MACROECONOMIC THEORY **T. J. KEHOE**
ECON 8105 **FALL 2018**

FINAL EXAMINATION

Answer ***two*** of the following four questions.

1. Consider an economy in which there are both a continuum of measure one of infinitely lived consumers and a continuum of measure one of two-period lived overlapping generations consumers born every period. There is a single good in every period $t, t = 1, 2, \ldots$. The representative infinitely lived consumer is born in period 1, has the utility function

$$\sum_{t=1}^{\infty} \beta^{t-1} \log c_t^1 ,$$

and is endowed with a sequence of goods

$$(w_1^1, w_2^1, w_3^1, w_4^1 \ldots) = (3,1,3,1,\ldots) .$$

The representative overlapping generations consumer born in every period $t, t = 1, 2, \ldots,$ has the utility function

$$\log c_t^{2t} + \log c_{t+1}^{2t}$$

and endowment $(w_t^{2t}, w_{t+1}^{2t}) = (3,1)$. The representative initial old generations consumer has the utility function

$$\log c_1^{20}$$

and endowment $w_1^{20} = 1$ of the good in period 1. There is no fiat money, and there is no production or storage.

Notice that the demographic structure of the economy is such that in every period $t, t = 1, 2, \ldots,$ the active economic agents are a continuum of measure one infinitely lived consumers, a continuum of measure one of old overlapping generations consumers, and a continuum of measure one of young overlapping generations consumers.

(a) Describe an Arrow-Debreu market structure for this economy, explaining when markets are open, who trades with whom, and so on. Define an Arrow-Debreu equilibrium for this economy.

(b) Describe a sequential market structures for this economy, explaining when markets are open, who trades with whom, and so on. Define a sequential markets equilibrium for this economy.

(c) Define a Pareto efficient allocation.

(d) Is the equilibrium allocation in part a Pareto efficient? Explain carefully why or why not.

2. Consider an overlapping generations economy in which the representative consumer born in period $t, t = 1, 2, \ldots,$ has the utility function over consumption of the single good in periods t and $t+1$

$$u(c_t^t, c_{t+1}^t) = \log c_t^t + \gamma \log c_{t+1}^t$$

and endowments $(w_t^t, w_{t+1}^t) = (w_1, w_2)$. Suppose that the representative consumer in the initial old generation has the utility function

$$u^0(c_1^0) = \gamma \log c_1^0$$

and endowment $w_1^0 = w_2$ of the good in period 1 and endowment m of fiat money.

(a) Describe an Arrow-Debreu market structure for this economy, explaining when markets are open, who trades with whom, and so on. Define an Arrow-Debreu equilibrium.

(b) Describe a sequential market structure for this economy, explaining when markets are open, who trades with whom, and so on. Define a sequential markets equilibrium.

(c) Suppose that $m = 0$. Calculate both the Arrow-Debreu equilibrium and the sequential markets equilibrium.

(d) Define a Pareto efficient allocation. Suppose that $\gamma = 3$ and $(w_1, w_2) = (3, 5)$. Is the equilibrium allocation in part c Pareto efficient? Explain carefully why or why not.

(e) Suppose now that, rather than endowments of consumption goods, the consumers have endowments of labor $(\bar{\ell}_t^t, \bar{\ell}_{t+1}^t) = (\bar{\ell}_1, \bar{\ell}_2)$ and $\bar{\ell}_1^0 = \bar{\ell}_2$. The representative consumer in the initial old generation has an endowment of capital $\bar{k}_1^0$ and an endowment m of fiat money. Final output, which can be consumed or invested is produced using the production function $\theta k_t^\alpha \ell_t^{1-\alpha}$, $\theta > 0$, $0 < \alpha < 1$, and a fraction δ, $0 \le \delta \le 1$, of capital depreciates every period. Define a sequential markets equilibrium for this economy.

3. Consider an economy in which the representative consumer lives forever. There is a good in each period that can be consumed or saved as capital as well as labor. The consumer's utility function is

$$\sum_{t=0}^{\infty} \beta^t c_t^{\rho} .$$

Here $0 < \beta < 1$ and $0 < \rho < 1$. The consumer is endowed with 1 unit of labor in each period and with $\bar{k}_0$ units of capital in period 0. Feasible allocations satisfy

$$c_t + k_{t+1} - (1-\delta)k_t \le \theta k_t^{\alpha} \ell_t^{1-\alpha}$$
$$c_t, k_t \ge 0 \,,\; 1 \ge \ell_t \ge 0 .$$

Here $\theta > 0$, $0 < \alpha < 1$, and $0 \le \delta \le 1$.

(a) Formulate the problem of maximizing the representative consumer's utility subject to feasibility conditions as a dynamic programming problem. Write down the appropriate Bellman's equation.

(b) Let $K = [0, \tilde{k}]$. Explain how you can use the feasibility condition to choose $\tilde{k}$ to be the maximum sustainable capital stock. Let $C(K)$ be the space of continuous bounded functions on K. Endow $C(K)$ with the topology induced by the sup norm

$$d(V,W) = \sup_{k \in K} \left| V(k) - W(k) \right| \text{ for any } V, W \in C(K) .$$

Define a contraction mapping $T : C(K) \to C(K)$.

(c) State Blackwell's sufficient conditions for T to be a contraction. (You do not need to prove that these conditions are sufficient for T to be a contraction.)

(d) Using the Bellman's equation from part a, define the mapping for the value function iteration algorithm,

$$V_{n+1} = T(V_n) ,$$

where $T : C(K) \to C(K)$; that is $V = T(V)$ is the Bellman's equation. (You do not need to prove that $T(V) \in C(K)$ for all $V \in C(K)$.) Prove that T satisfies Blackwell's sufficient conditions to be a contraction.

(e) Specify an economic environment for which the solution to the social planner's problem in part a is a Pareto efficient allocation/production plan. Define a sequential markets equilibrium for this environment. Explain how you could use the value function iteration algorithm $V_{n+1} = T(V_n)$ to calculate the unique sequential markets equilibrium. (You do not have to prove that this equilibrium is unique.)

4. Consider the problem faced by an unemployed worker searching for a job. Every period that the worker searches, she receives a job offer with the wage w drawn independently from the time invariant probability distribution $F(v) = \text{prob}(w \leq v)$, $v \in [0, B]$, $B > 0$. After receiving the wage offer w the worker faces the choice (1) to accept it or (2) to reject it, receive unemployment benefit b, and search again next period. That is,

$$y_t = \begin{cases} w & \text{if job offer has been accepted} \\ b & \text{if searching} \end{cases}$$

The worker solves

$$\max E \sum_{t=0}^{\infty} \beta^t y_t$$

where $1 > \beta > 0$. Once a job offer has been accepted, there are no fires or quits.

(a) Formulate the worker's problem as a dynamic programming problem by writing down Bellman's equation.

(b) Using Bellman's equation from part (a), characterize the value function $V(w)$ in a graph and argue that the worker's problem reduces to determining a reservation wage $\bar{w}$ such that she accepts any wage offer $w \geq \bar{w}$ and rejects any wage offer $w < \bar{w}$.

(c) Consider two economies with different unemployment benefits b_1 and b_2 but otherwise identical. Let $\bar{w}_1$ and $\bar{w}_2$ be the reservation wages in these two economies. Suppose that $b_2 > b_1$. Prove that $\bar{w}_2 > \bar{w}_1$. Provide some intuition for this result.

(d) Consider two economies with different wage distributions F_1 and F_2 but otherwise identical. Define what it means for F_2 to be a mean preserving spread of F_1.

(e) Suppose that F_2 is a mean preserving spread of F_1 and that the inequality that defines a mean preserving spread holds strictly in the case of F_2 and F_1. Let $\bar{w}_1$ be the reservation wage in the economy with wage distribution F_1 and $\bar{w}_2$ be the reservation wage in economy with wage distribution F_2. Prove that $\bar{w}_2 > \bar{w}_1$. Provide some intuition for this result.