Consider a small open economy whose government borrows from international bankers. In every period, the value of output is

$$y(z) = Z^{1-z} \overline{y}$$

where 1 > Z > 0 is a constant and z = 0 if the government defaults in that period or has defaulted in the past and \overline{y} is a constant. The government's tax revenue is $\theta y(z)$ where the tax rate $1 > \theta > 0$ is constant. The consumers in the economy consume $c = (1 - \theta)y(z)$. The government is benevolent and makes choices to maximize the expected discounted value of

$$u(c, g) = \log c + \gamma \log g$$

where $\gamma>0$ and $1>\beta>0$ is the discount factor. At the beginning of every period, the state of the economy is $s=(B,z_{-1},\zeta)$ where B is the level of government debt; $z_{-1}=0$ if the government has defaulted in the past, and $z_{-1}=1$ if not, and $\zeta\sim U[0,1]$ is the realization of a sunspot variable. The government first offers B' to international bankers. The intentional bankers have the same discount factor β as the government. They are also risk neutral and have deep pockets. These international bankers buy the bonds at a competitive auction that determines a price for B', q(B',s). The government finally chooses to default or not, which determines private consumption c. Government spending g is determined by the government's budget constraint

$$g + zB = \theta y(z) + q(B', s)B'$$
.

If the government defaults, setting z = 0, then assume that $z_{-1} = 0$ implies z = 0 thereafter; that is, the economy suffers from the default penalty 1 - Z forever. Furthermore, $z_{-1} = 0$ implies q(B', s) = 0; that is, the government is permanently excluded form credit markets.

- a) Define a recursive equilibrium.
- b) Assume that the bankers expect the government to default if $\zeta > 1-\pi$ and if such an expectation would be self-fulfilling, where $1 > \pi \ge 0$ is an arbitrary constant. Find a level of debt \overline{b} such that, if $B \le \overline{b}$, no default occurs in equilibrium, but that, if $B > \overline{b}$, default occurs in equilibrium.
- c) Suppose that $B_0 > \overline{b}$, and the government chooses to run down its debt to $B_T \leq \overline{b}$ in T periods. Prove that it cannot be optimal to set $B_T < \overline{b}$. Prove that it is optimal for the

government to set g_t constant as long as $B_t > \overline{b}$ and no crisis occurs. Find expressions for g_t and B_t that depend on B_0 and T. Find an expression for the expected discounted value of the utility of running down the debt that starts at B_0 to \overline{b} in T periods. Find the limit of these expressions when $T = \infty$.

- d) Using the answers to part c, write down a formula that determines a value of debt $\overline{B}(\pi)$ such that the government would choose to default if $B > \overline{B}(\pi)$ even if international bankers do not expect a default.
- e) Using the answers to parts a–d, construct a recursive equilibrium.
- f) Use this model to interpret events of the Mexican financial crisis of December 1994 through January 1995.
- g) Assume that Z=0.9, $\overline{y}=100$, $\theta=0.4$, $\gamma=0.5$, $\beta=0.95$, and $\pi=0.05$. Calculate \overline{b} . Calculate the expected discounted value of the utility of running down the debt that starts at B_0 to \overline{b} in T periods for T=1,2,3,4,5,6,7. Calculate $\overline{B}(0.05)$. Graph a policy function for government debt B'(B). Graph a policy function for government spending g(B).