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Growth and Redistribution with Heterogeneous Attitudes toward Risk

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ABSTRACT

We develop an endogenous growth model of overlapping generations in which agents leave warm-glow bequests. There are dynasties of risk loving investors and dynasties of risk averse investors. We start with a simple model in which risk averse investors can invest only in a safe asset while risk loving investors (entrepreneurs) can invest in a risky asset with a higher expected return. This simple structure allows us to analytically calculate the unique invariant distribution of relative wealth holdings in a balanced growth path. We define a social welfare function for this model and calculate tax and transfer policies that maximize social welfare in the invariant distribution. We show how to extend our results to models (1) where a fraction of risk averters have risk loving descendants and an equal fraction of risk lovers have risk averse descendants, (2) where risk averse investors can invest in the risky asset and (3) where agents have nonhomothetic preferences, which lead wealthier agents to leave a higher fraction of their income in bequests.

Keywords: Endogenous growth; Inequality; Redistribution; Overlapping generations; Invariant distribution; Social welfare function.

JEL Codes: C62, D51, H21, O4

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1. Introduction

In this paper, we develop an endogenous growth model to analyze optimal government policy when there is a trade-off between economic growth and redistribution. In the model, there are dynasties of two-period-lived overlapping generations, and there is a single good in each period. Some dynasties are composed of risk averse agents, others are composed of risk loving, or at least risk neutral, agents. When young, each agent chooses to invest in one of two types of assets, a safe asset or a risky asset that has a higher expected rate of return. During the second period of his or her life, each agent decides how much of the wealth accumulated from this investment to consume and how much to give as a warm-glow bequest to his or her single child, who has the same attitude towards risk as he or she does. There are no other sources of income or objects for spending. There is a government that can tax some agents on their wealth, their consumption, and their bequests when they are old and give subsidies to other agents on their wealth, their consumption, and their bequests. The government sets tax and subsidy policies to maximize a social welfare policy of the sort developed by Atkinson (1971). If we assume that there are subsidies high enough to provide positive income to the children of risk loving agents who have lost all the bequests that they received, then we can prove that, for any given initial distribution of assets in the economy, there is a unique equilibrium and this equilibrium converges to a unique invariant distribution of relative wealth levels in a balanced growth path. Our assumptions make the model simple enough so that we can provide a complete analytical characterization of this invariant distribution.

We analyze the trade-off that the government that faces between growth and redistribution by calculating the optimal taxes and subsidies on wealth, consumption, and bequests. As is frequent in optimal taxation studies, we find that the government needs to use only two of the three available sets of instruments to implement the optimal policy. When we restrict ourselves to taxes and subsidies on wealth and bequests, we find that the government often subsidies the bequests of the wealthy agents and taxes the bequests of the poor agents. The government does this because the wealthy agents are risk lovers and their investments have higher expected returns. The poor agents are both risk averse and risk lovers, but poor risk lovers leave less bequests than wealthy risk lovers. Furthermore, the government can use the wealth tax and subsidy to redistribute income. The parameter values for which the government chooses to tax both the bequests and the wealth of the wealthy agents are ones for which the government has a discount factor that is low compared to the fraction of wealth that agents want to leave to their children. In these cases, the government wants to lower the growth rate of the economy to increase the welfare of the current generation relative to future generations.

The simple structure of our model allows for many extensions. In this paper, we provide three. In the first extension, we allow dynasties to stochastically change their attitude towards risk. In particular, we assume that in every period a fraction of risk averters has risk loving descendants and an equal fraction of risk lovers has risk averse descendants. In the second extension, we allow risk averse investors to invest in the risky asset. In the third extension, we model agents as having nonhomothetic preferences, which lead wealthier agents to leave a higher fraction of their income in bequests.

The tension in our model between inequality and growth is due to investment in the risky asset being both the source of inequality and the source of growth. Since the risky asset has an uncertain return, it creates inequality among the risk loving agents, who invest in it and do not diversity. Since the risky asset has a higher expected return than does the safe asset, it creates inequality between the risk loving agents, who invest in it, and the risk averse agents, who invest in the safe asset. Furthermore, since the risky asset has a higher expected return than does the safe asset, a higher level of aggregate investment in the risky asset produces more growth.

The mechanism that generates inequality and growth in our model is inspired by the theory of entrepreneurship developed by Knight (1921): According to Knight, entrepreneurs take personal responsibility for economic outcomes by guaranteeing fixed payments to other factors of production — workers, suppliers, lenders — while accepting the residual, whether profit or loss. Since uncertainty cannot be eliminated through calculation, entrepreneurs must possess the psychological capacity to act decisively despite incomplete information. While others in the economy can often shift risks through contracts or insurance, entrepreneurs bear the ultimate uncertainty about whether their ventures will succeed. Knight argued that entrepreneurial profit, as distinct from wages, rent, or interest, emerges specifically from this function of bearing uncertainty. The entrepreneur's reward comes from successfully navigating unmeasurable uncertainty rather than simply managing calculable risks. We model this willingness to bear uncertainty as risk loving or risk neutrality. Knight also argued that an essential characteristic of an entrepreneur as the ability to organize and direct production, coordinating various inputs and market relationships. Knight saw entrepreneurs as the organizing intelligence that brings together land, labor, and capital. Our model does not include this organizational ability, but we can imagine extensions that would.

There is a vast related literature on economic growth and inequality, the seminal work for empirical work in this area is that of Kuznets (1955), who found evidence that growth rates and measures of inequality simultaneously increased as countries began to industrialize. Kuznets hypothesized that this correlation was driven by structural transformation. This empirical work is complementary to Knight's theory. The structural transformation from an economy based on agriculture, handicrafts, and mineral extraction to one based on industry gave entrepreneurs the opportunity to engage in the profitable, but uncertain, projects in Knight's theory and in ours.

The structure of our model is related to that of a number of models in the literature. We develop an overlapping generations model with warm-glow bequests as in Andreoni (1989). To analyze the impact of different production technologies on the accumulation of wealth, we use a model with bequests and idiosyncratic uncertainty on the technologies as in Piketty (1997). In our model, unlike Piketty's, we consider heterogenous agents: risk averse agents and risk loving agents. We study government policy in the form of redistributive taxes and subsidies as do Alesina and Rodrik (1994). Our analysis of the interaction of agents with different attitudes toward risk is related to that in Araujo, Gama and Kehoe (2025) and in Araujo, Chateauneuf, Gama and Novinski (2018).

Our results are related to those in the vast literature on wealth inequality. For example, Benhabib, Bisin, and Zhu (2011, 2015, 2016) categorize the distribution of wealth in terms of the underlying economic mechanisms generating skewness and thick tails in the wealth distribution. Furthermore, Beare and Toda (2022) characterize the Pareto exponent of distributions that come from returns heterogeneity.

The paper is organized as follows. In section 2, we define the basic model including the notion of equilibrium. In subsection 2.2, we define the basic properties of the model including the relationship between growth and inequality without taxes, and, in subsection 2.3, we analyze the basic properties with taxes. In section 3, we prove the existence of a growth path with an invariant distribution of wealth and its uniqueness and show some of the properties of the growth rates. In section 4, we analyze the existence of optimal taxes and subsidies on wealth using a social welfare function, and we also prove the basic properties of this function and of the optimal taxes. In subsection 5, we give some numerical examples to illustrate the properties of optimal taxes and subsidies on wealth. In section 6, we extend our analysis to taxes and subsidies on consumption and bequests. In section 7, we present three extensions to the model. Finally, in section 8, we summarize our results.

2. Model with segmentation and wealth taxes

We analyze an overlapping generation model with warm-glow bequests and uncertainty. Our model has two continuous types of agents *risk averters* and *risk lovers*, each of which has measure 1. We denote risk loving agents as $\{l_i\}_{i\in[0,1]}$ and risk averters as $\{a_i\}_{i\in[0,1]}$. There are two linear one-period technologies given by a constant value $R_S > 0$ for the safe technology and $\overline{R}_R = R_R > 0$ with probability $1/2$ and $\underline{R}_R = 0$ with probability $1/2$ for the risky technology. We choose the probability of the high return to be equal to $1/2$ for ease of computation and to be definitive. We could easily choose to be any $q \in (0,1)$ although, of course, this would change the formulas and the computational results.

We assume that

$$R_R > R_S > 0 \tag{2.1}$$

and

$$R_R/2 > \frac{1}{\beta} \geq R_S \tag{2.2}$$

where $\beta \in (0,1)$ is the natural bequest rate that we explain below.

To ensure that the markets for assets are segmented in the sense that risk averse agents purchase only the safe asset and risk loving agents purchase only the risky asset, we make the following assumption:

Assumption S1: The technology R_S, the safe one, is available for both types of agents, and R_R, the risky one, is available for the risk loving agents only.

We assume that the probability of the risky technology is independent among the agents. Consequently, there is no aggregate uncertainty in the economy.

There is a single consumption good at every date t, $t = 0,1,\dots$. Every agent is characterized at date t by his idiosyncratic state $s^i = (\eta_1^i, \eta_2^i, \dots, \eta_t^i)$, where $\eta_k^i = 1$ if his investment has the high return at date k and $\eta_k^i = 0$ if his investment has the low return at date k. Because there is no aggregate uncertainty, the specification of idiosyncratic states is only important for defining the maximization problems of individual agents and their solutions, not aggregate variables.

Any young generation uses the bequest from his parent to invest in the production technologies, and any old generation uses the production returns to consume, $c_{s^i}^i$, and to leave a bequest, $b_{s^i}^i$, to his child. All the agents leave a bequest that is a proportion of the agent's total wealth. In $t = 0$, the initial endowment of the old generation is w_0^i.

2.1. Wealth taxes and subsidies

At each date $t \geq 0$, the government imposes taxes on wealth. There is a tax on wealth of agents whose wealth exceeds a threshold W_t and a subsidy on wealth of agents whose wealth is less than W_t. For simplicity, the marginal tax rate is equal to the marginal subsidy rate, τ^W. Consequently, the net wealth tax on an agent with wealth w is $\tau^W \max(w - W_t, 0) - \tau^W \max(W_t - w, 0)$. There is also a tax on bequests that works similarly.

Notice that τ^W is naturally constrained to the set $[0,1]$. The marginal tax rate τ^W is exogenously defined by the central planner. To keep tax policy simple, we set the threshold W_t equal to the average wealth level in the economy. This assumption ensures a balanced government budget.

2.2. Utility maximization

Given an initial amount of wealth, $w_0^{a_i}$, the utility maximization problem of a risk averter a_i of the old generation in $t = 0$ is

$$\begin{aligned} \max_{(c,b)} \quad & (1-\beta)\log c + \beta \log b \\ s.t. \quad & 0 \leq c + b \leq w_0^{a_i} - \tau^W\left(w_0^{a_i} - W_0\right). \end{aligned}$$

Similarly, given the initial wealth, $w_0^{l_i}$, the utility maximization problem of a risk loving agent l_i of the old generation in $t = 0$ is

$$\begin{aligned} \max_{(c,b)} \quad & \left(c^{1-\beta} b^{\beta}\right)^{\gamma} \\ s.t. \quad & 0 \leq c + b \leq w_0^{l_i} - \tau^W\left(w_0^{l_i} - W_0\right), \end{aligned}$$

with $\gamma \geq 1$.

Given the bequest $b^{a_i}_{s^{a_i}}$ received from his parent at state s^{a_i} at date $t \geq 1$, the utility maximization problem of a young risk averter a_i is

$$\max_{(c,b,\theta)} \quad (1-\beta)\log c + \beta \log b$$
$$s.t. \quad \theta_S \leq b^{a_i}_{s^{a_i}},$$
$$0 \leq c + b \leq R_S\theta_S - \tau^W(R_S\theta_S - W_t).$$

Given the bequest $b^{l_i}_{s^{l_i}}$ received from his parent at state s^{l_i} at date $t \geq 1$, the utility maximization problem of a young risk loving agent l_i is

$$\max_{(c,b,\theta)} \quad \frac{1}{2}\left(c_1^{1-\beta}\, b_1^{\beta}\right)^{\gamma} + \frac{1}{2}\left(c_2^{1-\beta}\, b_2^{\beta}\right)^{\gamma}$$
$$s.t. \quad \theta_R + \theta_S \leq b^{l_i}_{s^{l_i}},$$
$$0 \leq c_1 + b_1 \leq R_R\theta_R + R_S\theta_S - \tau^W(R_R\theta_R + R_S\theta_S - W_t),$$
$$0 \leq c_2 + b_2 \leq R_S\theta_S - \tau^W(R_S\theta_S - W_t).$$

Note that, in our model, wealth taxes can be seen as income taxes since the capital is completely transformed into income in each state s. We focus mainly on interpretation of τ^W as wealth taxes.

Because of the form of the utility index and Equation 3.2, a risk loving agent l_i never invests in the safe technology, that is, $\theta^{l_i}_{S,s^{l_i}} = 0$ for all $i \in [0,1]$. Therefore, all agents invest in only one technology, which is the market segmentation that we refer to. The risk averters invest in the safe asset, the less productive one, and the risk loving agents invest in the risky asset, the productive one.

2.3. Equilibrium

Now, let us define the *equilibrium* for the economy as $((c^{a_i}, b^{a_i}, \theta^{a_i})_i, (c^{l_i}, b^{l_i}, \theta^{l_i})_i)$ such that (c^i, b^i, θ^i) solves the utility maximization problem defined above for any state s^i, and

$$0 = \int_0^1 \left(\tau^W\left(R_S\theta_{S,s^{a_i}} - W_t\right) + \tau^W\left((1/2)R_R\theta_{R,s^{l_i}} + R_S\theta_{S,s^{l_i}} - W_t\right)\right) di,$$
$$\bar{c}_{t+1} + \bar{b}_{t+1} = \bar{y}_{t+1} = \int_0^1 \left(R_S\theta^{a_i}_{S,s^{a_i}} + (1/2)R_R\theta^{l_i}_{R,s^{l_i}} + R_S\theta^{l_i}_{S,s^{l_i}}\right),$$

where $\bar{y}_t = (1/2)\left(\int_0^1 y_{s^{a_i}}^{a_i} di + \int_0^1 y_{s^{l_i}}^{l_i} di\right)$, $\bar{c}_t$, and $\bar{b}_t$ are the average net wealth, consumption, and bequests in date t, respectively, and, $y_{s^{a_i}}^{a_i}$, $y_{s^{l_i}}^{l_i}$ are the net wealth of the risk averter a_i in state s^{a_i} and net wealth of the risk loving agent l_i in state s^{l_i}. Notice that the first equilibrium condition is government budget balance, and the second is feasibility. As we have explained, the first condition is satisfied if we set $W_t = \bar{y}_t$.

From the first order conditions of the utility maximization problems, we know that

$$(1-\beta)b_{s^i}^i = \beta c_{s^i}^i$$

for all agents i. Combining this condition with the consumer's budget constraints yields

$$c_{s^i}^i = (1-\beta)y_{s^i}^i,$$
$$b_{s^i}^i = \beta y_{s^i}^i.$$

Integrating over all agents gives us the aggregate equilibrium conditions

$$\bar{c}_t = (1-\beta)\bar{y}_t,$$
$$\bar{b}_t = \beta\bar{y}_t.$$

Using the tax policy $\tau = (\tau^W)$ and the threshold W_t, the budget constraint of the risk averter a_i when he is old at date $t \geq 1$ can be written as

$$0 \leq c + b \leq (1-\tau^W)R_S\theta_S + \tau^W\bar{y}_t, \qquad (2.3)$$

and the budget constraint of the risk loving agent i when he is old date at $t \geq 1$ can be written as

$$0 \leq c_1 + b_1 \leq (1-\tau^W)R_R\theta_R + \tau^W\bar{y}_t, \qquad (2.4)$$
$$0 \leq c_2 + b_2 \leq \tau^W\bar{y}_t. \qquad (2.5)$$

Thus, each agent receives one transfer that depends on the average wealth. Additionally, the wealth tax reduces the agent's wealth by the proportion of $1-\tau^W$.

If $\bar{y}_{\tau,t}^a$ is the after-tax average wealth of the risk averters at date t, and $\bar{y}_{\tau,t}^l$ is the after-tax average wealth of the risk loving agents at date t, the average wealth at date $t+1$ is

$$\bar{y}_{\tau,t+1} = \frac{(R_R/2)\beta}{2}\bar{y}_{\tau,t}^l + \frac{R_S\beta}{2}\bar{y}_{\tau,t}^a. \qquad (2.6)$$

Given the tax policy τ^W, we define the growth rate from t to date $t+1$ to be $g_{\tau,t} = \bar{y}_{\tau,t+1}/\bar{y}_{\tau,t} - 1$. Moreover, if $\bar{x}_{\tau,t}^a$ is the before-tax average wealth of the risk averters at date t, and $\bar{x}_{\tau,t}^l$ is the before-tax average wealth of the risk loving agents at date t, the average wealth at date $t+1$ is

$$\overline{y}_{\tau,t+1} = \left(\frac{R_R\beta}{4}\right)\left((1-\tau^W)\overline{x}^l_{\tau,t} + \tau^W\overline{y}_t\right) + \frac{R_S\beta}{2}\left((1-\tau^W)\overline{x}^a_{\tau,t} + \tau^W\overline{y}_t\right). \quad (2.7)$$

2.4. A recursive specification of the model and equilibrium

At date $t = 0$, we specify the state of an initial old agent by the wealth he brings into the period. There is a distribution function for initial wealth $\left(G^W_{a,\tau}, G^W_{l,\tau}\right)$, where $G^W_{a,\tau}$ is the distribution of initial wealth for risk-averse agents and $G^W_{l,\tau}$ is the distribution of initial wealth for risk loving agents. The utility maximization problem of a risk averter of the initial old generation with wealth w is

$$\max_{(c,b')} \quad (1-\beta)\log c + \beta \log b'$$
$$s.t. \quad 0 \le c + b' \le w - \tau^W(w - W_0).$$

Similarly, the utility maximization problem of a risk loving agent of the initial old generation with wealth w is

$$\max_{(c,b')} \quad \left(c^{1-\beta}b'^{\beta}\right)^{\gamma}$$
$$s.t. \quad 0 \le c + b' \le w - \tau^W(w - W_0).$$
$$\frac{1}{\rho}\left((1-\delta)c^{\rho} + \delta b^{\rho}\right), \rho \in (0,1)$$

At date $t \geq 1$, we specify the state of a young agent by the bequest b he receives from his parent. The utility maximization problem of a young risk averter is

$$\max_{(c,b',\theta)} \quad (1-\beta)\log c + \beta \log b'$$
$$s.t. \quad \theta_S \le b,$$
$$0 \le c + b' \le R_S\theta_S - \tau^W(R_S\theta_S - W_t).$$

The utility maximization problem of a young risk loving agent is

$$\max_{(c,b',\theta)} \quad \frac{1}{2}\left(c_1^{1-\beta}\, b_1'^{\beta}\right)^{\gamma} + \frac{1}{2}\left(c_2^{1-\beta} b_2'^{\beta}\right)^{\gamma}$$
$$s.t. \quad \theta_R + \theta_S \le b = \beta w,$$
$$0 \le c_1 + b_1' \le R_R\theta_R + R_S\theta_S - \tau^W(R_R\theta_R + R_S\theta_S - W_t),$$
$$0 \le c_2 + b_2' \le R_S\theta_S - \tau^W(R_S\theta_S - W_t).$$

We can specify the state of the economy by distribution function for after-tax wealth $\left(F^W_{l,\tau,t-1}, F^W_{a,\tau,t-1}\right)$ of the immediate parents.

Given this recursive specification of the model and the initial distribution of wealth $\left(G^W_{a,\tau}, G^W_{l,\tau}\right)$, we define an equilibrium as policy functions $\left(c_{a,0}(w), b'_{a,0}(w)\right), \left(c_{l,0}(w), b'_{l,0}(w)\right)$ and $\left(c_{a,t}(w), b'_{a,t}(w), \theta_{a,t}(w)\right), \left(c_{l,t}(w), b'_{l,t}(w), \theta_{a,t}(w)\right)$, thresholds W_t and B_t, and equation of motion for the distribution functions $\Omega_{a,0}\left(G^W_{a,\tau}, G^W_{l,\tau}\right)$, $\Omega_{l,0}\left(G^W_{a,\tau}, G^W_{l,\tau}\right)$, $\Omega_a\left(F^W_{a,\tau,t}, F^W_{l,\tau,t}\right)$, and $\Omega_l\left(F^W_{a,\tau,t}, F^W_{l,\tau,t}\right)$. The policy functions solve the utility maximization problems and satisfy the feasibility conditions

$$\int_0^\infty c_{a,0}(w)dG^W_{a,\tau}(w) + \int_0^\infty c_{l,0}(w)dG^W_{l,\tau}(w) + \int_0^\infty b'_{a,0}(w)dG^W_{a,\tau}(w) + \int_0^\infty b'_{l,0}(w)dG^W_{l,\tau}(w)$$
$$= \int_0^\infty wdG^W_{a,\tau}(w) + \int_0^\infty wdG^W_{l,\tau}(w)$$

in period 0, and

$$\int_0^\infty c_{a,t}(w)dF^B_{a,\tau,t-1}(w) + \int_0^\infty c_{l,t}(w)dF^B_{l,\tau,t-1}(w) + \int_0^\infty b'_{a,t}(w)dF^B_{a,\tau,t-1}(w)$$
$$+ \int_0^\infty b'_{l,t}(w)dF^B_{l,\tau,t-1}(w)$$
$$= \int_0^\infty R_S\theta_{a,t}(w)dF^B_{a,\tau,t-1}(w) + \int_0^\infty \left(R_S\theta_{l,t}(w) + (1/2)R_R\theta_{l,t}(w)\right) F^B_{l,\tau,t-1}(w)$$

in period $t \geq 0$. The threshold W_t satisfies the conditions

$$W_0 = \int_0^\infty wdG^W_{a,\tau}(w) + \int_0^\infty wdG^W_{l,\tau}(w)$$

$$W_t = \int_0^\infty R_S\theta_{a,t}(w)dF^W_{a,\tau,t-1}(w) + \int_0^\infty \left(R_S\theta_{l,t}(w) + (1/2)R_R\theta_{l,t}(w)\right) F^B_{l,\tau,t-1}(w).$$

The distribution functions $\left(G^W_{a,\tau}, G^W_{l,\tau}\right)$, $\left(F^B_{a,\tau,t}, F^B_{l,\tau,t}\right)$ satisfy the equations of motion

$$\begin{bmatrix} F_{a,\tau,0}^W \\ F_{l,\tau,0}^W \end{bmatrix} = \begin{bmatrix} \Omega_{a,0}\left(G_{a,\tau}^W, G_{l,\tau}^W\right) \\ \Omega_{l,0}\left(G_{a,\tau}^W, G_{l,\tau}^W\right) \end{bmatrix}$$

and

$$\begin{bmatrix} F_{a,\tau,t+1}^W \\ F_{l,\tau,t+1}^W \end{bmatrix} = \begin{bmatrix} \Omega_{a}\left(F_{a,\tau,t}^W, F_{l,\tau,t}^W\right) \\ \Omega_{l}\left(F_{a,\tau,t}^W, F_{l,\tau,t}^W\right) \end{bmatrix}.$$

Notice that the distribution function for before-tax wealth $\left(G_{a,\tau}^W, G_{l,\tau}^W\right)$ induce distribution function for after-tax wealth as fraction of the aggregate wealth $\left(F_{a,\tau,0}^W, F_{l,\tau,0}^W\right)$. More specifically,

$$F_{a,\tau,0}^W(w) = G_{a,\tau}^W\left(\frac{W_0 w}{1-\tau^W} - \frac{\tau^W W_0}{2(1-\tau^W)}\right),$$

$$F_{l,\tau,0}^W(w) = G_{l,\tau}^W\left(\frac{W_0 w}{1-\tau^W} - \frac{\tau^W W_0}{2(1-\tau^W)}\right),$$

$$F_{a,\tau,t+1}^W(w) = F_{a,\tau,t}^W\left(\left(w - \frac{\tau^W}{2}\right)\frac{W_{t+1}}{\beta(1-\tau^W)R_S}\right),$$

$$F_{l,\tau,t+1}^W(w) = \begin{cases} 0, & w < \underline{w} \\ \dfrac{1}{2}\left(1 + F_{l,\tau,t}^B\left(\left(w - \dfrac{\tau^W}{2}\right)\dfrac{W_{t+1}}{\beta(1-\tau^W)R_R}\right)\right), & w \geq \underline{w}, \end{cases}$$

where

$$\underline{w} = \frac{\tau^W}{2},$$

is the minimum level of (proportional) wealth in period $t \geq 1$.

3. Existence and uniqueness of the invariant distribution and convergence

In this section we study the existence of the invariant distribution, its uniqueness and some of its basic properties.

In the absence of taxes, a small proportion of risk loving agents — those whose parents were always lucky — concentrate most of the wealth in the economy. Thus, there is no invariant distribution.

Proposition 1. If $\tau^W = 0$, there is no invariant distribution.

If, however, $\tau^W > 0$, we are able to prove the following result of existence, uniqueness of an invariant distribution and the convergence of all equilibria to this invariant distribution.

Theorem 1. For any distribution of initial endowments $\left(G^W_{a,\tau}, G^W_{l,\tau}\right)$ the distribution of after-tax wealth among the agents $\left(F^W_{a,\tau,t}, F^W_{l,\tau,t}\right)$ and the growth rate $g_{\tau,t}$ converge to invariant distribution $\left(F^W_{a,\tau}, F^W_{l,\tau}\right)$ and a constant g_τ. Moreover, g_τ is a strictly decreasing C^1 function in τ.

The proof of Theorem 1 is in Appendix A. The proof of an invariant distribution is done recursively, for that we take advantage of $\underline{R}_R = 0$ which implies that the wealth history of the risk loving agents is lost whenever they fail. Each risk loving agent dynasty is characterized by the number of consecutive times that it has been successful in its investments after the last time it was unsuccessful.

From the proof of Theorem 1, we have that

$$g_{\tau^W} = \frac{((R_R/2)\beta - 1)z_{\tau^W} + (R_S\beta - 1)}{z_{\tau^W} + 1}$$

where $z_{\tau^W} = \overline{x}^l_{\tau^W}/\overline{x}^a_{\tau^W}$ is the ratio between the after-tax wealth in hands of the risk loving agents and the after-tax wealth in hands of the risk averters, and it is the solution of

$$z_{\tau^W} = \frac{\overline{x}^l_{\tau^W}}{\overline{x}^a_{\tau^W}} = \frac{\left(1 - \frac{\tau^W}{2}\right)(R_R/2)z_\tau + \left(\frac{\tau^W}{2}\right)R_S}{\left(\frac{\tau^W}{2}\right)(R_R/2)z_\tau + \left(1 - \frac{\tau^W}{2}\right)R_S},$$

which is given by

$$z_{\tau^W} = \left(\frac{1}{\tau^W} - \frac{1}{2}\right)\left(1 - \frac{R_S}{R_R/2}\right) + \left(\left(\frac{1}{\tau^W} - \frac{1}{2}\right)^2\left(1 - \frac{R_S}{R_R/2}\right)^2 + \frac{R_S}{R_R/2}\right)^{1/2}.$$

Note that z_{τ^W} is a strictly decreasing function in τ^W such that $\lim_{\tau^W \to 0} z_{\tau^W} = \infty$ and $\lim_{\tau^W \to 1} z_{\tau^W} = 1$, so any increase in the tax rate τ^W implies a reduction in $z_{\tau^W} = \bar{x}^l_{\tau^W} / \bar{x}^a_{\tau^W}$. And since only the risk loving agents invest in the most productive technology, an increase in the tax rate implies a lower invariant growth rate.

We define $\left(F^W_{a,\tau}\right)^{-1}$ and $\left(F^W_{l,\tau}\right)^{-1}$ as the quantile functions of the distributions $F^W_{a,\tau}$ and $F^W_{l,\tau}$. The invariant distribution of wealth $F^W_{a,\tau}$ is a constant distribution

$$x^a_\tau = \left(F^W_{a,\tau}\right)^{-1}(1) = \sum_{k=0}^{\infty} (\beta\pi_a)^k = \frac{\left(1 + g_{\tau^W}\right)}{2(1 + g_{\tau^W}) - 2\beta(1 - \tau^W)R_S} \tau^W,$$

where $\pi_a = R_S(1 - \tau^I)/\left(1 + g_{\tau^I}\right)$.

Next, we provide the invariant distribution of the risk loving agents. For that, we denote $\pi_r = R_R(1 - \tau^W)/\left(1 + g_{\tau^W}\right)$.

- For the poorest risk loving agents — those who risk loving agents who failed — whose proportion is $1/2$,

$$x^{l,1}_\tau = \left(F^W_{l,\tau}\right)^{-1}(1/2) = \tau^W/2.$$

- For the second poorest risk loving agents — those who were lucky, and their parent failed — whose proportion is $1/4$,

$$x^{l,2}_\tau = \left(F^W_{l,\tau}\right)^{-1}(3/4) = \beta\pi_r\tau^W/2 + \tau^I/2.$$

- For the third poorest risk loving agents — those who were lucky, their parents were lucky, and their parent's parent failed — whose proportion is $1/8$,

$$x^{l,3}_\tau = \left(F^W_{l,\tau}\right)^{-1}(7/8) = (\beta\pi_r)^2\tau^W/2 + \beta\pi_r\tau^W/2 + \tau^W/2.$$

- For the poorest group n of risk loving agents — those who were lucky, and all their parents have been lucky in the last $n - 1$ periods and failed n periods ago — whose proportion is $1/2^n$,

$$x^{l,n}_\tau = \left(F^W_{l,\tau}\right)^{-1}(1 - 1/2^n) = \sum_{k=0}^{n-1} (\beta\pi_r)^k \tau^W/2.$$

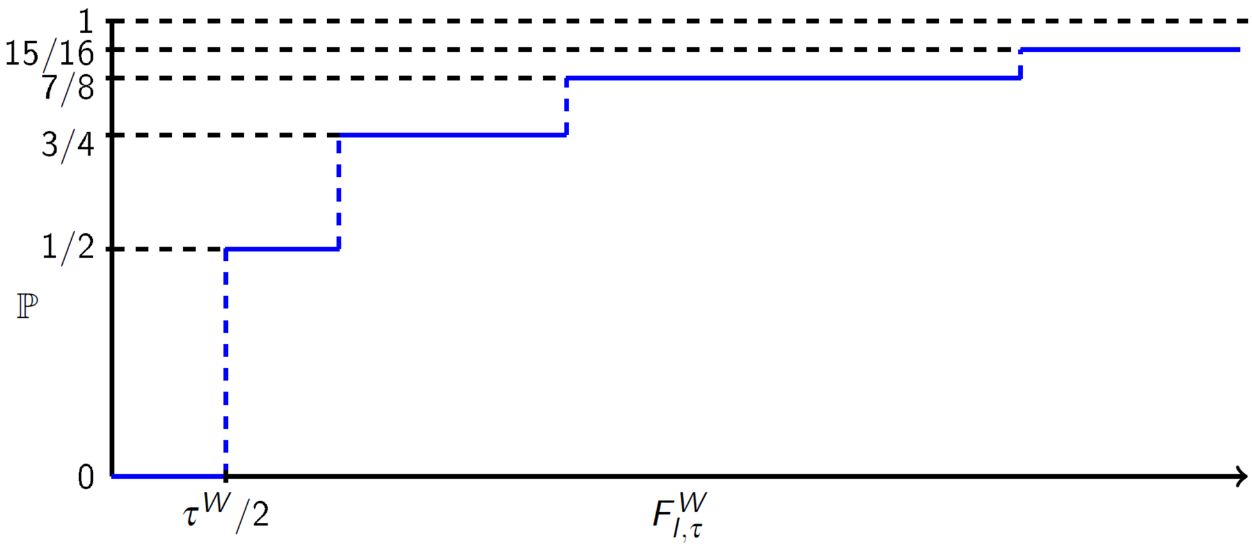

Figure 1: Invariant distribution of wealth of risk loving agents with a large bequest rate $\beta \geq (1 + g_{\tau^W})(R_R(1-\tau^W))^{-1}$.

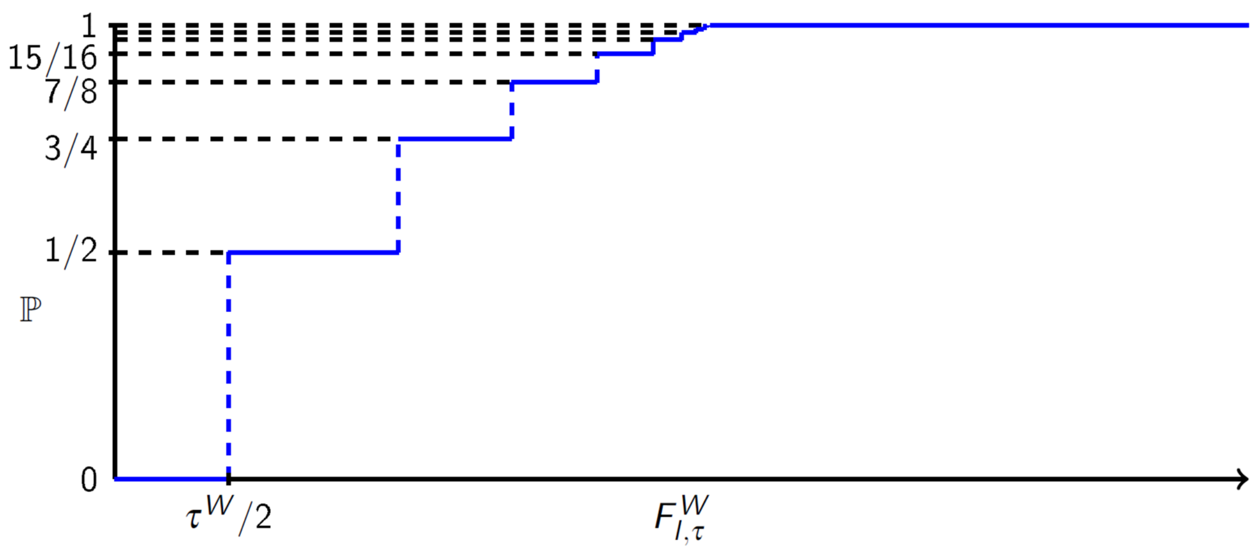

Figure 2: Invariant distribution of wealth of risk loving agents with a small bequest rate, $\beta < (1 + g_{\tau^W})(R_R(1-\tau^W))^{-1}$.

In our framework, if $\beta \geq (1 + g_{\tau^W})(R_R(1-\tau^W))^{-1}$, the invariant distribution of wealth has fat tails (see Figure 1). This will be the case if the bequest rate and the return of the risky technology are large enough and the wealth tax rates are not. Moreover, in this case the invariant distribution of wealth has exponential fat tails. For the definition of exponential fat tails and related theorems, see Araujo, Gama, and Kehoe (2024). If $\beta < \left(1+g_{\tau^W}\right)\left(R_R(1-\tau^W)\right)^{-1}$, that is, the bequest rate is small and the wealth tax rates are close to one, the dispersion of the wealth distribution is small in such a way that it is bounded from above (see Figure 2). In this case, risk loving agents whose parents were lucky n consecutive times have a wealth that satisfies that

$$x_\tau^{l,n+1} = \sum\nolimits_{k=0}^{n} (\beta\pi_r)^k \tau^W/2 < \frac{1+g_{\tau^W}}{2+2g_{\tau^W}-2\beta(1-\tau^W)R_R}\tau^W.$$

From Theorem 1, if a social planner increases taxes on wealth or bequests, the growth rate decreases. Furthermore, we observe that an increment in taxes increases the wealth of the poorest risk loving agents and decreases the wealth of the risk loving agents at the top of the distribution. Therefore, an increase in taxes reduces the dispersion of the invariant distribution of wealth. In the following section we explore more the implications of taxes in the invariant distribution and in the social welfare function.

4. The social welfare function and the optimal tax

The following welfare function W, which we use in the paper, captures the benefits of a lower inequality and a higher growth rate

$$W(c,b') \coloneqq (1-\delta)\Big(\int U^a\left(c_{a,0}(w), b_{a,0}(w)\right) dG_{a,\tau}^W(w) + \frac{1}{\gamma}\int \log U^l\left(c_{l,0}(w), b_{l,0}(w)\right) dG_{l,\tau}^W(w) + \sum\nolimits_{t=1}^{\infty}\delta^t\Big(\int U^a\left(c_{a,t}(w), b'_{a,t}(w)\right) dF_{a,\tau,t-1}^W(w) + \frac{1}{\gamma}\int \log U^l\left(c_{l,t}(w), b'_{l,t}(w)\right) dF_{l,\tau,t-1}^W(w)\Big)\Big) \tag{4.1}$$

where $\delta \in (0,1)$ is the discounted factor used by the social planner. This welfare function is a particular case of the CES welfare function (see Atkinson, 1970). In this case, the social welfare function has no problems related to the convergence of the series when the economy has a positive long-term growth rate since $U^a\left(c_{a,t}, b'_{a,t}\right)$ and $\log U^l\left(c_{l,t}, b'_{l,t}\right)$ are at most linear in t.

If $\tau^W = 0$, there is no invariant distribution of wealth since a small amount of risk loving agents concentrate all the aggregate wealth in the long run. If $\tau^W = 1$, the invariant distribution of wealth is constant, and the growth rate is considerably lower. The first and second cases imply that a large number of agents will not survive one way or another, which is hardly an optimal allocation for a central planner. The third case may be optimal if the social planner is more concerned about inequality. If a social planner is also concerned about the rate of growth, he is expected to have more intermediate taxation plans.

We define functions X, G, and a constant D as

$$X(\tau) := \log x_\tau^a + \sum_{n=2}^{\infty} \frac{1}{2^{n-1}\gamma}\left(\log\left(\frac{1}{2}\left(\left(x_\tau^{l,n}\right)^\gamma + \left(x_\tau^{l,1}\right)^\gamma\right)\right)\right),$$

$$G(d,\tau) := \frac{\delta}{(1-\delta)}\log(1+g_\tau),$$

and

$$D := \log\left(\beta^\beta(1-\beta)^{1-\beta}\right).$$

Notice that the first function depends on the invariant distribution of wealth. The second function only depends on the growth rate of the economy, and it is strictly decreasing in τ. The third function depends on the agents' bequest rate and the bequest tax. Lastly, the three terms depend on the discount factor of the social planner. Then, we have the following result.

Theorem 2. *(Decomposition of the welfare function)* In the equilibrium allocation, W can be written as

$$W\left(\left(U^i\right)_i, \left(c_\tau^i\right)_i, \left(b_\tau^i\right)_i\right) = X(\tau^W) + G(\delta, \tau^W) + D,$$

where G is differentiable in δ and τ^W, strictly increasing in d, and strictly decreasing in $(0,1)$, and $X(\tau^W) < 0$ for $\tau^W \in (0,1)$ and $\tau^W \in [0,1)$, $X(\tau^W) = 0$ for $\tau^W = 1$. Then, function X attains its maximum value when $\tau^W = 1$.

The characterization of the social welfare function given in Theorem 2 is extremely useful to understand the phenomena underlying the tax rate, inequality, the growth rate and the relationship between the discount factor and the bequest rate. When taxes are reduced, the growth rate and the dispersion of the invariant distribution increase. Since function X is a measure of wealth inequality, there is a tradeoff between growth and inequality.

To have an invariant distribution of wealth, it is necessary that $\tau^W > 0$. However, extremely low levels of taxes generate invariant distributions in which a large proportion of risk loving agents have almost no wealth.**.** Therefore, from now on, we assume that the marginal tax rate is bounded away from zero, that is, there is $\zeta > 0$ such that $\tau^W \geq \zeta$.

Note that only function G depends on the discount factor of the social planner. When the social planner discounts the future strongly, $\delta \approx 0$, the welfare function depends only on function X, which makes him concerned about reducing intragenerational inequality and not about the growth rate. When the social planner discounts the future weakly, $\delta \approx 1$, he is almost entirely worried about growth rate in relative terms since function X does not depend on d. Each case has implications on the optimal taxes, we explore these cases in Subsection 5.3.1.

Each type of taxation has different implications for growth and intragenerational inequality. Wealth taxes reduce dispersion the most, while generating changes in the growth rate. Consequently, the social planner must find a balance between low wealth taxes to have large economic growth and high taxes to reduce inequality. On the other hand, bequest taxes reduce the growth rate the most by increasing consumption at the earliest dates. Moreover, since bequest taxes have a small impact on the dispersion of the wealth distribution and a large negative impact on growth, bequest taxes may be zero in several cases.

5. Numerical examples in economies with only wealth taxes

In this section we analyze examples to illustrate the model and the theorems that we introduce above.

5.1. Numerical examples with changes in the discount factor

In the first example of this section, we assume that $R_R = 4.86$, $R_S = 1.6$, $\beta = 0.5$, $\gamma = 1$, $\varepsilon = 0.1$, and δ varies from 0.6 to 0.85.

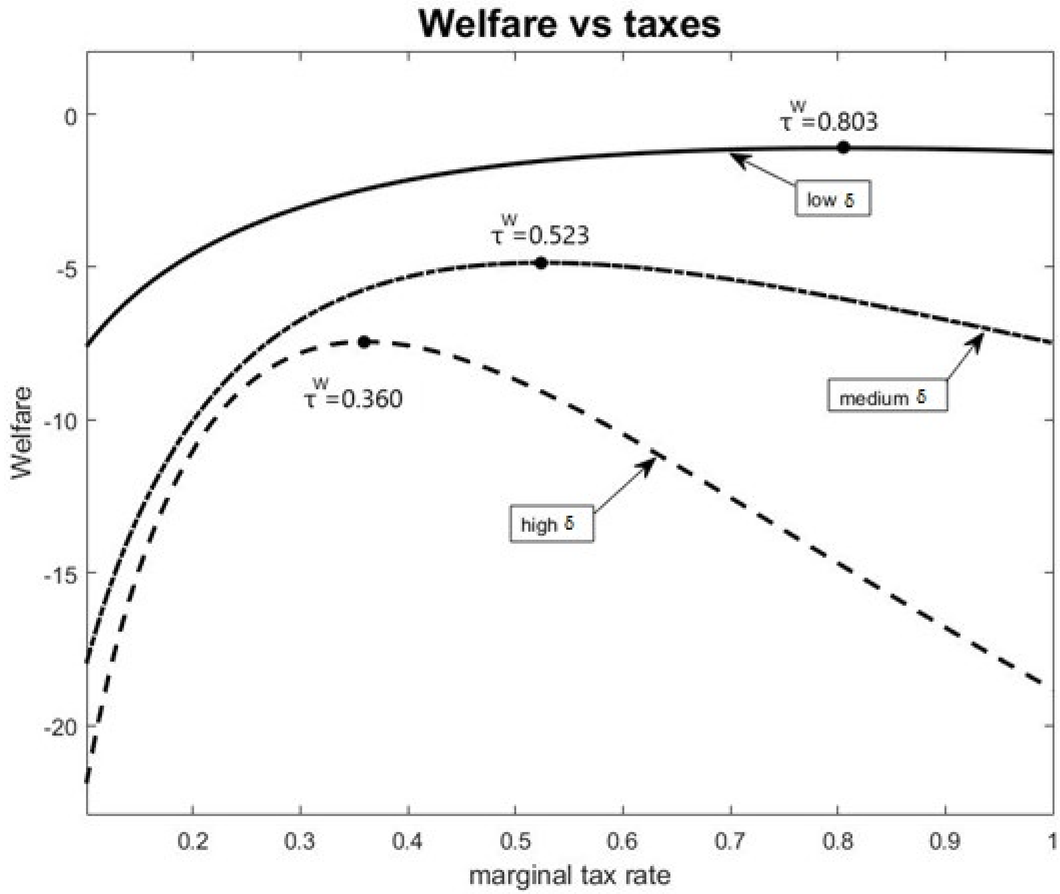

Figure 3: Social welfare function vs wealth taxes for different values of $\delta = 0.5, 0.75, 0.833$.

We can observe in Figure 3 that the optimal tax for $\delta = 0.5$ is 80.3%, and the growth rate of the economy is 1.018, which averages that the economy has a slightly positive growth. Moreover, if the social planner increases wealth tax to 1, the growth rate converges to $(\beta(R_R/2 + R_S)/2 - 1) \times 100 = 0.75$, and in the absence of taxes, the growth rate is $(\beta(R_R/2) - 1) \times 100 \ = 21.5$ (See Proposition 4).

If $\delta > \beta$, the optimal tax rate induces a larger growth rate. More concretely, if $\delta = 0.8$, the optimal marginal tax rate is approximately 52.4% and the growth rate is around 0.044. Moreover, if $\delta = 0.85$, the optimal marginal tax rate is approximately 36% and the growth rate is around 0.072.

Note that for all possible wealth taxes, the growth rate is positive. However, the growth rate increases as the social planner's discount factor increases. One explanation for this phenomenon is that the social planner is not too concerned about very distant consumption when the discount factor is low, which averages that low growth rates may be optimal. In this case, the social planner concern with inequality is more important than with consumption in the long run.

We also observe that, for all the numerical examples above, the optimal wealth tax is unique. Moreover, the social welfare function is strictly concave in the wealth tax rate for all the analyzed discount factors of the social planner.

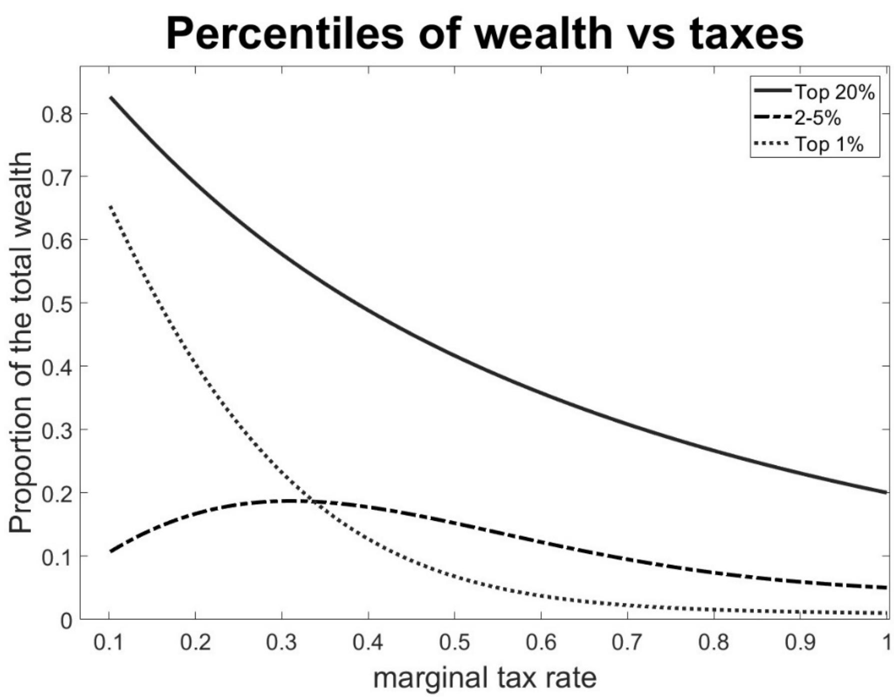

Figure 4: Income/wealth inequality for different wealth taxation rates.

We observe in Figures 3 and 4 that an increment in the discount factor of the social planner implies an increment in the growth rate and a more unequal distribution of wealth. Moreover, the level of inequality implemented by a social planner with a discount factor $\delta = 0.5$ is low ($\tau^W \approx 0.8$). In this case the top 1% has 2.85% of the total income, the top 10% has 20.7%, and the top 20% has 34.15%, which is more equal than Japan, a very equal country where the top 1% earns around 10% of the national income. For a social planner that with a discount factor equal to $\delta = 0.75$ (which implies $\tau^W \approx 0.52$), inequality is clearly larger with 9.69% for the top 1%, 34.85% for the top 10%, and 47.6% for the top 20% which is closer to Japan. Finally, for a social planner with a discount factor equal to $\delta = 0.833$ (which implies $\tau^W \approx 0.36$), inequality is clearly larger than the other two cases with 21.44% for the top 1%, 48.37% for the top 10%, and 59.09% for the top 20% which is closer to the US where the top 1% earns around 20% of the national income, see Rodriguez, Díaz-Giménez, Quadrini, and Ríos-Rull (2002).

As expected, the G function is a decreasing function and the X function is an increasing function on τ^W. Also, as δ increases, the values of the G function increases, the slope becomes more negative, and the X function does not change. As a result, the optimal marginal tax rate decreases when the discount factor of the social planner δ increases. For values of δ lower than 0.8, the welfare function is almost not affected by the G function, making the optimal tax rate to be equal to one. For values larger than 0.8, the importance of the G function becomes important enough to make the optimal wealth tax of the social planner to be lower than one. As δ goes to one, the optimal tax rate goes to zero.

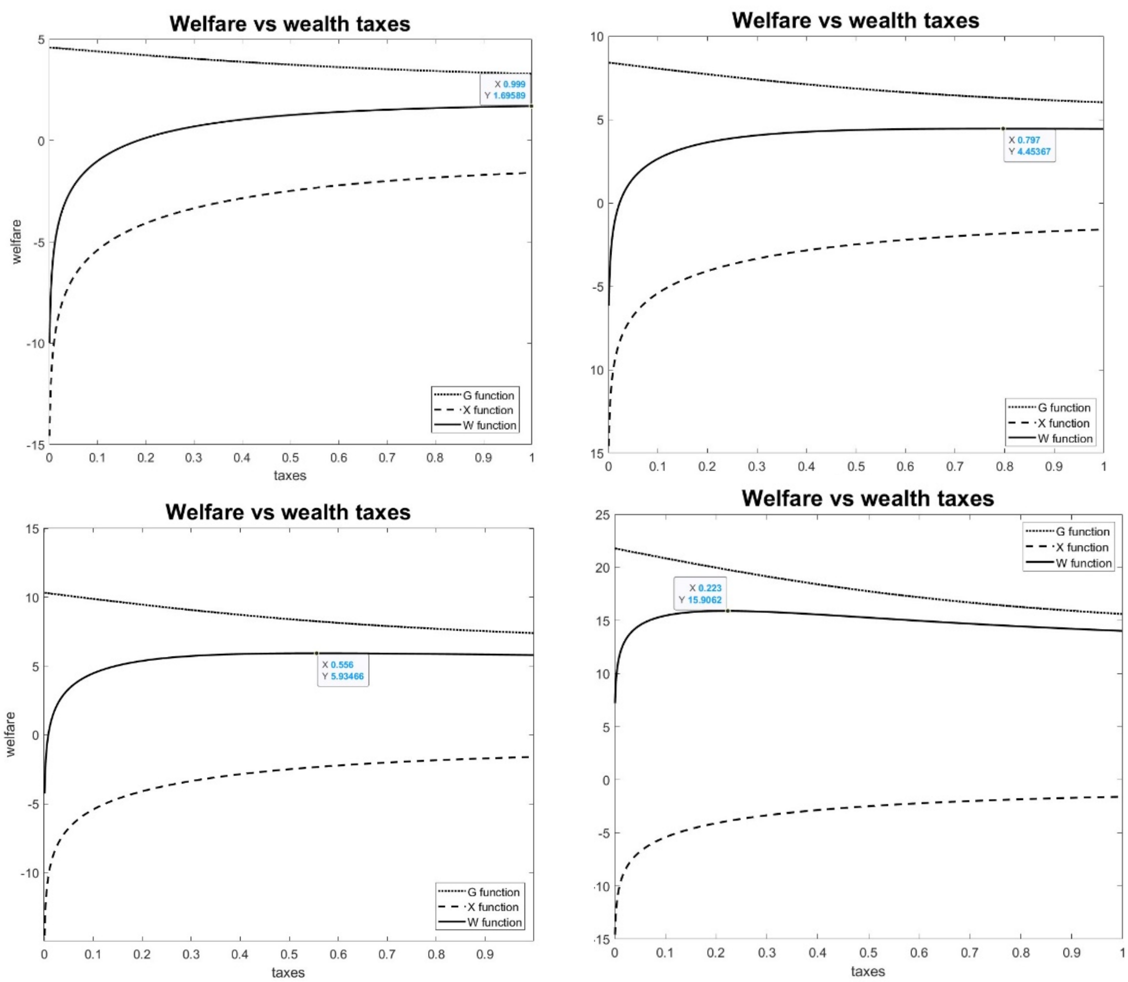

Figure 5: Welfare components with $\beta = 0.7$ and $\delta = 0.7, 0.8$ (top) and $\delta = 0.8, 0.95$ (bottom).

5.2. Additional properties of the optimal taxes

When the discount factor is large, the social planner has a large concern about future consumption plans. To have a large consumption in the long run, the growth rate must be also large. This implies that the risky technology must be largely used, generating a more disperse wealth distribution. Thus, a social planner who is more concerned about the future chooses a lower tax rate than a social planner who is not. If the social planner is more concerned about distant consumptions, that is, the discount factor moves from δ to $\delta + \epsilon$ with $\epsilon > 0$, functions X and D do not change. However, the variation of function G is positive and proportionally to $(1+\delta)/(1-\delta)^2$. Then, the importance of the growth function in the welfare function increases with the discount factor. This suggests that the optimal taxes are decreasing in d due to the negative correlation between growth and taxes.

The sensibility of the optimal taxes with the discount factor does not imply that the growth rate is necessarily positive or negative. Moreover, Theorem 4 shows that if there is a unique tax rate, the existence of a positive or a negative growth rate depends on the discount factor.

Proposition 2. *(Effect on growth and inequality of the social planner's discount factor)* If there is a unique optimal tax rate $\tau^W: (0,1) \to [0,1]$ with $\varepsilon = 0$, then

a. There exists $\delta_2 \in (0,1)$ such that, if $\delta > \delta_2$, then $g_{\tau(\delta)} > 0$. Moreover,

$$\lim_{\delta \to 1} \tau^W(\delta) = 0 \,, \lim_{\delta \to 1} g_{\tau^W(\delta)} = \beta(R_R/2) - 1 > 0,$$

and

$$\lim_{\delta \to 1} F^W_{a,\tau^W(\delta)}(w) = \lim_{\delta \to 1} F^W_{l,\tau^W(\delta)}(w) = 1 \text{ for } w > 0$$

b. Also, if

$$\beta\left(\frac{R_R/2 + R_S}{2}\right) < 1, \tag{5.1}$$

there exists $\delta_1 \in (0,1)$ such that, if $\delta < \delta_1$, then $g_{\tau^W(\delta)} < 0$. Therefore, the economy collapses. Moreover,

$$\lim_{\delta \to 0} \tau^W(\delta) = 1 \,, \lim_{\delta \to 0} g_{\tau^W(\delta)} = (\beta/2)\left(\frac{R_R}{2} + R_S\right) - 1 < 0,$$

and

$$\lim_{\delta \to 0} F^W_{a,\tau^W(\delta)}(w) = \lim_{\delta \to 0} F^W_{l,\tau^W(\delta)}(w) = 1 \text{ for all } w.$$

Numerically, when the optimal bequest taxes are zero, Proposition 5 holds. Moreover, as the discount factor increases, the optimal wealth tax decreases, and the growth rate increases (see Table 1). When the optimal bequest taxes are strictly positive, these properties do not always hold (see Table 2).

5.3.Numerical examples with changes in productivity

In the set of examples of this section, we assume that $R_R \in [3.9, 5.9]$, $R_S = 1.6$, $\gamma = 1$, $\varepsilon = 0.1$, and $\delta = \beta = 0.5$.

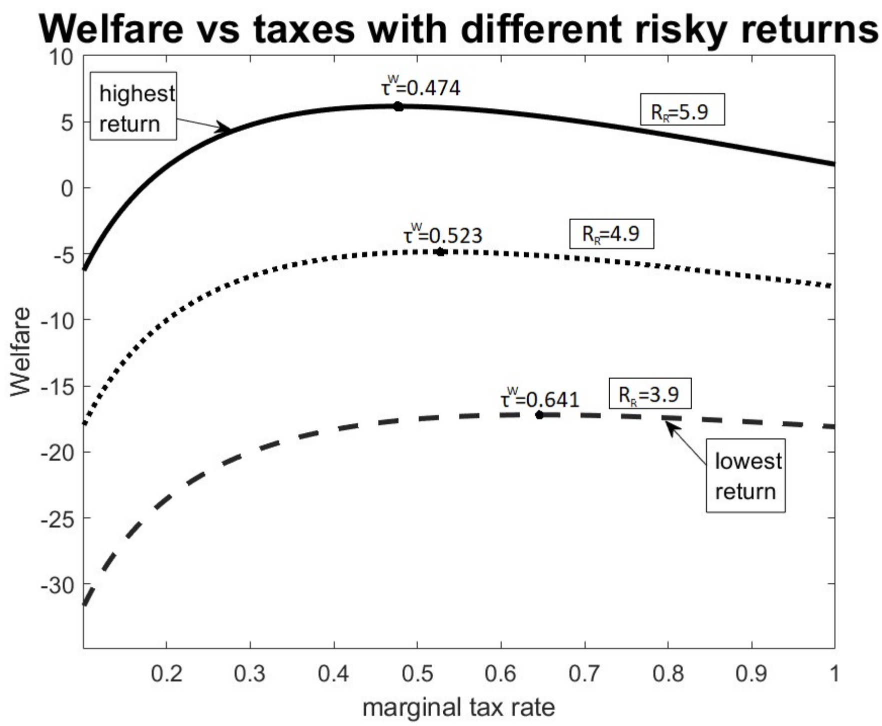

Figure 6: Welfare vs changes in productivity of R_R.

Note that, in this example, changes in the productivity of risky technology imply changes in the spread of the technology. Note that this phenomenon causes a change in optimal taxation. In this case, a more productive risky technology leads to a lower optimal tax rate. It is therefore optimal for a social planner to increase inequality due to the increase in productivity. This fact is also observed in a wide variety of economies around the world. The US has a higher level of productivity than the largest economies in Europe, such as Germany, France, and England, and also has considerably greater wealth and income inequality.

6. Model with bequest taxes and consumption taxes

At each date $t \geq 0$, the government imposes taxes on wealth, bequests, and consumption. There is a tax on wealth of agents whose wealth exceeds a threshold W_t and a subsidy on wealth of agents whose wealth is less than W_t. For simplicity, the marginal tax rate is equal to the marginal subsidy rate, τ^W. Consequently, the net wealth tax on an agent with wealth w is $\tau^W \max(w - W_t, 0) - \tau^W \max(W_t - w, 0)$. There is also a tax on bequests that works similarly. Agents whose bequests exceed a threshold B_t pay the tax and those whose bequests are less than B_t receive a subsidy. Again, we assume the marginal tax rate is equal to the marginal subsidy rate, τ^B, and the net bequest tax on an agent with bequests b is $\tau^B/(1-\tau^B)\max(b - B_t, 0) - \tau^B/(1 - \tau^B)\max(B_t - b, 0)$. We also assume the marginal tax rate is equal to the marginal subsidy rate, $\bar{\tau}^C$, and the net bequest tax on an agent with consumption c is $\tau^C/(1-\tau^C)\ \max(c - C_t, 0) - \tau^C/(1-\tau^C)\max(C_t - c, 0)$.

We specify a tax policy as $\tau = (\tau^W, \tau^B, \tau^C)$. The marginal tax rates τ^W, τ^B, τ^C are exogenously defined by the central planner such that the after-tax wealth of the poorest agents is bounded away from zero, that is,

$$\tau^W + \frac{\beta\tau^B}{(1-(1-\beta)\tau^C - \beta\tau^B)} + \frac{(1-\beta)\tau^C}{(1-(1-\beta)\tau^C - \beta\tau^B)} \geq \zeta.$$

Remark. When $\tau^B < 0$, the wealthiest agents are the ones who benefit more. However, in all these cases, the bequest tax cannot be too negative since the wealth in hands of the poorest agents, the risk loving agents who failed, must have a positive after-tax wealth.

To keep tax policy simple, we set the threshold W_t equals to the average wealth level in the economy, the threshold B_t equals to the average bequest level, and the threshold C_t equals to the average consumption level.

Given an initial amount of wealth, $w_0^{a_i}$, the utility maximization problem of a risk averter a_i of the old generation in $t = 0$ is

$$\max_{(c,b)} \quad (1-\beta)\log c + \beta \log b$$

$$s.t. \quad 0 \le c + \frac{\tau^C}{1-\tau^C}(c - C_0) + b + \frac{\tau^B}{1-\tau^B}(b - B_0) \le w_0^{a_i} - \tau^W\big(w_0^{a_i} - W_0\big).$$

Similarly, given the initial wealth, $w_0^{l_i}$, the utility maximization problem of a risk loving agent l_i of the old generation in $t = 0$ is

$$\max_{(c,b)} \quad \left(c^{1-\beta} b^{\beta}\right)^{\gamma}$$

$$s.t. \quad 0 \le c + \frac{\tau^C}{1-\tau^C}(c - C_0) + b + \frac{\tau^B}{1-\tau^B}(b - B_0) \le w_0^{l_i} - \tau^W\big(w_0^{l_i} - W_0\big).$$

Given the bequest $b_{s^{a_i}}^{a_i}$ received from his parent at state s^{a_i} at date $t \ge 1$, the utility maximization problem of a young risk averter a_i is

$$\max_{(c,b,\theta)} \quad (1-\beta)\log c + \beta \log b$$

$$s.t. \quad \theta_S \le b_{s^{a_i}}^{a_i},$$

$$0 \le c + \frac{\tau^C}{1-\tau^C}(c - C_t) + b + \frac{\tau^B}{1-\tau^B}(b - B_t) \le R_S\theta_S - \tau^W(R_S\theta_S - W_t).$$

Given the bequest $b_{s^{l_i}}^{l_i}$ received from his parent at state s^{l_i} at date $t \ge 1$, the utility maximization problem of a young risk loving agent l_i is

$$\max_{(c,b,\theta)} \quad \frac{1}{2}\left(c_1^{1-\beta}\, b_1^{\beta}\right)^{\gamma} + \frac{1}{2}\left(c_2^{1-\beta}\, b_2^{\beta}\right)^{\gamma}$$

$$s.t. \quad \theta_R + \theta_S \le b_{s^{l_i}}^{l_i},$$

$$0 \le c_1 + \frac{\tau^C}{1-\tau^C}(c_1 - C_t) + b_1 + \frac{\tau^B}{1-\tau^B}(b_1 - B_t) \le R_R\theta_R + R_S\theta_S - \tau^W(R_R\theta_R + R_S\theta_S - W_t),$$

$$0 \le c_2 + \frac{\tau^C}{1-\tau^C}(c_2 - C_t) + b_2 + \frac{\tau^B}{1-\tau^B}(b_2 - B_t) \le R_S\theta_S - \tau^W(R_S\theta_S - W_t).$$

Since bequest and consumption taxes affect the incentives that each agent has for consuming and leaving bequests, average consumption and average bequest depend on the marginal tax rates. Moreover, a higher bequest tax implies a higher level of consumption and a lower level of bequests, which decreases the wealth of all immediate descendants. More specifically, if $\overline{y}_{\tau,t}^{a}$ is the after-tax average wealth of the risk averters at date t, and $\overline{y}_{\tau,t}^{l}$ is the after-tax average wealth of the risk loving agents at date t, the average wealth at date $t+1$ is

$$\overline{y}_{\tau,t+1} = \frac{R_R\beta(1-\tau^B)}{4(1-(1-\beta)\tau^C-\beta\tau^B)}\overline{y}^l_{\tau,t} + \frac{R_S\beta(1-\tau^B)}{2(1-(1-\beta)\tau^C-\beta\tau^B)}\overline{y}^a_{\tau,t}.$$

Given the tax policy τ^W, we define the growth rate from t to date $t+1$ to be $g_{\tau,t} = \overline{y}_{\tau,t+1}/\overline{y}_{\tau,t} - 1$.

Proposition 3. If we consider two tax policies (τ^W, τ^B, τ^C) and $(\hat{\tau}^W, \hat{\tau}^B, \hat{\tau}^C)$ such that

$$\hat{\tau}^B = \frac{\tau^B - \tau^W}{1-\tau^W}, \qquad \hat{\tau}^C = \frac{\tau^C - \tau^W}{1-\tau^W}.$$

Then, both tax policies are equivalent. More specifically, when $\hat{\tau}^C = 0$, we have that

a. If $\tau^B = \tau^C$, $\hat{\tau}^B = 0$;

b. If $\tau^B > \tau^C$, $\hat{\tau}^B > 0$; and

c. If $\tau^B < \tau^C$, $\hat{\tau}^B < 0$.

Analogously, when $\hat{\tau}^B = 0$, we have that

a. If $\tau^B = \tau^C$, $\hat{\tau}^C = 0$;

b. If $\tau^B > \tau^C$, $\hat{\tau}^C < 0$; and

c. If $\tau^B < \tau^C$, $\hat{\tau}^C > 0$.

Since any tax policy can be written with only two taxes, from now on, we focus on tax policies with only wealth and bequest taxes.

Proposition 4. For any $\epsilon > 0$ and tax policy $\tau = (\tau^W, \tau^B)$ such that $\tau^W, \tau^B \leq 1-\epsilon$, any increment on the wealth tax rate (from τ^W to $\tau^W + \epsilon$) induces a higher growth rate than an increment on the bequest tax rate (from τ^B to $\tau^B + \epsilon$) at date $t+1$, $g_{(\tau^W+\epsilon,\tau^B),t} > g_{(\tau^W,\tau^B+\epsilon),t}$.

The following Theorem shows that Theorem 1 holds also with bequest taxes.

Theorem 3. For any distribution of initial endowments $\left(G^W_{a,\tau}, G^W_{l,\tau}\right)$ the distribution of after-tax wealth among the agents $\left(F^W_{a,\tau,t}, F^W_{l,\tau,t}\right)$ and the growth rate $g_{\tau,t}$ converge to invariant distribution $\left(F^W_{a,\tau}, F^W_{l,\tau}\right)$ and a constant g_τ. Moreover, g_τ is a strictly decreasing C^1 function in $\tau = (\tau^W, \tau^B)$.

6.1. Welfare function properties

We define functions X, G, and D as

$$X(\tau) \coloneqq \log x_\tau^a + \sum_{n=2}^{\infty} \frac{1}{2^{n-1}\gamma}\left(\log\left(\frac{1}{2}\left(\left(x_\tau^{l,n}\right)^\gamma + \left(x_\tau^{l,1}\right)^\gamma\right)\right)\right),$$

$$G(\delta,\tau) \coloneqq \frac{\delta}{(1-\delta)}\log(1+g_\tau),$$

and

$$D(\tau^B) \coloneqq \log\left(\left(\frac{\beta^\beta(1-\beta)^{1-\beta}(1-\tau^B)}{1-\beta\tau^B}\right)\right).$$

Then, we have the following result.

Theorem 4. *(Decomposition of the welfare function)* In the equilibrium allocation, W can be written as

$$W\left(\left(U^i\right)_i, \left(c_\tau^i\right)_i, \left(b_\tau^i\right)_i\right) = X(\tau) + G(\delta,\tau) + D(\tau^B),$$

where G is differentiable in δ and τ, strictly increasing in δ, and strictly decreasing in τ^W and τ^B, and $X(\tau) = 0$ for $\tau^W = 1$. If $\gamma = 1$, $X(\tau) < 0$ for $\tau^W \in (0,1)$ and τ^B, then, function X attains its maximum value when $\tau^W = 1$.

The following result shows that, if the bequest rate is small and the discount factor is large, positive bequest taxes induce lower welfare than taxes on wealth alone.

Proposition 5. *(Optimality of bequest subsidies)* There is $\bar{\beta} \in (0,0.5)$ and $\underline{\delta} \in (0,1)$ such that if $\beta \leq \bar{\beta}$ and $\underline{\delta} \leq \delta$, for any optimal taxation plan, $\tau^B \leq 0$.

In the numerical examples is Subsection 5.3, we observe that $\tau^B < 0$ occurs in almost all cases unless β is large and δ small.

Note that for low levels of the bequest rate, almost any level of bequest taxes generates lower welfare than tax policies with only wealth taxes. To have positive optimal bequest taxes, we must look for situations in which the bequest rate is large. In the following subsection, we analyze this case.

7. Numerical examples with all three taxes

We assume that $R_R = 9$, $R_S = 2$, $\gamma = 1$, $\zeta = 0.01$.

From Figure 7 as it was mentioned above, the G function is a decreasing function on τ^B. However, the $X + D$ function is not always an increasing function on τ^B. The reason for this phenomenon might be that bequest taxes only reduce inequality for low levels, and it can even increase the inequality when the bequest taxes are large. Another important phenomenon in this case is that the slope of the X function is completely different to the one seen with wealth taxes. The slope is slightly positive for negative and low positive values of the marginal bequest rate, and the slope is negative for large values. This generates that the impact of the G function is considerably larger when compared to taxes on wealth.

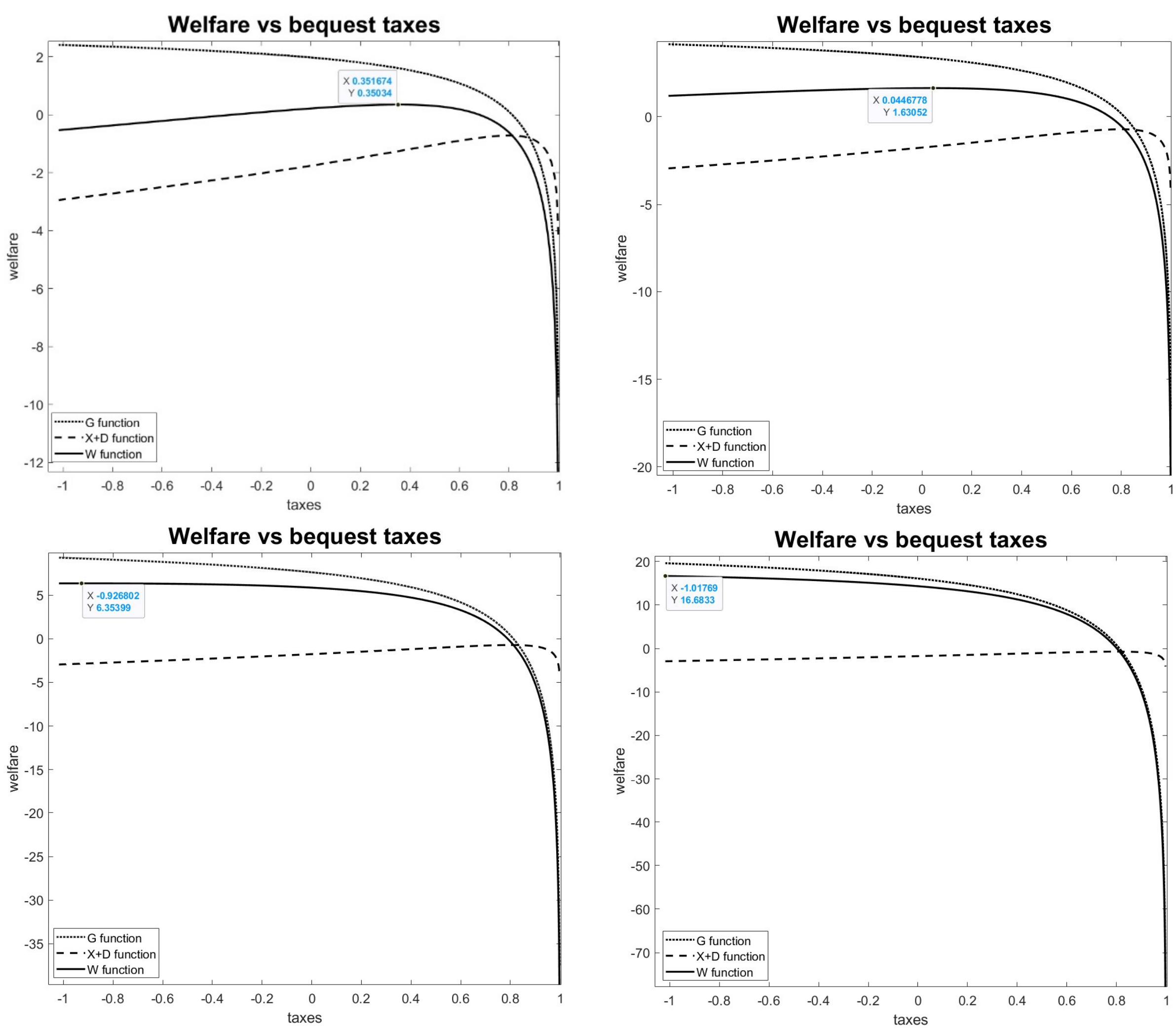

Figure 7. Welfare components with $\beta = 0.7$, $\delta = 0.7, 0.8$ (top) and $\delta = 0.88, 0.95$ (bottom), and $\tau^W = 0.85$.

As in the analysis of the wealth taxes, as δ increases, the values of the G function increases, the slope becomes more negative, and the $X + D$ function does not change. Note that the change of the X function is due to a change of scale. As a result, the optimal marginal bequest tax rate, assuming that the wealth tax is fixed in $\tau^W = 0.85$, decreases when the discount factor of the social planner δ increases. For values of δ lower than 0.9, the welfare function is less affected by the G function, making the optimal tax rate to be in the interior of the set of feasible bequest marginal tax rates. For values larger than 0.9, the importance of the G function becomes important enough to make it impossible to have an optimal bequest tax of the social planner.

Let us analyze the optimal taxation plans. To do so, we assume that $R_R = 9$, $R_S = 2$, $\gamma = 1$, $\zeta = 0.01$, $\beta = 0.5, 0.6, 0.9$, and $\delta = 0.6, 0.75, 0.9$.

$\beta = 0.5$	$\delta = 0.6$	$\delta = 0.75$	$\delta = 0.9$
(τ^W, τ^B)	(0.99, -3.00)	(0.89, -8.05)	(0.94, -15.50)
(τ^B, τ^C)	(0.96, 0.99)	(0.01, 0.89)	(0.01, 0.94)
(τ^W, τ^C)	(0.96, 0.75)	(0.01, 0.89)	(0.01, 0.94)
Growth rate	161%	233%	246%
Top 1%	1%	13%	11%
$\beta = 0.6$			
(τ^W, τ^B)	(0.99, -1.00)	(0.84, -5.22)	(0.90, -8.90)
(τ^B, τ^C)	(0.98, 0.99)	(0.01, 0.84)	(0.01, 0.90)
(τ^W, τ^C)	(0.98, 0.50)	(0.01, 0.84)	(0.01, 0.90)
Growth rate	144%	248%	257%
Top 1%	1%	23%	20%
$\beta = 0.9$			
(τ^W, τ^B)	(0.95, 0.80)	(0.32, -0.10)	(0.31, -0.13)
(τ^B, τ^C)	(0.99, 0.95)	(0.26, 0.32)	(0.21, 0.31)
(τ^W, τ^C)	(0.99, -4.00)	(0.26, 0.09)	(0.21, 0.11)
Growth rate	109%	251%	270%
Top 1%	1%	35%	43%

Table 1: Optimal taxation plans for low bequest rates, β, and discount factor of the social planner, δ.

We see in Table 1 that if the social planner has a discount factor is small, the optimal wealth tax should be high to reduce inequality. Furthermore, when the social planner discounts the future less, he is more concerned with growth, which implies a lower optimal tax rate. Therefore, the growth rate is positively related to the discount factor. Notice that the bequest rate has no influence on optimal taxes (changes in the bequest rate have no impact on the optimal tax rate), but it does have a strong effect on the growth rate. Increases in the bequest rate or discount factor have a positive impact on growth.

Note that in Table 1, the optimal bequest tax is negative when bequest rate is small (see Proposition 3). We observe strictly negative bequest taxes independently of the discount factor if the bequest rate $\beta \leq 0.5$, and it also increases as the bequest rate increases. However, it only occurs when the discount factor of the social planner is lower than the bequest rate. Thus, when the bequest rate is negative, it is optimal for the social planner to generate incentives to the investors to increase their bequests. Moreover, it is observed that bequest taxes are positively related to the bequest rate. This may be due to the social planner's intention to reduce the proportion of wealth that is given to the next generation. Positive bequest taxes occur only when the discount factor of the social planner is low, and the bequest rate is large. This occurs since this is the only case in which the social planner wants to reduce the bequest rate of the economy. In this case, bequest taxes reduces the incentives for bequests that are given to future generations.

We also observe in Table 1 that wealth taxes are always extremely high, regardless of the bequest rate, when the social planner's discount factor is small. When the social planner heavily discounts the future, he is almost no concerned about growth. When the social planner discounts the future less, wealth taxes decrease and the wealth share of the top 1% increases.

However, this fact is not present in all cases: for discount factors close to zero, function X is almost identical to the social planner's welfare function, and for discount factors close to one, the social planner is mostly concerned with growth in relative terms. The case where the discount factor is close to zero implies extremely high optimal wealth taxes, and the case where the discount factor is close to one implies extremely low wealth taxes.

For intermediate levels of the discount factor, the importance of inequality is not particularly great and the social planner's optimal bequest rate are low compared to the agent's bequest rate. Furthermore, it is observed numerically that the bequest taxes are positive if δ/β is less than 1.

7. Extensions

7.1. Model without segmentation

Let us define a model based on Section 2 in which both agents have access to both technologies.

Assumption S2: Risk averse and risk loving investors have access to the safe technology and to risky one, but not simultaneously.

Note that if an agent decides to invest in one of the technologies, he cannot invest in the other one. This may happen when each agent has a limited capacity to manage investments with different type of properties at the same time.

Under these assumptions, risk loving agents will continue investing only in the risky one. From now on, we assume Equation 5.1 and $\tau^B = 0$.

The taxation rate and the level of wealth of the agent affect the optimal production strategy of the risk averters. More precisely, we have that:

Proposition 6. Given a marginal wealth tax rate $\tau^W \in (0,1)$, there is a constant

$$\alpha^*_{\tau^W} = \frac{\tau^W (R_R - 2R_S)}{\beta R_S^2 (1 - \tau^W)} \tag{6.1}$$

such that:

1. if the after-tax wealth y_t^a is such that $y_t^a > \alpha^*_{\tau^W} \overline{y}_{t+1}$, the risk averter invests in the safe technology at date $t+1$,

2. if $y_t^a \in \left(0, \alpha^*_{\tau^W} \overline{y}_{t+1}\right)$, the risk averter invests in the risky technology at date $t+1$, and

3. if $y_t^a = \alpha^*_{\tau^W} \overline{y}_{t+1}$, the risk averter is indifferent between both type of investments at date $t+1$.

Therefore, in the invariant distribution, taxation can have a positive impact on growth since it leads a portion of risk-averse agents to invest in the risky technology. In addition, low tax levels average that the wealth invested in risky technology by risk-averse investors is low, which implies a lower growth rate. However, if taxes increase, the gap mentioned in Proposition 5 becomes large and the proportion of wealth invested in risky technology increases.

7.2. Model with changing attitudes towards risk within a dynasty

Let us suppose that there is a probability $p \in [0,1)$ of a risk loving agent having a risk averter child, and vice versa. The utility of a risk loving agent who has a risk loving agent child and the utility of a risk averter who has a risk averter child is the same as in the basic model. However, a risk loving agent who has a risk averter child has the utility function

$$\widehat{U}^l(c,b) = c^\gamma,$$

and a risk averter who has a risk loving agent child has the utility function

$$\widehat{U}^a(c,b) = \log c.$$

Therefore, all agents who switch their attitude towards risk receive zero bequest, and they are unable to invest in any technology. Therefore, their after-tax wealth when they are old is

$$\left(\tau^W + \frac{\hat{\beta}\tau^B}{\left(1-(1-\hat{\beta})\tau^C - \beta\tau^B\right)} + \frac{(1-\hat{\beta})\tau^C}{\left(1-(1-\hat{\beta})\tau^C - \hat{\beta}\tau^B\right)}\right)\bar{y},$$

where $\hat{\beta} = \beta(1-p) \in (0,1)$ is the average bequest rate. As in previous cases, the marginal taxes must such that

$$\tau^W + \frac{\hat{\beta}\tau^B}{\left(1-(1-\hat{\beta})\tau^C - \beta\tau^B\right)} + \frac{(1-\hat{\beta})\tau^C}{\left(1-(1-\hat{\beta})\tau^C - \hat{\beta}\tau^B\right)} \geq \zeta.$$

Note that the proof of existence and convergence to an invariant distribution with a tax policy with $\tau^C = 0$ is analogous to Theorem 1. The form of the invariant after-tax distribution for risk loving agents, $F_{l,\tau}^W(w)$, is like in the basic model. The difference is that the mass of agents at each level of wealth. Moreover, the distribution for risk loving agents is

$$x_\tau^{l,n} = \sum_{k=0}^{n-1} \frac{\frac{1}{2}\left(\tau^W + \frac{\hat{\beta}\tau^B}{\left(1-\hat{\beta}\tau^B\right)}\right) R_R^k (1-\tau^W)^k \frac{\beta^k(1-\tau^B)^k}{(1-\beta\tau^B)^k}}{(1+g_\tau)^k}$$

for the n^{th} poorest group of risk loving agents with weight $\left((1+p)/2\right)\left((1-p)/2\right)^{n-1}$ for $n \in \mathbb{N}$. On the other hand, the distribution for the risk averters, $F_{a,\tau}^W(w)$, is

$$x_\tau^{a,n} = \sum_{k=0}^{n-1} \frac{\frac{1}{2}\left(\tau^W + \frac{\hat{\beta}\tau^B}{(1-\hat{\beta}\tau^B)}\right) R_S^k (1-\tau^W)^k \frac{\beta^k (1-\tau^B)^k}{(1-\beta\tau^B)^k}}{(1+g_\tau)^k}$$

for the n^{th} poorest group of risk averters with weight $p(1-p)^{n-1}$ for $n \in \mathbb{N}$.

The following welfare function W, which we use in the paper, captures the benefits of a lower inequality and a higher growth rate

$$W(c,b') \coloneqq (1-\delta)\left(\int U^a\left(c_{a,0}(w), b_{a,0}(w)\right) dG_{a,\tau}^W(w) + \frac{1}{\gamma}\int \log U^l\left(c_{l,0}(w), b_{l,0}(w)\right) dG_{l,\tau}^W(w) + \sum_{t=1}^{\infty} \delta^t \left(\int U^a\left(c_{a,t}(w), b'_{a,t}(w)\right) dF_{a,\tau,t-1}^W(w) + \frac{1}{\gamma}\int \log U^l\left(c_{l,t}(w), b'_{l,t}(w)\right) dF_{l,\tau,t-1}^W(w)\right)\right)$$

where $\delta \in (0,1)$ is the discounted factor used by the social planner.

We define functions X, G, and a constant D as

$$X(\tau) \coloneqq \sum_{n=1}^{\infty} p(1-p)^{n-1}\log(x_\tau^{a,n}) + p\log(x_\tau^{l,1}) + \frac{1-p}{2\gamma}\left(\log\left(\frac{1}{2}\left(\left(x_\tau^{l,2}\right)^\gamma + \left(x_\tau^{l,1}\right)^\gamma\right)\right)\right)$$
$$+\left(\sum_{n=2}^{\infty} \frac{(1+p)(1-p)^{n-1}}{2^n\gamma}\left(\log\left(\frac{1}{2}\left(\left(x_\tau^{l,n+1}\right)^\gamma + \left(x_\tau^{l,1}\right)^\gamma\right)\right)\right)\right),$$

$$G(d,\tau) \coloneqq \frac{\delta}{(1-\delta)}\log(1+g_\tau),$$

and

$$D(\tau^B) \coloneqq (1-p)\log\left(\left(\frac{\beta^\beta(1-\beta)^{1-\beta}(1-\tau^B)}{1-\beta\tau^B}\right)\right).$$

Note that g_τ in this case is defined by

$$g_\tau = \frac{\left(\frac{(R_R/2)\hat{\beta}(1-\tau^B)}{1-\hat{\beta}\tau^B} - 1\right) z_\tau + \left(\frac{R_S\hat{\beta}(1-\tau^B)}{1-\hat{\beta}\tau^B} - 1\right)}{z_\tau + 1},$$

where $z_{\tau^W} = \overline{x}_{\tau^W}^l / \overline{x}_{\tau^W}^a$ is the ratio between the after-tax wealth in hands of the risk loving agents and the after-tax wealth in hands of the risk averters.

Theorem 5. *(Decomposition of the welfare function)* In the equilibrium allocation, W can be written as

$$W\left(\left(U^i\right)_i, \left(c_\tau^i\right)_i, \left(b_\tau^i\right)_i\right) = X(\tau) + G(\delta, \tau) + D(\tau^B),$$

where G is differentiable in δ and τ, strictly increasing in δ, and strictly decreasing in τ^W and τ^B, and $X(\tau) = 0$ for $\tau^W = 1$. If $\gamma = 1$, $X(\tau) < 0$ for $\tau^W < 1$ and τ^B, then, function X attains its maximum value when $\tau^W = 1$.

In this case, inequality is larger than in the case without switching due to inequality among risk averters. In this case, the distribution of the risk averters is not constant, but it is bounded from above. On the other hand, inequality among risk loving agents is still large. Moreover, the distribution of these agents has exponential fat tails as in the case without switching with slightly thinner tails. Therefore, inequality function is more negative than in the case without switching but with similar properties as Figure 8 shows.

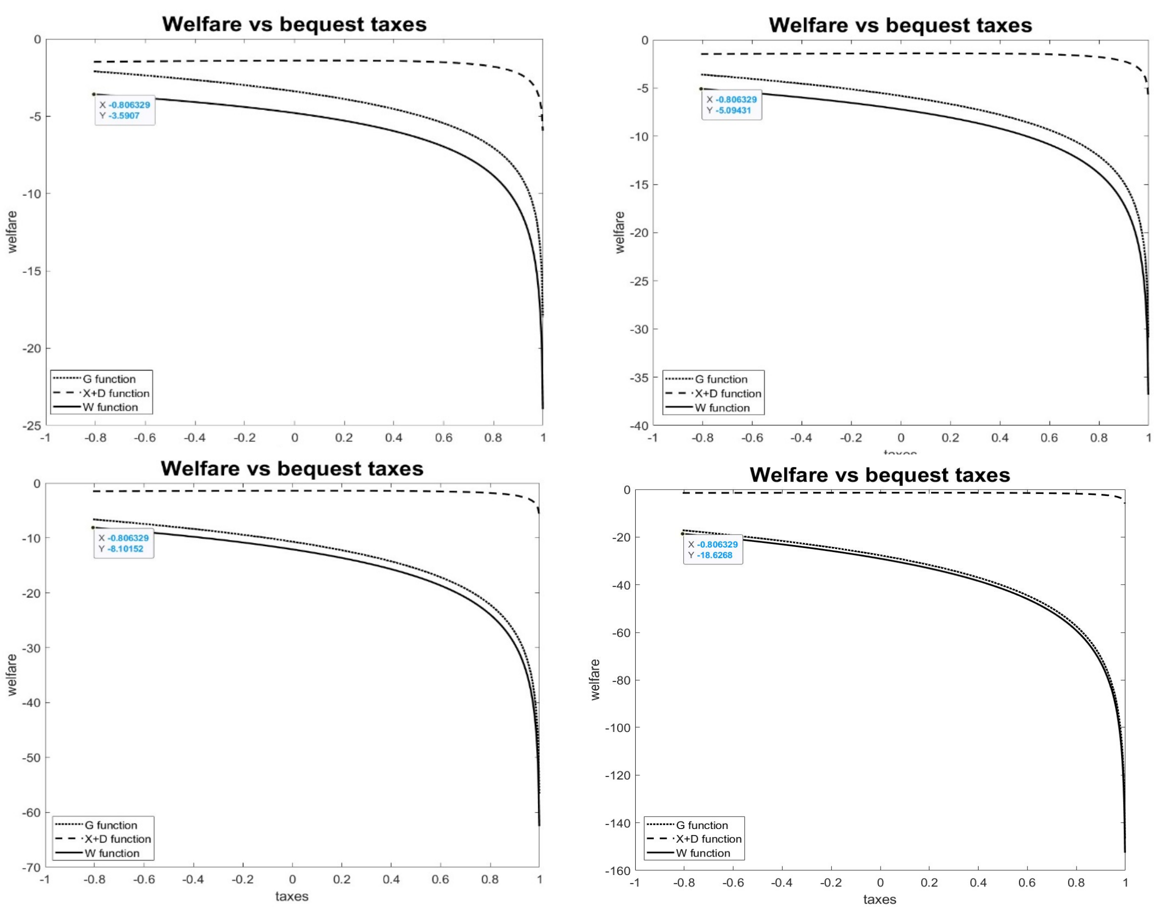

Figure 8. Welfare components with $\beta = 0.7$, $p = 0.1$, $\delta = 0.7, 0.8$ (top) and $\delta = 0.88, 0.95$ (bottom), and $\tau^W = 0.85$.

Since the average bequest rate is lower than in the case without switching, the growth rate and G function are also lower in this case. Since negative bequest taxes can be used to increase the growth rate of the economy, the social planner can use a negative bequest tax to compensate the reduction of the growth rate and the increment of inequality, which implies that the social planner might have more negative taxes on bequests. Figure 8 shows that with a positive probability of switching, the optimal bequest taxes are more negative in this case when they are compared with the ones without switching

7.3.Model with non-homothetic preferences

Let us consider a model in which a risk averse investor has a utility function in date t is given by

$$u^a(c,b) = \log\left(\frac{1}{\rho}\left(\frac{c}{\overline{b}_{t-1}}\right)^{\rho} + \beta b\right)$$

where $\rho \in (0,1)$ and $\overline{b}_{t-1}$ is the average bequest in date $t-1$, and a risk loving investor has a utility function in date t is given by

$$u^l(c,b) = \left(\frac{1}{\rho}\left(\frac{c}{\overline{b}_{t-1}}\right)^{\rho} + \beta b\right)^{\gamma}.$$

Note that if the after-tax wealth of an agent i in a state s at date t, y_s^i, is lower or equal to $\left(\beta^{\frac{1}{\rho-1}}/(1-\beta)\right)\overline{b}_{t-1}$, we have that the optimal consumption is lower or equal to $\left(\beta^{\frac{1}{\rho-1}}\right)\overline{b}_{t-1}$ and the optimal bequest is 0. If the after-tax wealth of an agent i in a state s at date t is larger than $\left(\beta^{\frac{1}{\rho-1}}/(1-\beta)\right)\overline{b}_{t-1}$, we have that the optimal consumption is equal to $\left(\beta^{\frac{1}{\rho-1}}\right)\overline{b}_{t-1}$ and the optimal bequest is positive. Therefore, within a generation, the consumption allocation is increasing on the level of wealth but bounded from above when compared to the basic model.

In this model, the average bequest rate of the economy might change over time based on the wealth distribution of the agents in the generation. However, the convergence to an invariant distribution is still holds.

8. Conclusions

We have developed an overlapping generation model with an endogenous growth rate and heterogeneous production technologies. In this model, taxes and redistribution has a negative impact on growth if the more productive technologies involve larger amount of idiosyncratic risk. Moreover, in absence of taxes, the most productive technologies will dominate the economy in the long run, and the long run inequality will depend on the risk that it involves. In the presence of taxes, taxes ensure the existence of an invariant distribution of wealth among the agents and an invariant growth rate of the economy. We also have shown that there is no poverty trap among the agents with the most productive technology. Among the agents that do not have access to the most productive technologies, wealth may not reach the top in any future date.

Redistributive taxes has a negative effect in growth rate and inequality. We analyze a social planner who sets taxes to maxima social welfare function. We have shown that the social welfare function can be written as the sum of three independent functions, one depending on growth, one depending on inequality, and one depending on the difference of the discount factors of the agent and the social planner. The first function is comonotonic with the growth rate of the economy, implying that this function might be an increasing function on taxes. The second one is anticomonotonic with the inequality of the invariant distribution which implies that this function might be a decreasing function on taxes.

We also found that, for a fixed discount rate for every agent in the economy, the optimal taxation is strictly decreasing on how the social planner discounts the future. Moreover, the optimal tax will be such that the invariant wealth distribution tends to an equal one if the social planner strongly discounts the future, and, on the other hand, the optimal tax is zero when the social planner does not discount the future at all. The intuition behind these results is that, if a social planner discounts the future strongly, the weight of distant dates becomes almost irrelevant, and analogously with the growth rate of the economy. Therefore, the social welfare function will be dominated by the inequality effect. If a social planner almost does not discount the future, however, the weight of future consumption will dominate the inequality effect even when both effects are increasing. Additionally, our model suggests that changes in the tax policy may be based on changes on the form of the social planner discounts the future compared to how the other agents do so.

The optimal bequest taxes are positive when the bequest rate is large, and the ratio of the discount factor of the social planner with the bequest rate is lower than 1. This is due to the social planner's intention to decrease the proportion of the wealth that is given to the next generation. However, this concern does not occur in all cases, for discount factors close to zero, inequality tends to dominate social planner optimal tax policy, and, for discount factor close to one, the social planner is more concerned about growth. Then, for intermediate levels of the discount factor, the importance of inequality is not particularly dominant, and the social planner's optimal "levels of savings" is low compared to the agent actual bequest rate.

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Appendix A. Proofs

For any initial distribution of wealth of the risk loving agents, a small proportion of them concentrates all the wealth in the long run in the absence of taxes. Then, there is only one invariant distribution of wealth. In this invariant distribution, all risk loving agents have zero wealth.

A.1. Proof of Theorem 1 and Theorem 3

We focus on this subsection to prove Theorem 3 since Theorem 1 is a particular case of Theorem 3. Let us prove a preliminary result that ensures that for any initial distribution $\left(w_0^i\right) \gg 0$, the aggregate wealth in hands of the risk loving agents over the aggregate net wealth in hands of the risk averse agents, $\frac{\overline{y}_{\tau,t}^l}{\overline{y}_{\tau,t}^a}$, converge to a positive constant.

Lemma 1. For taxes defined by a nonnegative marginal tax rate $\tau = (\tau^W, \tau^B)$ with $\tau^W + \beta\tau^B \big/ (1-\beta\tau^B) > 0$ and with technology returns such that satisfy Equations 3.1 and 3.2, $\lim_{t\to\infty} \overline{y}_{\tau,t}^l / \overline{y}_{\tau,t}^a = z_\tau$ where $z_\tau \in [1, \infty)$.

Proof. To simplify the proof, we assume that $\tau > 0$. We define z_τ^t as the proportion of the net wealth of the l agents and the a agents with the tax rate τ , $\overline{y}_{\tau,t}^l / \overline{y}_{\tau,t}^a$. Then, we have that

$$z_\tau^{t+1} = \frac{\overline{y}_{\tau,t+1}^l}{\overline{y}_{\tau,t+1}^a} = f(z_\tau^t) = \frac{\left(1 - \frac{\tau^W}{2} + \frac{\beta\tau^B}{2(1-\beta\tau^B)}\right)(R_R/2)z_\tau^t + \left(\frac{\tau^W}{2} + \frac{\beta\tau^B}{2(1-\beta\tau^B)}\right)R_S}{\left(\frac{\tau^W}{2} + \frac{\beta\tau^B}{2(1-\beta\tau^B)}\right)(R_R/2)z_\tau^t + \left(1 - \frac{\tau^W}{2} + \frac{\beta\tau^B}{2(1-\beta\tau^B)}\right)R_S}.$$

Note that $f(0) > 0$,

$$\lim_{z\to\infty} f(z) = \left(\frac{R_R/2}{R_S}\right)\left(\frac{\left(2-\tau^W\right)(1-\beta\tau^B) + \beta\tau^B}{\tau^W(1-\beta\tau^B) + \beta\tau^B}\right),$$

$f'(z) > 0\ \forall z \in (0, \infty)$, $f'(\infty) = 0$, f' is a decreasing function, and

$$f\big([0,\infty)\big) \subseteq \left[0, \left(\frac{R_R/2}{R_S}\right)\left(\frac{\left(2-\tau^W\right)(1-\beta\tau^B) + \beta\tau^B}{\tau^W(1-\beta\tau^B) + \beta\tau^B}\right)\right].$$

Then, using the intermediate value function Theorem, the function f has a fixed point. Moreover, there is only one fixed point z_τ such that $f(z_\tau) > 0$, and it is given by

$$z_\tau = \left(\frac{2(1-\beta\tau^B) - \tau^W(1-\beta\tau^B) + \beta\tau^B}{2(\tau^W(1-\beta\tau^B)+\beta\tau^B)}\right)\left(1-\frac{2R_S}{R_R}\right) + \left(\left(\frac{2(1-\beta\tau^B) - \tau^W(1-\beta\tau^B) + \beta\tau^B}{2(\tau^W(1-\beta\tau^B)+\beta\tau^B)}\right)^2\left(1-\frac{2R_S}{R_R}\right)^2 + \frac{R_S}{R_R/2}\right)^{1/2} \qquad (\mathrm{A.1})$$

and, for each $z^0 \in (0,\infty)$, $z_\tau^t = \bar{y}_{\tau,t}^l/\bar{y}_{\tau,t}^a$ converge to z_τ.

From Proof of Lemma 1, z_τ is a C^1 function for τ . Additionally, we have that

$$\frac{\partial}{\partial\tau^W} z_\tau = -\left(\frac{4(1-\beta\tau^B)\left(1-\frac{2R_S}{R_R}\right)}{2(\tau^W(1-\beta\tau^B)+\beta\tau^B)^2}\right) - \frac{8(1-\beta\tau^B)\left(1-\frac{R_S}{R_R/2}\right)^2(2(1-\beta\tau^B)-\tau^W(1-\beta\tau^B)+\beta\tau^B)}{\left(2(\tau^W(1-\beta\tau^B)+\beta\tau^B)\right)^3\left(\left(\left(1-\frac{2R_S}{R_R}\right)^2\left(\frac{2(1-\beta\tau^B)-\tau^W(1-\beta\tau^B)+\beta\tau^B}{2(\tau^W(1-\beta\tau^B)+\beta\tau^B)}\right)^2\right)+\frac{2R_S}{R_R}\right)^{1/2}}$$

and

$$\frac{\partial}{\partial\tau^B} z_\tau = -\left(\frac{\left(1-\frac{2R_S}{R_R}\right)\left((1-\tau^W)2\beta(\tau^W(1-\beta\tau^B)+\beta\tau^B)+2((1-\beta)\tau^W)\left(2-\beta\tau^B-\tau^W(1-\beta\tau^B)\right)\right)}{\left(2(\tau^W(1-\beta\tau^B)+\beta\tau^B)\right)^2}\right) - \frac{2\left(1-\frac{2R_S}{R_R}\right)^2\left(\left((1-\tau^W)2\beta(\tau^W(1-\beta\tau^B)+\beta\tau^B)+2((1-\beta)\tau^W)\left(2-\beta\tau^B-\tau^W(1-\beta\tau^B)\right)\right)\right)}{\left(2(\tau^W(1-\beta\tau^B)+\beta\tau^B)\right)^3\left(\left(\left(1-\frac{2R_S}{R_R}\right)^2\left(\frac{2(1-\beta\tau^B)-\tau^W(1-\beta\tau^B)+\beta\tau^B}{2(\tau^W(1-\beta\tau^B)+\beta\tau^B)}\right)^2\right)+\frac{2R_S}{R_R}\right)^{1/2}}.$$

Then, $z_.$ is strictly decreasing in τ^W for any τ^B, is strictly decreasing in τ^B for any τ^W, $z_\tau = 1$ for $\tau^W = 1$ and $\tau^B \in [0,1]$, and $z_\tau = \infty$, if and only if $\tau = (0,0)$. In this case, $\frac{\partial}{\partial\tau^W} z_\tau = \frac{\partial}{\partial\tau^B} z_\tau = \infty$.

Since the aggregate production depends on aggregate wealth of each group, the convergence of the ratio of the risk loving agents and risk averters aggregate wealth ensures the convergence of the growth path.

Lemma 2. For any fixed tax rate $\tau = (\tau^W, \tau^B)$ with $\tau^W + {\beta\tau^B}/{(1-\beta\tau^B)} > 0$, the growth rate of the economy, $g_{\tau,t}$ converges to g_τ when t goes to infinity where

$$g_\tau = \frac{\left(\frac{(R_R/2)\beta(1-\tau^B)}{1-\beta\tau^B} - 1\right) z_\tau + \left(\frac{R_S\beta(1-\tau^B)}{1-\beta\tau^B} - 1\right)}{z_\tau + 1}.$$

Proof of Lemma 2. Since

$$g_{\tau,t} = \frac{\left(\frac{(R_R/2)\beta(1-\tau^B)}{1-\beta\tau^B} - 1\right) \bar{y}_t^l + \left(\frac{R_S\beta(1-\tau^B)}{1-\beta\tau^B} - 1\right) \bar{y}_t^a}{\bar{y}_t^l + \bar{y}_t^a}$$
$$= \frac{\left(\frac{(R_R/2)\beta(1-\tau^B)}{1-\beta\tau^B} - 1\right) z_\tau^t + \left(\frac{R_S\beta(1-\tau^B)}{1-\beta\tau^B} - 1\right)}{z_\tau^t + 1},$$

and z_τ^t converges to z_τ when $t \to \infty$, we obtain that $g_{\tau,t}$ converges to g_τ, which concludes the proof.

Due to the convergence of how each group invest in each technology, the growth rate of the economy will also converge. Then, the proportion of wealth of the poorest risk loving agent converges, which implies that the proportion of wealth of a risk loving agent that has received at least once the lower return $\underline{R}_R = 0$ also converges.

Proof of Theorem 3. Let us define after-tax wealth distributions $F_{a,\tau}^W(w)$ and $F_{l,\tau}^W(w)$. The distribution for the risk averters is a constant distribution with the level of wealth x_τ^a given by

$$x_\tau^a = \sum_{k=0}^{\infty} \left(\frac{\frac{1}{2}\left(\tau^W + \frac{\beta\tau^B}{(1-\beta\tau^B)}\right) R_S^k (1-\tau^W)^k \frac{\beta^k (1-\tau^B)^k}{(1-\beta\tau^B)^k}}{(1+g_\tau)^k} \right),$$

and the distribution for the risk loving agents is

$$x_{\tau}^{l,n} = \sum_{k=0}^{n-1} \frac{\frac{1}{2}\left(\tau^W + \frac{\beta\tau^B}{(1-\beta\tau^B)}\right) R_R^k (1-\tau^W)^k \frac{\beta^k (1-\tau^B)^k}{(1-\beta\tau^B)^k}}{(1+g_\tau)^k}$$

for the n^{th} poorest group of risk loving agents with weight $1/2^{n+1}$ for $n \in \mathbb{N}$.

Since $\bar{x}_\tau^l / \bar{x}_\tau^a = z_\tau$ in this case, the distribution functions $F_{a,\tau}^W(w)$ and $F_{l,\tau}^W(w)$ are invariant distribution of after-tax wealth with the invariant growth rate g_τ.

Let us suppose that the initial distribution of after taxes wealth are $F_{a,\tau,0}^W(w)$ and $F_{l,\tau,0}^W(w)$. Given at date t, for the risk averse agents, the proportion of the wealth is given by

$$x_{\tau,t}^a = \sum_{k=0}^{t-1} \left(\frac{\frac{1}{2}\left(\tau^W + \frac{\beta\tau^B}{(1-\beta\tau^B)}\right) R_S^k (1-\tau^W)^k \frac{\beta^k (1-\tau^B)^k}{(1-\beta\tau^B)^k}}{\prod_{l=1}^k (1+g_{\tau,t-l})} \right) + (1-\tau^W)^t R_S^t \frac{\beta^t (1-\tau^B)^t}{(1-\beta\tau^B)^t} \frac{w_0^{a_i}}{\left(\prod_{k=1}^t (1+g_{\tau,t-k})\right)\overline{w}_0}. \qquad (A.3)$$

Since $R_S < R_R/2$, we have that

$$(1-\tau^W)\, R_S \frac{\beta(1-\tau^B)}{(1-\beta\tau^B)} < 1 + g_{\tau,t}$$

for all $t \geq 0$, which proves that $x_{\tau,t}^a \to x_\tau^{a_i}$ when $t \to \infty$.

The proportion of the wealth of the poorest l agents is $x_{\tau,t}^l = (\tau^W + \beta\tau^B/(1-\beta\tau^B))/2 = x_\tau^{l,1}$, and the weight of this group is 1/2. The wealth of the second poorest group of l agents only depends on the average wealth and the wealth of the poorest l agents in the previous period. Therefore, the proportion of the second poorest group of l agents is

$$x_{\tau,t}^l = \frac{\frac{1}{2}\left(\tau^W + \frac{\beta\tau^B}{1-\beta\tau^B}\right) R_R (1-\tau^W)\, \frac{\beta(1-\tau^B)}{(1-\beta\tau^B)}}{(1+g_{\tau,t-1})} + \frac{1}{2}\left(\tau^W + \frac{\beta\tau^B}{1-\beta\tau^B}\right),$$

and its weight is 1/4. If we continue this process, we obtained that proportion of the n^{th} poorest group of l agents is

$$x_{\tau,s}^l = \sum_{k=0}^{n-1} \frac{\frac{1}{2}\left(\tau^W + \frac{\beta\tau^B}{(1-\beta\tau^B)}\right) R_R^k (1-\tau^W)^k \frac{\beta^k (1-\tau^B)^k}{(1-\beta\tau^B)^k}}{\prod_{j=1}^k (1+g_{\tau,t-k})}, \qquad (A.2)$$

and the weight of this group is $1/2^{n+1}$ for $t \geq n$. Since $z_\tau^t \to z_\tau \in [1, \infty)$; when $t \to \infty$, $g_{\tau,t} \to g_\tau$ when $t \to \infty$. Then, $x_{\tau,t}^l \to x_\tau^l$ when $t \to \infty$, which concludes the proof.

A.2. Proof of Theorem 2 and Theorem 4

In this subsection we prove Theorem 4 since Theorem 2 is a particular case of Theorem 4. We denote the consumption and bequest in equilibrium in the invariant after-tax wealth distributions $F_{a,\tau}^{W}$ and $F_{l,\tau}^{W}$ at the tax rate τ as $c_{a,t}(w), b_{a,t}'(w)$. Using Theorem 1, the equilibrium consumption and bequest plan $c_{a,t}(w), b_{a,t}'(w)$ at date t and tax rate τ can be written as

$$\left(c_{a,t}(w), b_{a,t}'(w)\right) = \left(\left(\frac{(1-\beta)}{1-\beta\tau^{B}}\right) w(1+g_{\tau})^{t}, \left(\frac{\beta(1-\tau^{B})}{1-\beta\tau^{B}}\right) w(1+g_{\tau})^{t}\right).$$

Proof of Theorem 4. Therefore, the welfare function can be rewritten as

$$\begin{aligned}
W(c,b') &= (1-\delta)\left(\int U^{a}\left(c_{a,0}(w), b_{a,0}(w)\right) dG_{a,\tau}^{W}(w) + \int \log U^{l}\left(c_{l,0}(w), b_{l,0}(w)\right) dG_{l,\tau}^{W}(w)\right. \\
&\quad + \sum_{t=1}^{\infty} \delta^{t}\left(\int U^{a}\left(c_{a,t}(w), b_{a,t}'(w)\right) dF_{a,\tau,t-1}^{W}(w)\right. \\
&\quad \left.\left. + \int \log U^{l}\left(c_{l,t}(w), b_{l,t}'(w)\right) dF_{l,\tau,t-1}^{W}(w)\right)\right) \\
&= (1-\delta)\sum_{t=0}^{\infty} \delta^{t} \sum_{n=2}^{\infty} \frac{\beta}{2^{n-1}\gamma} \log\left(\left(\frac{\beta(1-\tau^{B})}{1-\beta\tau^{B}}\right)\frac{1}{2}\left(\left(x_{\tau}^{l,n}\right)^{\gamma} + \left(x_{\tau}^{l,1}\right)^{\gamma}\right)(1+g_{\tau})^{t}\right) \\
&\quad + (1-\delta)\sum_{t=0}^{\infty} \delta^{t} \sum_{n=2}^{\infty} \frac{1-\beta}{2^{n-1}\gamma} \log\left(\left(\frac{(1-\beta)(1-\tau^{B})}{1-\beta\tau^{B}}\right)\frac{1}{2}\left(\left(x_{\tau}^{l,n}\right)^{\gamma}\right.\right. \\
&\quad \left.\left. + \left(x_{\tau}^{l,1}\right)^{\gamma}\right)(1+g_{\tau})^{t}\right) + (1-\delta)\sum_{t=0}^{\infty} \delta^{t}\beta \log\left(\left(\frac{\beta(1-\tau^{B})}{1-\beta\tau^{B}}\right)(x_{\tau}^{a})(1+g_{\tau})^{t}\right) \\
&\quad + (1-\delta)\sum_{t=0}^{\infty} \delta^{t}(1-\beta) \log\left(\left(\frac{(1-\beta)}{1-\beta\tau^{B}}\right)(x_{\tau}^{a})(1+g_{\tau})^{t}\right).
\end{aligned}$$

Then,

$$
\begin{aligned}
W(c,b') &= \log x_\tau^a + \sum\nolimits_{n=1}^{\infty} \frac{1}{2^{n-1}\gamma}\left(\log\left(\frac{1}{2}\left(\left(x_\tau^{l,n}\right)^\gamma + \left(x_\tau^{l,1}\right)^\gamma\right)\right)\right) \\
&\qquad + (1-\delta)\sum\nolimits_{t=0}^{\infty} \delta^t t \log(1+g_\tau) + \log\left(\left(\frac{\beta^\beta(1-\beta)^{1-\beta}(1-\tau^B)}{1-\beta\tau^B}\right)\right) \\
&\qquad = \sum\nolimits_{n=2}^{\infty} \frac{1}{2^{n-1}\gamma}\left(\log\left(\frac{1}{2}\left(\left(x_\tau^{l,n}\right)^\gamma + \left(x_\tau^{l,1}\right)^\gamma\right)\right)\right) + \log x_\tau^a + \frac{\delta}{1-\delta}\log(1+g_\tau) \\
&\qquad + \log\left(\left(\frac{\beta^\beta(1-\beta)^{1-\beta}(1-\tau^B)}{1-\beta\tau^B}\right)\right)
\end{aligned}
$$

We can define $X(\cdot)$ as

$$
X(\tau) := \log x_\tau^a + \sum\nolimits_{n=2}^{\infty} \frac{1}{2^{n-1}\gamma}\left(\log\left(\frac{1}{2}\left(\left(x_\tau^{l,n}\right)^\gamma + \left(x_\tau^{l,1}\right)^\gamma\right)\right)\right),
$$

$G(\cdot,\cdot)$ as $G(\delta,\tau) \coloneqq \delta/(1-\delta)\log(1+g_\tau)$ which is clearly a decreasing function in τ, and $D(\cdot)$ as

$$
D(\tau^B) \coloneqq \log\left(\left(\frac{\beta^\beta(1-\beta)^{1-\beta}(1-\tau^B)}{1-\beta\tau^B}\right)\right).
$$

In this case, each function depends directly or indirectly on δ since the bequest rate affects the distribution of the invariant distribution and the growth rate of the economy by increasing inequality and the growth rate when β increases.

The properties of G are a consequence of the properties of the invariant growth explained in the proof of Theorem 3.

Additionally, $X(\tau)$ can be written as

$$X(\tau) := \sum_{n=2}^{\infty} \frac{1}{2^{n-1}\gamma} \left(\log \left(\frac{1}{2} \left(\left(\sum_{k=0}^{n-1} \frac{\frac{1}{2}\left(\tau^W + \frac{\beta\tau^B}{(1-\beta\tau^B)}\right) R_R^k (1-\tau^W)^k \frac{\beta^k (1-\tau^B)^k}{(1-\beta\tau^B)^k}}{(1+g_\tau)^k} \right)^{\gamma} \right.\right.\right.$$

$$\left.\left.\left. + \left(\tau^W + \frac{\beta\tau^B}{(1-\beta\tau^B)}\right)^{\gamma} \right)\right)\right)$$

$$+ \log\left(\sum_{k=0}^{\infty} \left(\frac{\frac{1}{2}\left(\tau^W + \frac{\beta\tau^B}{(1-\beta\tau^B)}\right) R_S^k (1-\tau^W)^k \frac{\beta^k (1-\tau^B)^k}{(1-\beta\tau^B)^k}}{(1+g_\tau)^k} \right)\right).$$

For $\gamma = 1$, since $\sum_{n=1}^{\infty}(1/2^n)x_\tau^{l,n} + x_\tau^a = 1$, Jensen's inequality and the dominated convergence theorem ensure that $X(\tau) < 0$ for $\tau^I \in (0,1)$ and τ^B such that $\tau^W + {\beta\tau^B}/{(1-\beta\tau^B)} > 0$, the growth rate of the economy, $g_{\tau,t}$, and function X is a continuous function. For $\tau^W = 1$, we have that $X(\tau) = 0$. Then, function attains its maximum value when $\tau^W = 1$.

A.3. Proof of Proposition 5

Let us prove a preliminary result that establishes a relationship between different tax policies with small tax rates.

Proposition 7. Any taxation plan τ with positive bequest taxes is dominated by any taxation τ' such that $1 \geq {\tau'}^W > \tau^W + \beta\tau^B/(1-\beta\tau^B)$ and ${\tau'}^B = 0$.

Proof of Proposition 7. Consider a taxation plan $\tau' \in \mathcal{T}$ such that $1 > {\tau'}^W > \tau^W + \beta\tau^B/(1-\beta\tau^B)$. Since we analyze the welfare function in the invariant distribution, we can assume that $\bar{y}_0 = 1$ in both cases.

For the poorest *risk loving agents*, we have that their level of after tax income with the taxation plan (τ^W, τ^B) is given by $\tau^W = \bar{y}_0\tau^W + \bar{y}_0(\tau^B/(1-\tau^B))\big(\beta(1-\tau^B)/(1-\beta\tau^B)\big) =$

$\bar{y}_0\big(\tau^W + \beta\tau^B/(1-\beta\tau^B)\big) < \bar{y}_0{\tau'}^W = {\tau'}^W$ which is the after tax income with the new taxation plan.

Note that that since the function $f_k(x) = x/(1-x)^k$ is an increasing function for $x \in [0,1)$ for all $k, l \in \mathbb{N}$ and $\tau^W < \tau'^W$, we have that $\tau^W(1-\tau^W)^k(1-\tau^B)^l < \tau^W(1-\tau^W)^k < \tau'^W(1-\tau'^W)^k$. For the second poorest group of risk loving agents, we have that their level of after tax income with the taxation plan (τ^W, τ^B) is given by

$$\begin{aligned}
&\big(R_R\tau^W\bar{y}_0\beta(1-\tau^B)\big)(1-\tau^W) + \bar{y}_0\tau^W + \left(\frac{\bar{y}_0\tau^B}{1-\tau^B}\right)\left(\frac{\beta(1-\tau^B)}{1-\beta\tau^B}\right)\\
&\qquad = \bar{y}_0\left(R_R\delta\tau^W(1-\tau^B)(1-\tau^W) + \tau^W + \frac{\tau^B\beta}{1-\beta\tau^B}\right)\\
&\qquad < \bar{y}_0\big(R_R\beta\tau'^W(1-\tau'^W) + \tau'^W\big).
\end{aligned}$$

For the *n*-poorest group of risk loving agents, we have

$$\sum_{k=0}^{n-1}\big(R_R^k\beta^k(1-\tau^B)^k\big)(1-\tau^W)^k\left(\tau^W\bar{y}_0 + \frac{\tau^B\beta}{1-\beta\tau^B}\bar{y}_0\right) = \bar{y}_0\left(\sum_{k=0}^{n-1}\left(R_R^k\beta^k(1-\tau^B)^k(1-\tau^W)^k\left(\tau^W + \frac{\tau^B\beta}{1-\beta\tau^B}\right)\right)\right) < \bar{y}_0\left(\sum_{k=0}^{n-1}\left(R_R^k\beta^k(1-\tau^W)^k\left(\tau^W + \frac{\tau^B\beta}{1-\beta\tau^B}\right)\right)\right) <$$

$$\bar{y}_0\left(\sum_{k=0}^{n-1}\left(R_R^k\beta^k(1-\tau^W)^k\left(\tau^W + \frac{\tau^B\beta}{1-\beta\tau^B}\right)\right)\right) < \bar{y}_0\left(\sum_{k=0}^{n-1}\big(R_R^k\beta^k(1-\tau'^W)^k\tau'^W\big)\right).$$

Then, the income of the invariant distribution in each period is always lower with the taxation plan (τ^W, τ^B) than with $(\tau'^W, 0)$. For the risk averse agents, the result is also true because of the convergence of an analogous series as the one described above. To conclude the proof, notice that the utility of a risk averse agent with an after-taxes wealth w is

$$\begin{aligned}
u^a\big(c(w/\bar{y}_t), b'(w/\bar{y}_t)\big) &= \log\left(\big(c(w/\bar{y}_t)\big)^{1-\beta}\big(b'(w/\bar{y}_t)\big)^{\beta}\right)\\
&= \log\left(\big((1-\delta)w\big)^{1-\beta}(\delta(1-\tau^B)w)^{\beta}\right) = \log\left(\big((1-\delta)\big)^{1-\beta}\delta^{\delta}(1-\tau^B)^{\beta}w\right)\\
&< \log\left(\big((1-\delta)\big)^{1-\beta}\beta^{\beta}w\right).
\end{aligned}$$

Proof of Proposition 5. From Proof of Theorem 3, we have that g_τ is a C^1 function, strictly decreasing in τ^W for any τ^B such that $\tau^W + {\beta\tau^B}/{(1-\beta\tau^B)} > 0$, strictly decreasing in τ^B for any $\tau^W \in [0,1)$, $g_\tau = 0$ for $\tau^B = 1$. Then, function $G(\delta,\cdot)$ attains its maximum value when $\tau = 0$.

Due to Theorem 4 and the form of the invariant distribution, if δ goes to 1, the welfare function converges to function G uniformly in τ when taxes are bounded away from zero and one. Then, optimal taxes δ goes to 1 converges to zero, that is, $(0,0)$. Then, there is $\bar{\beta} \in (0,1/2)$, $\underline{\delta} \in (0,1)$ such that any optimal tax rate τ satisfies that $1 > \tau^W + \beta/(1-\beta) \geq \tau^W + \beta\tau^B/(1-\beta\tau^B)$ for $\beta \leq \bar{\beta}$ and $\delta \geq \underline{\delta}$. Finally, using Proposition 6 we conclude that $\tau^B = 0$, which concludes the proof.

A.4. Proof of Proposition 2

Proof of Proposition 2. Note that D is constant. The first part holds since when δ goes to 1, the welfare function $(1-\delta)W$ converges to function $(1-\delta)G$ uniformly in τ^W when taxes are bounded away from zero. Then, the optimal tax rate converges to zero. Since $R_R/2 > 1/\beta$, for δ large enough, the invariant growth rate is positive, which concludes the first part.

To prove the second part, it is enough to analyze asymptotic behavior of W when δ goes to zero. When δ goes to zero, W converges to X uniformly in τ^W when taxes are bounded away from zero. Then, optimal taxes converge to 1. Using Equation 5.1, we obtain that the growth rate is negative if δ is small enough, which concludes the second part.

Appendix B. Model without segmentation

Proof of Proposition 6. We denote w_t^{*a} as the levels of wealth after-taxes for the risk averters such that the agent is indifferent between investing in the safe technology and investing in the risky one in period $t+1$, that is,

$$\log(\beta w_t^{*a} R_S(1-\tau^W) + \tau^W \bar{y}_{t+1}) = \frac{1}{2}(\log(\delta\beta R_R(1-\tau^W) + \tau^W \bar{y}_{t+1}) + \log(\tau^W \bar{y}_{t+1})),$$

$$\log\left(\beta \frac{w_t^{*a}}{\bar{y}_{t+1}} R_S(1-\tau^W) + \tau^W\right) = \frac{1}{2}\left(\log\left(\beta \frac{w_t^{*a}}{\bar{y}_{t+1}} \bar{R}_R(1-\tau^W) + \tau^W\right) + \log(\tau^W)\right),$$

$$\left(\beta \frac{w_t^{*a}}{\bar{y}_{t+1}} R_S(1-\tau^W) + \tau^W\right)^2 = \tau^W\left(\beta \frac{w_t^{*a}}{\bar{y}_{t+1}} R_R(1-\tau^W) + \tau^W\right),$$

Then, we have that

$$\frac{w_t^{*a}}{\bar{y}_{t+1}} = \frac{\tau^W(R_R - 2R_S)}{\beta R_S^2(1-\tau^W)}$$

which concludes the proof.

B.1. Invariant Distribution

Proposition 6 ensures that a process similar to the one made in the other models can be done in this case since the wealth of each risk averter (more precisely, almost every risk averter) can be computed recursively.

Numerically, it requires only to compute a preliminary proportion of agents that invest in each technology in each period t by using an increasing function $\alpha_{\cdot}^{*}$ defined above. Then, we compute $\alpha_{\tau}^{*}\frac{\bar{y}_{t+1}}{\bar{y}_{t}}$ and the distribution of wealth step by step. Now, we restart the process with the proportion of investment induced by the distribution that we have just found.

By doing this, we can find the invariant distribution of wealth invested in each technology, and, therefore, the invariant growth rate of the economy, which are the only things that we need to know the invariant concentration of wealth among the agents.

Proposition 8. Given a positive marginal wealth tax rates τ, the growth rate $g_{\tau,t}$ converges to g_{τ} when $t \to \infty$. Moreover, there is an invariant distribution such that the growth rate is equal to g_{τ}.

This result implies the existence of the invariant concentration of wealth, and it also suggests that if the distribution is considerably close to the invariant one, it converges in the long run to the invariant distribution. In the following subsection, we explore numerically the invariant distribution.

B.2. Numerical examples

Based on the numerical examples defined before, we consider $R_R = 4.5$, $R_S = 1.6$, and $\beta = 0.5$. We start with a distribution constant distribution of wealth among the agents.

In this case, increments on taxes generate different effects on growth depending on the marginal tax rate that we start on. For very low levels of taxes, increment in the tax rates might decrease the growth rate due to the transfers from risk loving agents to risk averters, see for example the reduction of the growth rate from $\tau^W = 0.1$ to $\tau^W = 0.3$, $g_{0.1} = 7.6\%$ and $g_{0.3} = 5.2\%$. The risk loving agents are investing completely in the risky and more productive type of investment, and the risk averters are investing part in the risky and part in the safe investment.

Therefore, the increment in the marginal taxation rate implies less investments in the risky and more investments in the safe one.

For marginal tax rates above $\tau^W = 0.3$, the growth rate increases when the marginal tax rate increases, see for example the increment of the growth rate between, $\tau^W = 0.3$ and $\tau^W = 0.6$, $g_{0.3} = 5.2\%$, $g_{0.5} = 10.8\%$, and $g_{0.6} = 12.5\%$. In this case, the risk averse agents are investing considerably more in the risky than in the safe one since the indifference threshold, α^*_τ, reduces. When the marginal tax on wealth increases, the threshold also increases, making poor risk averters have incentives to invest in the risky technology. Intuitively, poor risk averters see taxes on wealth as a large enough insurance of the lowest return of the risky technology. In this case, given an increment of the marginal tax rate, the proportion of agents that decide to invest in the risky one compensates for the transfers of wealth from the risk loving agents to the risk averters who decide to invest in the safe one.

For marginal taxation rate above 0.6, the wealth distribution of risk averters and risk averters are the same. The reason for this is that there is $\tau^W \in (0.5, 0.6)$ such that the invariant distribution of wealth is bounded, and all the levels of wealth are lower than the threshold α_τ^*. Then, the growth rate of the economy is at its maximum, that is, $g_\tau w = R_R \beta/2 - 1 = 12.5\%$.

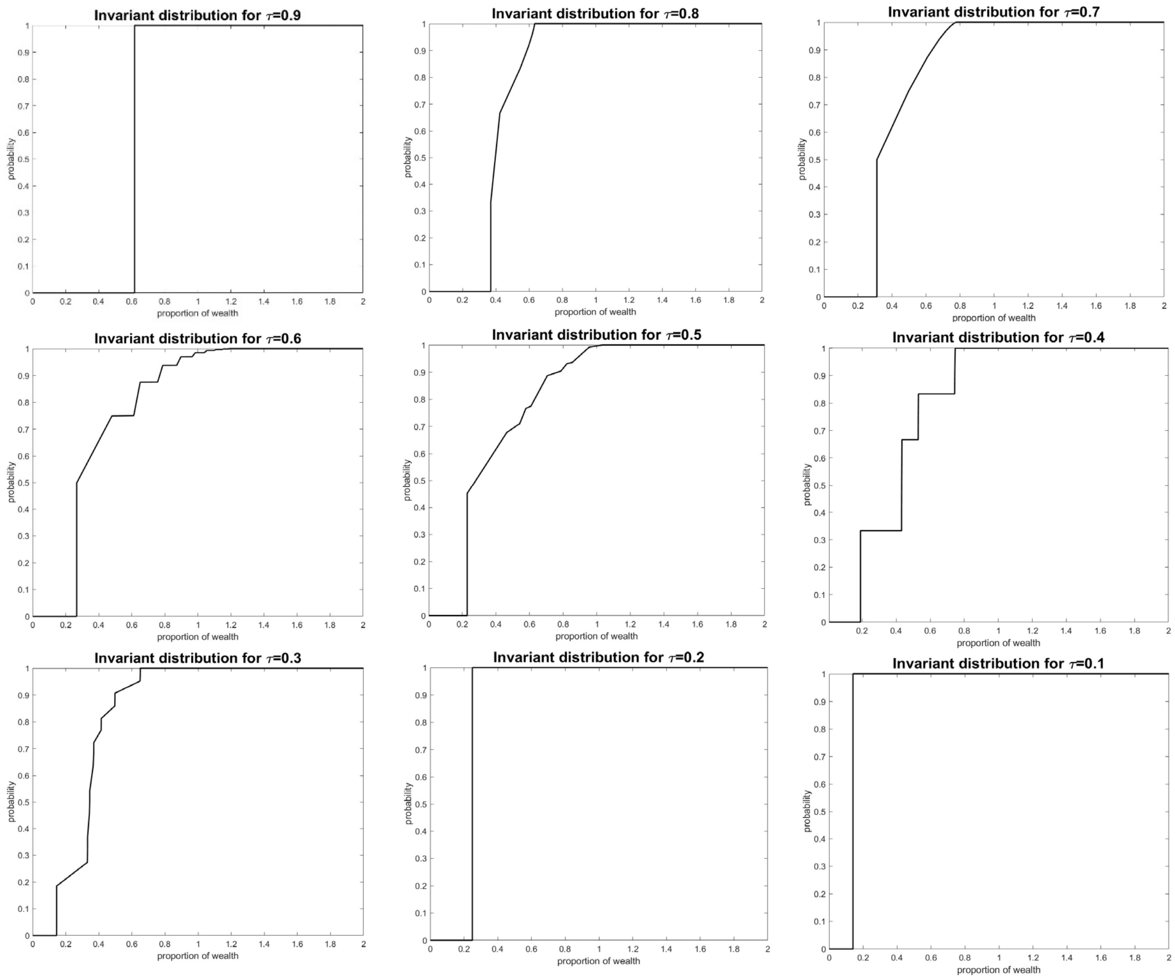

Figure 9: Invariant distribution of wealth for the risk averters for different income tax rates $\tau = 0.9, 0.8, 0.7, 0.6, 0.5, 0.4, 0.3, 0.2, 0.1$.

From Figure 9, we noticed that for low marginal tax rate, all risk loving agent agents invest in the safe one implying a constant distribution. When $\tau = 0.3$, 0.4, and 0.5 the averter agents invest in the risky one when they are poor and in the safe one when they are above the threshold α_τ^*. The biggest difference between these two distributions is that the threshold is considerably higher when $\tau = 0.4$ (moreover when $\tau = 0.5$) implying that a larger proportion of wealth is being invested in the risky in this case. This causes the phenomenon mentioned before, an increment on the growth rate. This happens because this effect overcome the transfers made from the risk loving agent agents to the risk averters that invest in the safe one a proportion of agents that is small in this case and decreases every time that the marginal tax rate increases.

When $\tau = 0.6$ and 0.7, all risk averters invest in the risky one all the time since there is no over-accumulation of wealth (fat tails) in this case. Therefore, a risk averse agent that is infinitely successful by investing in the risky one has a wealth in period t bounded by $2.6\overline{y}_{t+1}$ for $\tau = 0.6$, and $2.6\overline{y}_{t+1}$ for $\tau = 0.7$.