frac{r_{\text{us}} + \delta}{r_{\text{mex}} + \delta} \right)^{\alpha_{1-\alpha}} \]

Let \( \alpha = 0.30, \quad \delta = 0.05 \)
\[ \frac{y_{\text{mex}}}{y_{\text{us}}} = \left( \frac{0.10}{0.18} \right)^{0.3} = 0.8383 \]

while, in the data,

\[ \frac{y_{\text{mex}}}{y_{\text{us}}} = 0.4442 \]

Difference in capital per worker explains 30 percent of difference in output per worker.
We can calibrate different productivity parameters $A_{mex}$ and $A_{us}$:

\[ k_j = \frac{\alpha y_j}{r_j + \delta} \]

\[ A_j = \frac{y_j}{k_j^\alpha} \]

$k_{mex} = 27,288, A_{mex} = 764.43$

$k_{us} = 110,577, A_{us} = 1130.95$

Productivity in the United States is 48 percent higher than in Mexico.
Notice that

\[
\frac{k_{\text{mex}}}{k_{\text{us}}} = \frac{27,288}{110,577} = 0.2468
\]

while, in the Summers-Heston data set,

\[
\frac{k_{\text{mex}}}{k_{\text{us}}} = \frac{21,985}{59,011} = 0.3726
\]

This would imply a much smaller difference in real interest rates:

\[
\frac{r_{\text{mex}} + \delta}{r_{\text{us}} + \delta} = \frac{k_{\text{us}} / y_{\text{us}}}{k_{\text{mex}} / y_{\text{mex}}} = \frac{1.6010}{1.3428} = 1.1923
\]

\[r_{\text{us}} = 0.05, \; \delta = 0.05 \text{ imply } r_{\text{mex}} = 0.07\]
HOW LARGE SHOULD CAPITAL FLOWS BE?

Equate returns on capital marginal products

\[ \alpha A_{mex} k_{mex}^{\alpha-1} - \delta = \alpha A_{us} k_{us}^{\alpha-1} - \delta \]

\[ N_{mex} k_{mex} + N_{us} k_{us} = N_{mex} 27,288 + N_{us} 110,577 \]

\[ N_{mex} = 27,302,000, \ N_{us} = 121,863,000. \]

Free capital flows result in \( k_{mex} = 56,019, \ k_{us} = 98,027. \)

Mexican capital stock would have to increase by 28,731, which is 175 percent of Mexican GDP, 105 percent of Mexican capital stock.
MODEL

Consumers

\[
\max \sum_{t=t_0}^{\infty} \beta^t \left[ \varepsilon \left( \frac{c_{Tt}}{n_t} \right)^{\rho} + (1 - \varepsilon) \left( \frac{c_{Nt}}{n_t} \right)^{\rho} - 1 \right] / \rho
\]

subject to

\[
p_{Ti} c_{Ti} + p_{Nt} c_{Nt} + a_{t+1} = w_t \ell_t + (1 + r_t) a_t + T_t
\]

\[
a_t \geq -A
\]

where

\[
a_t = q_{t-1} k_t + b_t,
\]

\[
k_0, b_0 \text{ given.}
\]

Here \( \ell_t \) is working-age population and \( n_t = 0.5 \ell_t + 0.5 \text{pop}_t \) is adult-equivalent population.
Production functions

Domestically produced traded good

\[ y_{Dt} = \min \left[ \frac{z_{TDt}}{a_{TD}}, \frac{z_{NDt}}{a_{ND}}, A_D k_{Dt}^{\alpha_D} \ell_{Dt}^{1-\alpha_D} \right] \]

Nontraded good

\[ y_{Nt} = \min \left[ \frac{z_{TNt}}{a_{TN}}, \frac{z_{NNt}}{a_{NN}}, A_N k_{Nt}^{\alpha_N} \ell_{Nt}^{1-\alpha_N} \right] \]

Investment good

\[ i_t = G z_{Tlt}^{\gamma} z_{Nlt}^{1-\gamma} \]

Armington aggregator

\[ y_{Tt} = M \left[ \mu x_{Dt}^{\xi} + (1 - \mu) m_t^{\xi} \right]^{\frac{1}{\xi}}. \]
**Market clearing**

Domestically produced traded good

\[ x_{Dt} + x_{Ft} = y_{Dt} \]

Composite traded good

\[ c_{Tt} + z_{TIt} + z_{TDt} + z_{TNt} = y_{Tt} \]

Nontraded good

\[ c_{Nt} + z_{NIt} + z_{NDt} + z_{NNt} = y_{Nt} \]

Investment good

\[ k_{t+1} - (1 - \delta)k_t = i_t \]

Factor markets

\[ k_{Dt} + k_{Nt} = k_t, \quad \ell_{Dt} + \ell_{Nt} = \ell_t \]
Balance of payments

\[ m_t + b_{t+1} = p_{Dt} x_{Ft} + (1 + r_t) b_t \]

Foreign demand

\[ x_{Ft} = D \left[ (1 + \tau_{Ft}) p_{Dt} \right]^{\frac{-1}{1-\zeta}} \]

Transfer of tariff revenue

\[ T_t = \tau_{Dt} m_t \]
Profit maximization

Domestically produced traded good

\[ w_t = (p_{Dt} - a_{TD} p_{Tt} - a_{ND} p_{Nt}) (1 - \alpha_D) A_D (k_{Dt} / \ell_{Dt})^{\alpha_D} \]

\[ 1 + r_t = \left[ (p_{Dt} - a_{TD} p_{Tt} - a_{ND} p_{Nt}) \alpha_D A_D (\ell_{Dt} / k_{Dt})^{1-\alpha_D} + (1 - \delta) q_t \right] / q_{t-1} \]

Nontraded good

\[ w_t = (p_{Nt} - a_{TN} p_{Tt} - a_{NN} p_{Nt}) (1 - \alpha_N) A_N (k_{Nt} / \ell_{Nt})^{\alpha_N} \]

\[ 1 + r_t = \left[ (p_{Nt} - a_{TN} p_{Tt} - a_{NN} p_{Nt}) \alpha_N A_N (\ell_{Nt} / k_{Nt})^{1-\alpha_N} + (1 - \delta) q_t \right] / q_{t-1} \]
Investment good

\[ p_{Tt} = q_t \gamma G \left( \frac{z_{NIt}}{z_{TIt}} \right)^{1-\gamma} \]

\[ p_{Nt} = q_t (1-\gamma) G \left( \frac{z_{TIt}}{z_{NIt}} \right)^{\gamma} \]

Armington aggregator

\[ p_{Dt} = p_{Tt} \mu M^\zeta \left( \frac{y_{Tt}}{x_{Dt}} \right)^{1-\zeta} \]

\[ 1 + \tau_{Dt} = p_{Tt} (1-\mu) M^\zeta \left( \frac{y_{Tt}}{m_t} \right)^{1-\zeta} \]

where

\[ p_{Tt} = \left( 1/M \right) \left[ \mu^{1-\zeta} p_{Dt}^{-\zeta} + (1-\mu)^{1-\zeta} (1 + \tau_{Dt})^{-\zeta} \right]^{-\frac{1}{\zeta}} \]
CAPITAL ADJUSTMENT FRICTIONS

\[ i_{Dt} + i_{Nt} \leq Gz^\gamma T^\gamma \]

\[ k_{Dt_{t+1}} \leq \phi \left( \frac{i_{Dt}}{k_{Dt}} \right) k_{Dt} + (1 - \delta) k_{Dt} \]

\[ k_{Dt_{t+1}} \leq \phi \left( \frac{i_{Dt}}{k_{Dt}} \right) k_{Dt} + (1 - \delta) k_{Dt} \]

\[ \phi'(i / k) > 0, \quad \phi''(i / k) < 0, \quad \phi(\delta) = \delta, \quad \phi'(\delta) = 1 \]

\[ \left( \phi \left( i / k \right) = \left[ \delta^{1-\eta} \left( i / k \right)^\eta - (1-\eta)\delta \right] / \eta, \quad 0 < \eta \leq 1 \right) \]

Adjusting the sector specific capital stock rapidly is costly. Capital in the traded goods sector has a different price, \( q_{Dt} \), than capital in the nontraded goods sector, \( q_{Nt} \).

(In simulations \( \eta = 0.9 \).)
LABOR ADJUSTMENT FRICTIONS

\[ \ell_{D_t} \leq \lambda \ell_{D_{t-1}} \]
\[ \ell_{N_t} \leq \lambda \ell_{N_{t-1}} \]

There is a limit to how fast sector specific labor can adjust. Labor in the traded goods sector receives a different wage, \( w_{D_t} \), than labor in the nontraded goods sector, \( w_{N_t} \).

(In simulations \( \lambda = 1.03 \).)
## Input-Output Matrix for Mexico 1989 (percent of GDP)

<table>
<thead>
<tr>
<th></th>
<th>traded</th>
<th>nontraded</th>
<th>$C + G$</th>
<th>$I$</th>
<th>$X$</th>
<th>total</th>
</tr>
</thead>
<tbody>
<tr>
<td>traded</td>
<td>27.24</td>
<td>9.02</td>
<td>24.48</td>
<td>11.13</td>
<td>14.98</td>
<td>86.85</td>
</tr>
<tr>
<td>nontraded</td>
<td>9.76</td>
<td>19.42</td>
<td>52.49</td>
<td>11.90</td>
<td>0.00</td>
<td>93.57</td>
</tr>
<tr>
<td>$w\ell$</td>
<td>18.05</td>
<td>44.51</td>
<td></td>
<td></td>
<td></td>
<td>62.56</td>
</tr>
<tr>
<td>$(r + \delta)k$</td>
<td>13.07</td>
<td>20.62</td>
<td></td>
<td></td>
<td></td>
<td>33.69</td>
</tr>
<tr>
<td>$w\ell + (r + \delta)k$</td>
<td>31.12</td>
<td>65.13</td>
<td></td>
<td></td>
<td></td>
<td>96.25</td>
</tr>
<tr>
<td>$m$</td>
<td>14.98</td>
<td>0.00</td>
<td></td>
<td></td>
<td></td>
<td>14.98</td>
</tr>
<tr>
<td>$\tau m$</td>
<td>3.75</td>
<td>0.00</td>
<td></td>
<td></td>
<td></td>
<td>3.75</td>
</tr>
<tr>
<td>total</td>
<td>86.85</td>
<td>93.57</td>
<td>76.97</td>
<td>23.03</td>
<td>14.98</td>
<td></td>
</tr>
</tbody>
</table>
Principal Ingredients in Numerical Experiments

Demographic differences

Differences in initial real interest rates

Financial liberalization

Trade liberalization
SUDDEN STOP!

\[ b_t = b_{t-1} + \bar{b}, \ t = T, ..., T + N \]

Domestic interest rate is endogenously determined, although interest payments on foreign debt \(-b_t\) are made at international interest rate.
Real GDP

\[ Y_t = p_{Dt} y_{Dt} - p_{Tt} z_{TDt} - p_{Nt} z_{NDt} \]

\[ + p_{Nt} y_{Dt} - p_{Tt} z_{TNt} - p_{Nt} z_{NNt} + \tau_{Dt} m_t \]

Real Investment

\[ I_t = p_{Tt} z_{Tlt} + p_{Nt} z_{Nlt} \]

Real capital stock

\[ K_{t+1} = (1 - \delta) K_t + I_t \]

Total factor productivity

\[ TFP_t = \frac{Y_t}{K_t^\alpha (\ell_{Dt} + \ell_{Nt})^{1-\alpha}} \]
Base Case: Trade Balance

\begin{itemize}
  \item Data
  \item Model
\end{itemize}
Sudden Stop: Real Exchange Rate
Data: Gross Output by Sector


Index (1988=100):
- Traded
- Nontraded
Sudden Stop: Gross Output by Sector

Year

Index (1988=100)

Nontraded

Traded