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Growth and Redistribution with Heterogeneous Attitudes toward Risk

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ABSTRACT

We develop an endogenous growth model of overlapping generations in which agents leave warm-glow bequests. There are dynasties of risk neutral investors and dynasties of risk averse investors. We start with a simple model in which risk averse investors can invest only in a safe asset while risk neutral investors, whom we often refer to as entrepreneurs, can invest in a risky asset with a higher expected return. This simple structure allows us to analytically calculate the invariant distributions of wealth holdings. We define a social welfare function for this model and calculate tax and transfer policies that maximize social welfare in the invariant distribution. We extend our results to models where risk averse investors can invest in the risky asset and where there are labor and capital and endogenous wages and rental rates.

Keywords: Endogenous growth, Inequality, Redistribution, Overlapping generations, Invariant distribution, Social welfare function.

JEL Codes: C62, D51, H21, O4

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1. Introduction

We consider three models, one without growth and two with endogenous growth rate and heterogeneous production technologies — one with segmentation and one with risk lovers — to analyze the dynamics of taxation in OLG economies with risk lovers. In the latter models, taxes and redistribution have a negative impact on growth if the more productive technologies involve larger amount of idiosyncratic risk. However, taxation on bequest and income have different effects on growth and inequality. Moreover, in absence of taxes, the most productive technologies will dominate the economy in the long run, and inequality in the long run will depend mainly on the risk that the most productive one involves. In the presence of taxes and different expected technology returns, bequest or income taxes ensure the existence of an invariant distribution of wealth among the agents and an invariant growth rate of the economy. We also show that the invariant distribution with a single positive type of tax with constant marginal tax rate is also ergodic as in Piketty (1997), but only among the agents of the same type. Therefore, there is no poverty trap among the agents with the most productive and most risky technologies. Among the agents that do not have access to the most productive technologies, their wealth might not reach the top in any future date.

The study of the models are closely related. The distribution and convergence to the invariant distribution of the former model is strongly related to the latter ones. Additionally, the second one can be seen as a restricted case of the latter if the risky technology is more productive than the safe one as we assume in most part of the article, allowing us to have a better understanding of the latter model.

We study optimal taxation by introducing a central planner who chooses taxes to maximize the social welfare of the economy. We show that the social welfare function of the social planner can be written as the sum of three independent functions. The first one depends only on growth, the second one depends only on inequality, and third one depends only on the difference of the discount factors of the agent and the social planner. The first one is positively related to the growth rate of the economy, implying that the presence of low taxes might be optimal in some cases. The second function is negatively related to the inequality of the invariant distribution which implies that high taxes might be optimal in some cases. We prove that bequest taxes are always worse than income taxes for discount rate of the agents and the social planner.

We also find that, for a fixed discount rate of the consumers[1], the optimal taxation is strictly decreasing on how the social planner discounts the future. Moreover, the optimal tax is such that the invariant wealth distribution tends to a completely equal one if the social planner strongly discounts the future. On the other hand, the optimal tax is zero if the social planner does not discount the future at all. The intuition behind these results is that, if a social planner discounts the future strongly, the weight of distant dates and the growth rate become almost irrelevant. Therefore, the social welfare function is dominated by the inequality effect. However, if a social planner discounts the future very weakly, the weight of future consumption will dominate the inequality effect even when both effects are considerably large. Note that, in the presence of a social planner that almost does not discount the future, inequality affects strongly the social welfare function due to the presence of a very unequal distribution of wealth that causes a high impact in the social welfare function.

We developed an overlapping generations model with bequest as in Galor and Zeira (1993). To analyze the impact of different production technologies on the accumulation of wealth, we use a model with bequest and idiosyncratic uncertainty on the technologies as in Piketty (1997). However, we consider frictions on the use of the technologies separating the agents in two groups: skilled and unskilled, and risk neutrals and risk averters. As it was mentioned before, we also consider redistributive taxes as in Alesina and Rodrik (1994). The impact of different technologies with different levels of idiosyncratic uncertainty can also be related to models with different attitudes toward risk as in Araujo, Gama and Kehoe (2024) and Araujo, Chateauneuf, Gama and Novinski (2018).

The idea that high taxes should be imposed due to the existing spread between asset returns and real returns has been explored by Piketty and Saez (2014) and also Saez and Zucman (2016). They argued that changes in growth rate in western economies, mainly in the US, has a strong impact on inequality due to the gap among the interest rate and the real GDP growth rate. Other authors as Lindert and Williamson (2016) studied long term data to analyze the inequality in the US economy since colonial times. Bargain et al. (2015) made a deeper analysis of the tax policy and inequality from 1979-2007 in which changes in the tax policy increased income inequality

[1] We consider the discount rate of the agent as the bequest rate since, as we will show latter on, the bequest is a form of how the agent is concerned about his/her descendants' future consumptions.

causing more accumulation among the top one percent. Weide and Milanovic (2014) showed with a study on micro data of the US economy from 1960 to 2010 that high levels of inequality reduce the economic growth of the poor, but it might enhance the economic growth of the rich. Benhabib, Bisin, and Zhu (2011, 2016), Benhabib and Bisin (2018), and Jones (2015) analyze income or wealth distribution with taxation. Gabaix, Lasry, Lions, and Moll (2016) (analytically), and Aoki and Nirei (2017) (numerically) analyze the dynamics of income distribution with taxes. And Garcia-Peñalosa and Wen (2008) analyze the effect of taxation on growth with risk averse agents. Most of works mentioned above support the idea analyzes Pareto distribution of wealth or income. Moreover, Benhabib, Bisin, and Zhu (2011, 2016) and Benhabib and Bisin (2018) found conditions to generate fat tails for transformation processes induced by investment risk. On the other hand, Beare and Toda (2018) show that tails of wealth distribution decay exponentially in a heterogeneous-agent dynamic general equilibrium model with idiosyncratic endowment risk. Our model has larger similarities with the former than with the latter. However, we analyze the effect of different marginal taxation rate on growth, showing that there is a trade off between growth and taxation for middle and high marginal taxation rates from which there is no empirical evidence against it.

Our results support some of the ideas mentioned above since low taxes imply higher growth rates and high levels of inequality. At the same time, a reduction of income taxes in our model will change the invariant distribution to a more unequal one, then it will cause a gradual increment of the inequality supporting several empirical studies mentioned above. Moreover, our model suggest that changes in the tax policy may be based on changes on how the social planner discounts the future compared with the other agents do so.

There is an important exception to our result that taxes reduce growth: We identify parameter values for the model where agents are restricted to invest in only one type of assets in which high enough taxes and transfers insure risk averters and induce poor risk averters to invest in the risky asset. Those risk averters who are lucky and accumulate a large enough level of wealth choose to switch to investing in the safe asset. In this case, increasing taxes and increases growth and the welfare of risk averters although it decreases the welfare of risk lovers.

The paper is organized as follows. In section 2, we define the basic model including the notion of equilibrium. In subsection 2.2, we define the basic properties of the model including the relationship between growth and inequality without taxes, and, in subsection 2.3, we analyze the basic properties with taxes. In section 2.4, we show the existence of an invariant growth rate, an invariant distribution of wealth among the agents and their basic properties. In section 2.5, we analyze the existence of optimal taxes by a social welfare function, and we also prove the basic properties of this function and of the optimal taxes. In subsection 2.6, we give some numerical examples that help us to the analysis made in section 2.5. In section 3, we analyze the case with effort cost and the extension to a model with capital and labor. Finally, in section 4, we give some concluding remarks.

2. Model with production, segmentation, and taxation

We analyze an overlapping generation model with warm-glow bequests and uncertainty. Our model has two continuous types of agents *risk averse* and *risk neutral agents*, each of which has measure 1. We denote risk neutrals as $\{l_i\}_{i\in[0,1]}$ and risk averters as $\{a_i\}_{i\in[0,1]}$. There are two linear one-period technologies given by a constant value $R_S > 0$ for the safe technology and $\overline{R}_R = R_R > 0$ with probability $1/2$ and $\underline{R}_R = 0$ with probability $1/2$ for the risky technology. We choose the probability of the high return to be equal to $1/2$ for ease of computation and to be definitive. We could easily choose to be any $p \in (0,1)$ although, of course, this would change the formulas and the computational results.

We assume that

$$R_R > R_S > 0 \qquad (2.1)$$

and

$$R_R/2 > \frac{1}{\delta} \geq R_S \qquad (2.2)$$

where $\delta \in (0,1)$ is the natural bequest rate that we explain below.

To ensure that the markets for assets are segmented in the sense that risk averse agents purchase only the safe asset and risk neutral agents purchase only the risky asset, we make the following assumption:

Assumption S1: The technology R_S, the safe one, is available for both types of agents, and R_R, the risky one, is available for the risk neutral agents only.

We assume that the probability of the risky technology is independent among the agents. Consequently, there is no aggregate uncertainty in the economy.

There is a single consumption good at every date t, $t = 0,1,\dots$. Every agent is characterized at date t by his idiosyncratic state $s^i = (\eta_1^i, \eta_2^i, \dots, \eta_t^i)$, where $\eta_k^i = 1$ if his investment has the high return at date k and $\eta_k^i = 0$ if his investment has the low return at date k. Because there is no aggregate uncertainty, the specification of idiosyncratic states is only important for defining the maximization problems of individual agents and their solutions, not aggregate variables.

Any young generation uses the bequest from his predecessor to invest in the production technologies, and any old generation uses the production returns to consume, $c_{s^i}^i$, and to leave a bequest, $b_{s^i}^i$, to his successor. All the agents leave a bequest that is a proportion of the agent's total wealth. In $t = 0$, the initial endowment of the old generation is w_0^i.

2.1. Taxes

At each date $t \geq 0$, the government imposes taxes on wealth and bequests. There is a tax on wealth of agents whose wealth exceeds a threshold W_t and a subsidy on wealth of agents whose wealth is less than W_t. For simplicity, the marginal tax rate is equal to the marginal subsidy rate, τ^W. Consequently, the net wealth tax on an agent with wealth w is $\tau^W \max(w - W_t, 0) - \tau^W \max(W_t - w, 0)$. There is also a tax on bequests that works similarly. Agents whose bequests exceed a threshold B_t pay the tax and those whose bequests are less than B_t receive a subsidy. Again, we assume the marginal tax rate is equal to the marginal subsidy rate, $\bar{\tau}^B$, and the net bequest tax on an agent with bequests b is $\bar{\tau}^B \max(b - B_t, 0) - \bar{\tau}^B \max(B_t - b, 0)$.

Notice that τ^W is naturally constrained to the set $[0,1]$ while $\bar{\tau}^B$ is constrained to the set $[0, \infty]$. To keep the two taxes symmetric in our analysis, we define the (gross) bequest tax rate to be τ^B where $\tau^B = \bar{\tau}^B/(1 + \bar{\tau}^B)$. Notice that the tax on a bequest b satisfies $\tau^B/(1 - \tau^B)b = \bar{\tau}^B b$ and that τ^B is naturally in the set $[0,1]$. Given these definitions, we specify a tax policy as $\tau = (\tau^W, \tau^B)$. The marginal tax rates τ^W and τ^B are exogenously defined by the central planner. To keep tax policy simple, we set the threshold W_t equal to the average wealth level in the economy and the threshold B_t equal to the average bequest level. This assumption ensures a balanced government budget.

2.2. Utility maximization

Given an initial amount of wealth, $w_0^{a_i}$, the utility maximization problem of a risk averter a_i of the old generation in $t = 0$ is

$$\max_{(c,b)} \quad (1-\delta)\log c + \delta \log b$$
$$s.t. \quad 0 \le c + b + \frac{\tau^B}{1-\tau^B}(b - B_0) \le w_0^{a_i} - \tau^W\left(w_0^{a_i} - W_0\right).$$

Similarly, given the initial wealth, $w_0^{l_i}$, the utility maximization problem of a risk neutral l_i of the old generation in $t = 0$ is

$$\max_{(c,b)} \quad c^{1-\delta} b^{\delta}$$
$$s.t. \quad 0 \le c + b + \frac{\tau^B}{1-\tau^B}(b - B_0) \le w_0^{l_i} - \tau^W\left(w_0^{l_i} - W_0\right).$$

Given the bequest $b_{s^{a_i}}^{a_i}$ received from his predecessor at state s^{a_i} at date $t \ge 1$, the utility maximization problem of a young risk averter a_i is

$$\max_{(c,b,\theta)} \quad (1-\delta)\log c + \delta \log b$$
$$s.t. \quad \theta_S \le b_{s^{a_i}}^{a_i},$$
$$0 \le c + b + \frac{\tau^B}{1-\tau^B}(b - B_t) \le R_S\theta_S - \tau^W(R_S\theta_S - W_t).$$

Given the bequest $b_{s^{l_i}}^{l_i}$ received from his predecessor at state s^{l_i} at date $t \ge 1$, the utility maximization problem of a young risk neutral agent l_i is

$$\max_{(c,b,\theta)} \quad \frac{1}{2}c_1^{1-\delta}\, b_1^{\delta} + \frac{1}{2}c_2^{1-\delta} b_2^{\delta}$$
$$s.t. \quad \theta_R + \theta_S \le b_{s^{l_i}}^{l_i},$$
$$0 \le c_1 + b_1 + \frac{\tau^B}{1-\tau^B}(b_1 - B_t) \le R_R\theta_R + R_S\theta_S - \tau^W(R_R\theta_R + R_S\theta_S - W_t),$$
$$0 \le c_2 + b_2 + \frac{\tau^B}{1-\tau^B}(b_2 - B_t) \le R_S\theta_S - \tau^W(R_S\theta_S - W_t).$$

Note that, in our model, wealth taxes can be seen as income taxes since the capital is completely transformed into income in each state s. We focus mainly on interpretation of τ^W as wealth taxes.

Because of the form of the utility index and Equation 3.2, a risk neutral agent l_i never invests in the safe technology, that is, $\theta^{l_i}_{S,s^{l_i}} = 0$ for all $i \in [0,1]$. Therefore, all agents invest in only one technology, which is the market segmentation that we refer to. The risk averters invest in the safe asset, the less productive one, and the risk neutral agents invest in the risky asset, the productive one.

2.3. Equilibrium

Now, let us define the *equilibrium* for the economy as $((c^{a_i}, b^{a_i}, \theta^{a_i})_i, (c^{l_i}, b^{l_i}, \theta^{l_i})_i)$ such that $\left(c^i, b^i, \theta^i\right)$ solves the utility maximization problem defined above for any state s^i, and

$$\int_0^1 \left(\frac{\tau^B}{1-\tau^B}\left(b^{a_i}_{s^{a_i}} - B_t\right) + \frac{\tau^B}{1-\tau^B}\left(b^{l_i}_{s^{l_i}} - B_t\right)\right) di$$
$$= -\int_0^1 \left(\tau^W\left(R_S\theta_{S,s^{a_i}} - W_t\right) + \tau^W\left((1/2)R_R\theta_{R,s^{l_i}} + R_S\theta_{S,s^{l_i}} - W_t\right)\right) di,$$
$$\bar{c}_{t+1} + \bar{b}_{t+1} = \bar{y}_{t+1} = \int_0^1 \left(R_S\theta^{a_i}_{S,s^{a_i}} + (1/2)R_R\theta^{l_i}_{R,s^{l_i}} + R_S\theta^{l_i}_{S,s^{l_i}}\right),$$

where $\bar{y}_t = (1/2)\left(\int_0^1 y^{a_i}_{s^{a_i}} di + \int_0^1 y^{l_i}_{s^{l_i}} di\right)$, $\bar{c}_t$, and $\bar{b}_t$ are the average net wealth, consumption, and bequests in date t, respectively, and, $y^{a_i}_{s^{a_i}}$, $y^{l_i}_{s^{l_i}}$ are the net wealth of the risk averter a_i in state s^{a_i} and net wealth of the risk neutral agent l_i in state s^{l_i}. Notice that the first equilibrium condition is government budget balance, and the second is feasibility. As we have explained, the first condition is satisfied if we set $B_t = \bar{b}_t$ and $W_t = \bar{y}_t$.

2.4. Properties of the sequential markets equilibrium

From the first order conditions of the utility maximization problems, we know that

$$b^i_{s^i} = \frac{\delta\left(1-\tau^B\right)}{(1-\delta)} c^i_{s^i}$$

for all agents i. Combining this condition with the consumer's budget constraints yields

$$c^i_{s^i} = \frac{1-\delta}{1-\delta\tau^B} y^i_{s^i},$$

$$b^i_{s^i} = \frac{\delta(1-\tau^B)}{1-\delta\tau^B} y^i_{s^i}.$$

Integrating over all agents gives us the aggregate equilibrium conditions

$$\bar{c}_t = \frac{1-\delta}{1-\delta\tau^B}\bar{y}_t,$$

$$\bar{b}_t = \frac{\delta(1-\tau^B)}{1-\delta\tau^B}\bar{y}_t.$$

Notice that this final condition implies that the threshold B_t is proportional to the threshold W_t, which further implies that an agent pays positive bequest taxes if and only if he pays positive wealth taxes.

Using the tax policy $\tau = (\tau^W, \tau^B)$ and the thresholds W_t and B_t, the budget constraint of the risk averter a_i when he is old at date $t \geq 1$ can be written as

$$0 \leq c + \left(1 + \frac{\tau^B}{1-\tau^B}\right) b \leq (1-\tau^W) R_S \theta_S + \tau^W \bar{y}_t + \frac{\delta\tau^B \bar{y}_t}{(1-\delta\tau^B)}, \tag{2.3}$$

and the budget constraint of the risk neutral i when he is old date at $t \geq 1$ can be written as

$$0 \leq c_1 + \left(1 + \frac{\tau^B}{1-\tau^B}\right) b_1 \leq (1-\tau^W) R_R \theta_R + \tau^W \bar{y}_t + \frac{\delta\tau^B \bar{y}_t}{(1-\delta\tau^B)}, \tag{2.4}$$

$$0 \leq c_2 + \left(1 + \frac{\tau^B}{1-\tau^B}\right) b_2 \leq \tau^W \bar{y}_t + \frac{\delta\tau^B \bar{y}_t}{(1-\delta\tau^B)}. \tag{2.5}$$

Thus, each agent receives one transfer that depends on the average wealth. Additionally, the wealth tax reduces the agent's wealth by the proportion of $1-\tau^W$, and the bequest tax makes it more expensive to leave part of his wealth as a bequest.

Since bequest taxes affect the incentives that each agent has for consuming and leaving bequests, average consumption and average bequest depend on the marginal tax rates. Moreover, a higher bequest tax implies a higher level of consumption and a lower level of bequests, which decreases the wealth of all immediate descendants. More specifically, if $\bar{y}^a_{\tau,t}$ is the after-tax average wealth of the risk averters at date t, and $\bar{y}^l_{\tau,t}$ is the after-tax average wealth of the risk-neutral agents at date t, the average wealth at date $t+1$ is

$$\bar{y}_{\tau,t+1} = \frac{(R_R/2)\delta(1-\tau^B)}{(1-\delta\tau^B)2}\bar{y}^l_{\tau,t} + \frac{R_S\delta(1-\tau^B)}{(1-\delta\tau^B)2}\bar{y}^a_{\tau,t}. \tag{2.6}$$

Given the tax policy $\tau = (\tau^I, \tau^B)$, we define the growth rate from t to date $t+1$ to be $g_{\tau,t} = \bar{y}_{\tau,t+1}/\bar{y}_{\tau,t} - 1$.

Proposition 1. For any $\epsilon > 0$ and tax policy (τ^W, τ^B) such that $\tau^W, \tau^B \leq 1 - \epsilon$, any increment on the wealth tax rate (from τ^W to $\tau^W + \epsilon$) induces a higher growth rate than an increment on the bequest tax rate (from τ^B to $\tau^B + \epsilon$) at date $t+1$, $g_{(\tau^W+\epsilon,\tau^B),t} > g_{(\tau^W,\tau^B+\epsilon),t}$.

2.5. An alternative tax policy

It is worth noting that we can easily extend our analysis to tax policies that do not involve subsidies on bequests. In this case, the net bequest tax on an agent with bequests b is $\bar{\tau}^B b = \tau^B/(1-\tau^B)b$. To maintain budget balance in this case, the threshold W_s is given by

$$W_s = \bar{y}_t + \frac{\tau^B}{(1-\tau^B)\tau^W}\bar{b}_t = \bar{y}_t\left(1 + \frac{\delta\tau^B}{(1-\delta\tau^B)\bar{\tau}^W}\right).$$

Given this specification of the threshold, this tax policy is equivalent to the one in the previous section. In particular, Equations 2.3, 2.4, and 2.5 hold, and the rest of the analysis in this paper goes through.

2.6. A recursive specification of the model and equilibrium

At date $t = 0$, we specify the state of an initial old agent by the wealth he brings into the period. There is a distribution function for initial wealth $\left(F^W_{a,\tau}, F^W_{l,\tau}\right)$, where $F^W_{a,\tau}$ is the distribution of initial wealth for risk-averse agents and $F^W_{l,\tau}$ is the distribution of initial wealth for risk-neutral agents. The utility maximization problem of a risk averter of the initial old generation with wealth w is

$$\max_{(c,b')} \quad (1-\delta)\log c + \delta \log b'$$
$$s.t. \quad 0 \leq c + b' + \frac{\tau^B}{1-\tau^B}(b' - B_0) \leq w - \tau^W(w - W_0).$$

Similarly, the utility maximization problem of a risk neutral agent of the initial old generation with wealth w is

$$\max_{(c,b')} \quad c^{1-\delta} b'^{\delta}$$
$$s.t. \quad 0 \leq c + b' + \frac{\tau^B}{1-\tau^B}(b' - B_0) \leq w - \tau^W (w - W_0).$$

At date $t \geq 1$, we specify the state of a young agent by the bequest b he receives from his predecessor. The utility maximization problem of a young risk averter is

$$\max_{(c,b',\theta)} \quad (1-\delta)\log c + \delta \log b'$$
$$s.t. \quad \theta_S \leq b,$$
$$0 \leq c + b' + \frac{\tau^B}{1-\tau^B}(b' - B_t) \leq R_S\theta_S - \tau^W (R_S\theta_S - W_t).$$

The utility maximization problem of a young risk-neutral agent is

$$\max_{(c,b',\theta)} \quad \frac{1}{2} c_1^{1-\delta} b_1'^{\delta} + \frac{1}{2} c_2^{1-\delta} b_2'^{\delta}$$
$$s.t. \quad \theta_R + \theta_S \leq b,$$
$$0 \leq c_1 + b_1' + \frac{\tau^B}{1-\tau^B}(b_1' - B_t) \leq R_R\theta_R + R_S\theta_S - \tau^W (R_R\theta_R + R_S\theta_S - W_t),$$
$$0 \leq c_2 + b_2' + \frac{\tau^B}{1-\tau^B}(b_2' - B_t) \leq R_S\theta_S - \tau^W (R_S\theta_S - W_t).$$

We can specify the state of the economy by distribution function for bequests $\left(F^B_{l,\tau,t}, F^B_{a,\tau,t}\right)$.

Given this recursive specification of the model and the initial distribution of wealth $\left(F^W_{a,\tau}, F^W_{l,\tau}\right)$, we define an equilibrium as policy functions $\left(c_{a,0}(w), b'_{a,0}(w)\right), \left(c_{l,0}(w), b'_{l,0}(w)\right)$ and $\left(c_{a,t}(b), b'_{a,t}(b), \theta_{a,t}(b)\right), \left(c_{l,t}(b), b'_{l,t}(b), \theta_{a,t}(b)\right)$, thresholds W_t and B_t, and equation of motion for the distribution functions $\Omega_{a,0}\left(F^W_{a,\tau}, F^W_{l,\tau}\right)$, $\Omega_{l,0}\left(F^W_{a,\tau}, F^W_{l,\tau}\right)$, $\Omega_a\left(F^B_{a,\tau,t}, F^B_{l,\tau,t}\right)$, and $\Omega_l\left(F^B_{a,\tau,t}, F^B_{l,\tau,t}\right)$. The policy functions solve the utility maximization problems and satisfy the feasibility conditions

$$\int_0^\infty c_{a,0}(w) dF^W_{a,\tau}(w) + \int_0^\infty c_{l,0}(w) dF^W_{l,\tau}(w) + \int_0^\infty b'_{a,0}(w) dF^W_{a,\tau}(w) + \int_0^\infty b'_{l,0}(w) dF^W_{l,\tau}(w)$$
$$= \int_0^\infty w dF^W_{a,\tau}(w) + \int_0^\infty w dF^W_{l,\tau}(w)$$

in period 0, and

$$\int_0^\infty c_{a,t}(b)dF^B_{a,\tau,t}(b) + \int_0^\infty c_{l,t}(b)dF^B_{l,\tau,t}(b) + \int_0^\infty b'_{a,t}(b)dF^B_{a,\tau,t}(b) + \int_0^\infty b'_{l,t}(b)dF^B_{l,\tau,t}(b)$$

$$= \int_0^\infty R_S\theta_{a,t}(b)dF^B_{a,\tau,t}(b) + \int_0^\infty \left(R_S\theta_{l,t}(b) + (1/2)R_R\theta_{l,t}(b)\right) F^B_{l,\tau,t}(b)$$

in period $t \geq 1$. The thresholds W_t and B_t satisfy the conditions

$$W_0 = \int_0^\infty w dF^W_{a,\tau}(w) + \int_0^\infty w dF^W_{l,\tau}(w)$$

$$B_0 = \int_0^\infty b'_{a,0}(w)dF^W_{a,\tau}(w) + \int_0^\infty b'_{l,0}(w)dF^W_{l,\tau}(w)$$

$$W_t = \int_0^\infty R_S\theta_{a,t}(b)dF^B_{a,\tau,t}(b) + \int_0^\infty \left(R_S\theta_{l,t}(b) + (1/2)R_R\theta_{l,t}(b)\right) F^B_{l,\tau,t}(b)$$

$$B_t = \int_0^\infty b'_{a,t}(b)dF^B_{a,\tau,t}(b) + \int_0^\infty b'_{l,t}(b)dF^B_{l,\tau,t}(b).$$

The distribution functions $\left(F^W_{a,\tau}, F^W_{l,\tau}\right)$, $\left(F^B_{a,\tau,t}, F^B_{l,\tau,t}\right)$ satisfy the equations of motion

$$\begin{bmatrix} F^B_{a,\tau,1} \\ F^B_{l,\tau,1} \end{bmatrix} = \begin{bmatrix} \Omega_{a,0}\left(F^W_{a,\tau}, F^W_{l,\tau}\right) \\ \Omega_{l,0}\left(F^W_{a,\tau}, F^W_{l,\tau}\right) \end{bmatrix}$$

and

$$\begin{bmatrix} F^B_{a,\tau,t+1} \\ F^B_{l,\tau,t+1} \end{bmatrix} = \begin{bmatrix} \Omega_{a}\left(F^B_{a,\tau,t}, F^B_{l,\tau,t}\right) \\ \Omega_{l}\left(F^B_{a,\tau,t}, F^B_{l,\tau,t}\right) \end{bmatrix}.$$

Notice that the distribution function for before-tax wealth $\left(F^W_{a,\tau}, F^W_{l,\tau}\right)$ induce distribution function for after-tax wealth as fraction of the aggregate wealth $\left(\tilde{F}^W_{a,\tau,0}, \tilde{F}^W_{l,\tau,0}\right)$, and the distribution function for bequests $\left(F^B_{a,\tau,t} F^B_{l,\tau,t}\right)$ induce the distribution function for after-tax wealth as fraction of aggregate wealth $\left(\tilde{F}^W_{a,\tau,t}, \tilde{F}^W_{l,\tau,t}\right)$ for $t \geq 1$. More specifically,

$$\tilde{F}^W_{a,\tau,0}(w) = F^W_{a,\tau}\left(\frac{W_0 w}{1-\tau^W} - \frac{\tau^W W_0}{2(1-\tau^W)} - \frac{\delta\tau^B W_0}{2(1-\tau^W)(1-\tau^B)}\right),$$

$$\tilde{F}^W_{l,\tau,0}(w) = F^W_{l,\tau}\left(\frac{W_0 w}{1-\tau^W} - \frac{\tau^W W_0}{2(1-\tau^W)} - \frac{\delta\tau^B W_0}{2(1-\tau^W)(1-\tau^B)}\right),$$

$$F^B_{a,\tau,t+1}(b) = F^B_{a,\tau,t}\left(\frac{(1-\delta\tau^B)\left(b-\underline{b}_t\right)}{\delta(1-\tau^W)(1-\tau^B)R_S}\right),$$

$$F^B_{l,\tau,t+1}(b) = \begin{cases} 0, & b < \underline{b}_t \\ \frac{1}{2}\left(1 + F^B_{l,\tau,t}\left(\frac{(1-\delta\tau^B)\left(b-\underline{b}_t\right)}{\delta(1-\tau^W)(1-\tau^B)R_R}\right)\right), & b \geq \underline{b}_t, \end{cases}$$

where

$$\underline{b}_t = \frac{\delta(1-\tau^B)}{1-\delta\tau^B}\left(\tau^W + \frac{\delta\tau^B}{1-\tau^B}\right)\frac{W_t}{2},$$

is the minimum level of bequests in period t.

$$\tilde{F}^W_{a,\tau,t}(w) = F^B_{a,\tau,t}\left(\frac{\left(w-\underline{w}\right)W_t}{(1-\tau^W)R_S}\right),$$

$$\tilde{F}^W_{l,\tau,t}(w) = \begin{cases} 0, & w < \underline{w}, \\ \frac{1}{2}\left(1 + F^B_{l,\tau,t}\left(\frac{\left(w-\underline{w}\right)W_t}{(1-\tau^W)R_R}\right)\right), & b \geq \underline{w}, \end{cases}$$

where

$$\underline{w} = \frac{\tau^W}{2} + \frac{\delta\tau^B}{2(1-\tau^B)},$$

is the minimum level of (proportional) wealth in period $t \geq 1$.

3. Existence and uniqueness of the invariant distribution and convergence

TBA

3.1. Invariant distribution

TBA

Proposition 3. If $\tau = (0,0)$, there is no invariant distribution.

If, however, $\tau^W > 0$ or $\tau^B > 0$, we are able to prove the following result of existence, uniqueness of an invariant distribution and the convergence of all equilibria to this invariant distribution.

Theorem 1. For any distribution of initial endowments $\left(F_{a,\tau}^W, F_{l,\tau}^W\right)$ the distribution of after-tax wealth among the agents $\left(\tilde{F}_{a,\tau,t}^W, \tilde{F}_{l,\tau,t}^W\right)$ and the growth rate $g_{\tau,t}$ converge to invariant distribution $\left(\tilde{F}_{a,\tau}^W, \tilde{F}_{l,\tau}^W\right)$ and a constant g_τ. Moreover, g_τ is a strictly decreasing C^1 function in τ.

The proof of Theorem 1 is in Appendix A. The proof of an invariant distribution is done recursively, for that we take advantage of $\underline{R}_R = 0$ which implies that the wealth history of the risk neutral agents is lost whenever they fail. Each risk neutral dynasty is characterized by the number of consecutive times that it has been successful in its investments after the last time it was unsuccessful.

From the proof of Theorem 1 when $\bar{\tau}^B = 0$, we have that

$$g_{\tau^W} = \frac{((R_R/2)\delta - 1)z_{\tau^W} + (R_S\delta - 1)}{z_{\tau^W} + 1}$$

where $z_{\tau^W} = \bar{x}_{\tau^W}^l / \bar{x}_{\tau^W}^a$ is the ratio between the after-tax wealth in hands of the risk neutrals and the after-tax wealth in hands of the risk averters, and it is the solution of

$$z_{\tau^W} = \frac{\bar{x}_{\tau^W}^l}{\bar{x}_{\tau^W}^a} = \frac{\left(1 - \frac{\tau^W}{2}\right)(R_R/2)z_\tau + \left(\frac{\tau^W}{2}\right)R_S}{\left(\frac{\tau^W}{2}\right)(R_R/2)z_\tau + \left(1 - \frac{\tau^W}{2}\right)R_S},$$

which is given by

$$z_{\tau^W} = \left(\frac{1}{\tau^W} - \frac{1}{2}\right)\left(1 - \frac{R_S}{R_R/2}\right) + \left(\left(\frac{1}{\tau^W} - \frac{1}{2}\right)^2 \left(1 - \frac{R_S}{R_R/2}\right)^2 + \frac{R_S}{R_R/2}\right)^{1/2}.$$

Note that z_{τ^W} is a strictly decreasing function in τ^W such that $\lim_{\tau^W \to 0} z_{\tau^W} = \infty$ and $\lim_{\bar{\tau}^W \to 1} z_{\tau^W} = 1$, so any increase in the tax rate τ^W implies a reduction in $z_{\tau^W} = \bar{x}_{\tau^W}^l / \bar{x}_{\tau^W}^a$. And since only the risk neutrals invest in the most productive technology, an increase in the tax rate implies a lower invariant growth rate.

We define $\left(\tilde{F}_{a,\tau}^W\right)^{-1}$ and $\left(\tilde{F}_{l,\tau}^W\right)^{-1}$ as the quantile functions of the distributions $\tilde{F}_{a,\tau}^W$ and $\tilde{F}_{l,\tau}^W$. The invariant distribution of wealth $\tilde{F}_{a,\tau}^W$ is a constant distribution

$$x_\tau^a = \left(\tilde{F}_{a,\tau}^W\right)^{-1}(1) = \sum_{k=0}^{\infty}(\delta\pi_a)^k = \frac{\left(1+g_{\tau^W}\right)}{2(1+g_{\tau^W}) - 2\delta(1-\tau^W)R_S}\tau^W,$$

where $\pi_a = R_S(1-\tau^I)/\left(1+g_{\tau^I}\right)$.

Next, we provide the invariant distribution of the risk neutrals. For that, we denote $\pi_r = R_R(1-\tau^W)/\left(1+g_{\tau^W}\right)$.

- For the poorest risk-neutral agents — those who risk neutrals who failed — whose proportion is $1/2$,

$$x_\tau^{l,1} = \left(\tilde{F}_{l,\tau}^W\right)^{-1}(1/2) = \tau^W/2.$$

- For the second poorest risk-neutral agents — those who were lucky, and their predecessor failed — whose proportion is $1/4$,

$$x_\tau^{l,2} = \left(\tilde{F}_{l,\tau}^W\right)^{-1}(3/4) = \delta\pi_r\tau^W/2 + \tau^I/2.$$

- For the third poorest risk-neutral agents — those who were lucky, their predecessors were lucky, and their predecessor's predecessor failed — whose proportion is $1/8$,

$$x_\tau^{l,3} = \left(\tilde{F}_{l,\tau}^W\right)^{-1}(7/8) = (\delta\pi_r)^2\tau^W/2 + \delta\pi_r\tau^W/2 + \tau^W/2.$$

- For the poorest group n of risk-neutral agents — those who were lucky, and all their predecessors have been lucky in the last $n-1$ periods and failed n periods ago — whose proportion is $1/2^n$,

$$x_\tau^{l,n} = \left(\tilde{F}_{l,\tau}^W\right)^{-1}(1-1/2^n) = \sum_{k=0}^{n-1}(\delta\pi_r)^k\tau^W/2.$$

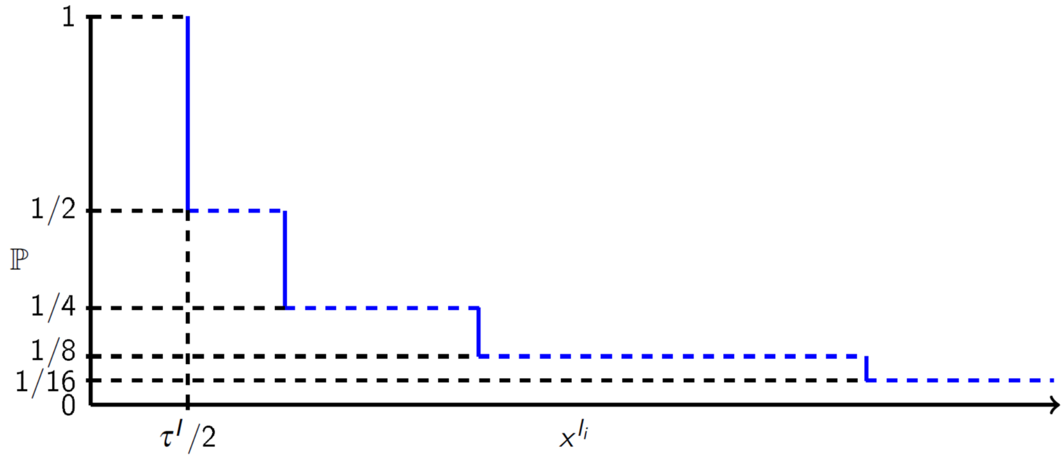

Figure 1: Invariant distribution of wealth of the risk neutrals with a large bequest rate $\delta \geq \left(1 + g_{\tau^W}\right)\left(R_R(1 - \tau^W)\right)^{-1}$ in absence of bequest taxes.

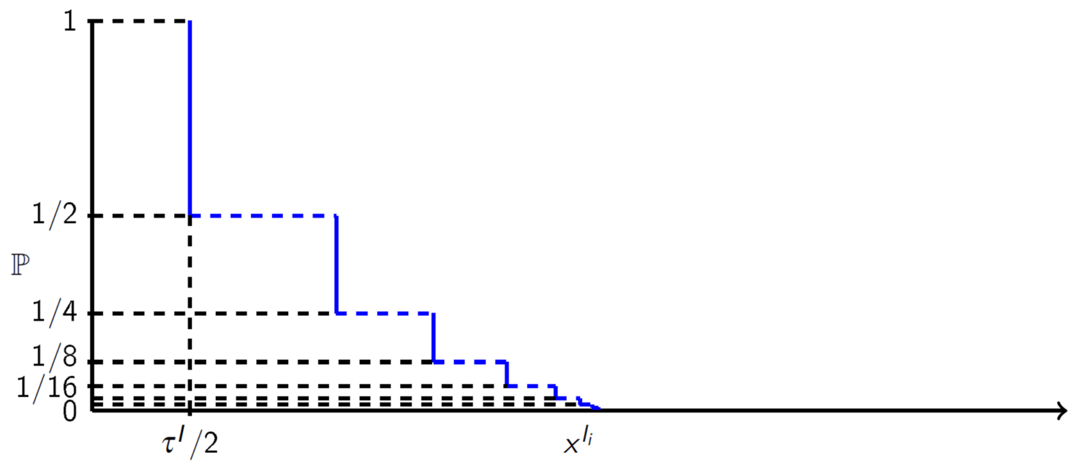

Figure 2: Invariant distribution of wealth of the risk with a small bequest rate, $\delta < \left(1 + g_{\tau^W}\right)\left(R_R(1 - \tau^W)\right)^{-1}$ in absence of bequest taxes.

In our framework, if $\delta \geq \left(1 + g_{\tau^W}\right)\left(R_R(1 - \tau^W)\right)^{-1}$, the invariant distribution of wealth has fat tails (see Figure 1). This will be the case if the bequest rate and the return of the risky technology are large enough and the wealth tax rates are not. Moreover, in this case the invariant distribution of wealth has exponential fat tails. For the definition of exponential fat tails and related theorems, see Araujo, Gama, and Kehoe (2024). If $\delta < \left(1 + g_{\tau^W}\right)\left(R_R(1 - \tau^W)\right)^{-1}$, that is, the bequest rate is small and the wealth tax rates are close to one, the dispersion of the wealth distribution is small in such a way that it is bounded from above (see Figure 2). In this case, risk neutrals whose predecessors were lucky n consecutive times have a wealth that satisfies that

$$x_{\tau}^{l,n+1} = \sum_{k=0}^{n} (\delta\pi_r)^k \tau^W/2 < \frac{1 + g_{\tau^W}}{2 + 2g_{\tau^W} - 2\delta(1-\tau^W)R_R}\tau^W.$$

From Theorem 1, if a social planner increases taxes on wealth or bequests, the growth rate decreases. Furthermore, we observe that an increment in taxes increases the wealth of the poorest risk neutral agents and decreases the wealth of the risk neutrals at the top of the distribution. Therefore, an increase in taxes reduces the dispersion of the invariant distribution of wealth. In the following section we explore more the implications of taxes in the invariant distribution and in the social welfare function.

4. **The social welfare function and the optimal tax**

The following welfare function W, which we use in the paper, captures the benefits of a lower inequality and a higher growth rate

$$W(c,b') := (1-d)\Big(\int U^a\left(c_{a,0}(w), b_{a,0}(w)\right) dF_{a,\tau}^W(w) + \int \log U^l\left(c_{l,0}(w), b_{l,0}(w)\right) dF_{l,\tau}^W(w) + \sum_{t=1}^{\infty} d^t \Big(\int U^a\left(c_{a,t}(b), b'_{a,t}(b)\right) dF_{a,\tau,t}^B(b) + \int \log U^l\left(c_{l,t}(b), b'_{l,t}(b)\right) dF_{l,\tau,t}^B(b)\Big)\Big) \quad (4.1)$$

where $d \in (0,1)$ is the discounted factor used by the social planner. This welfare function is a particular case of the CES welfare function (see Atkinson, 1970). In this case, the social welfare function has no problems related to the convergence of the series when the economy has a positive long-term growth rate since $U^a\left(c_{a,t}, b'_{a,t}\right)$ and $\log U^l\left(c_{l,t}, b'_{l,t}\right)$ are at most linear in t.

If $\tau^W, \tau^B = 0$, a small amount of risk neutral agents concentrate all the aggregate wealth in the long run. If $\tau^B = 1$, any agent does not leave bequests to the next generation. If $\tau^W = 1$, the invariant distribution of wealth is constant, and the growth rate is considerably lower. The first and second cases imply that a large number of agents will not survive one way or another, which is hardly an optimal allocation for a central planner. The third case may be optimal if the social planner is more concerned about inequality. If a social planner is also concerned about the rate of growth, he is expected to have more intermediate taxation plans.

We define functions X, G, and D as

$$X(\tau) \coloneqq \log x_\tau^a + \sum_{n=2}^{\infty} \frac{1}{2^{n-1}} \left(\log \left(\frac{1}{2} \left(x_\tau^{l,n} + x_\tau^{l,1} \right) \right) \right),$$

$$G(d,\tau) \coloneqq \frac{d}{(1-d)} \log(1+g_\tau),$$

and

$$D(\tau^B) \coloneqq \log \left(\left(\frac{\delta^\delta (1-\delta)^{1-\delta} (1-\tau^B)}{1-\delta\tau^B} \right) \right).$$

Notice that the first function depends on the invariant distribution of wealth. The second function only depends on the growth rate of the economy, and it is strictly decreasing in τ. The third function depends on the agents' bequest rate and the bequest tax. Lastly, the three terms depend on the discount factor of the social planner. Then, we have the following result.

Theorem 2. *(Decomposition of the welfare function)* In the equilibrium allocation, W can be written as

$$W\left(\left(U^i\right)_i, \left(c_\tau^i\right)_i, \left(b_\tau^i\right)_i \right) = X(\tau) + G(d,\tau) + D(\tau^B),$$

where G is differentiable in d and τ, strictly increasing in d, and strictly decreasing in $\tau^W \in (0,1)$ and $\tau^B \in [0,1)$, and $X(\bar{\tau}) < 0$ for $\tau^W \in (0,1)$ and $\tau^W \in [0,1)$, $X(\tau) = 0$ for $\tau^W = 1$. Then, function X attains its maximum value when $\tau^W = 1$.

The characterization of the social welfare function given by Equation 4.2 is extremely useful to understand the phenomena underlying the tax rate, inequality, the growth rate and the relationship between the discount factor and the bequest rate. When taxes are reduced, the growth rate and the dispersion of the invariant distribution increase. Since function X is a measure of wealth inequality, there is a tradeoff between growth and inequality.

Note that only function G depends on the discount factor of the social planner. When the social planner discounts the future strongly, $d \approx 0$, the welfare function depends only on function X, which makes him concerned about reducing intragenerational inequality and not about the growth rate. When the social planner discounts the future weakly, $d \approx 1$, he is almost entirely worried about growth rate in relative terms since function X does not depend on d. Each case has implications on the optimal taxes, we explore these cases in Subsection 5.3.1.

Each type of taxation has different implications for growth and intragenerational inequality. Wealth taxes reduce dispersion the most, while generating changes in the growth rate. Consequently, the social planner must find a balance between low wealth taxes to have large economic growth and high taxes to reduce inequality. On the other hand, bequest taxes reduce the growth rate the most by increasing consumption at the earliest dates. Moreover, since bequest taxes have a small impact on the dispersion of the wealth distribution and a large negative impact on growth, bequest taxes may be zero in several cases. The following result shows that, if the bequest rate is small and the discount factor is large, positive bequest taxes induce lower welfare than taxes on wealth alone.

Proposition 4. *(Non-optimality of bequest taxes)* There is $\bar{\delta} \in (0,0.5)$ and $\underline{d} \in (0,1)$ such that if $\delta \leq \bar{\delta}$ and $\underline{d} \leq d$, for any optimal taxation plan, $\tau^B = 0$.

In the numerical examples is Subsection 5.3, we observe that $\tau^B = 0$ occurs when $\bar{\delta} \approx 0.5$ and $\underline{d} \approx 0$.

Note that for low levels of the bequest rate, almost any level of bequest taxes generates lower welfare than tax policies with only wealth taxes. To have positive optimal bequest taxes, we must look for situations in which the bequest rate is large. In the following subsection, we analyze this case.

5. Numerical Examples

In this section we analyze examples to illustrate the model and the theorems that we introduce above.

5.1.Examples with fixed bequest rate and no taxes on bequests

In the first example of this section, we assume that $R_R = 4.86$, $R_S = 1.6$, $\delta = 0.5$, and $\tau^B = 0$ and d varies from 0.6 to 0.85.

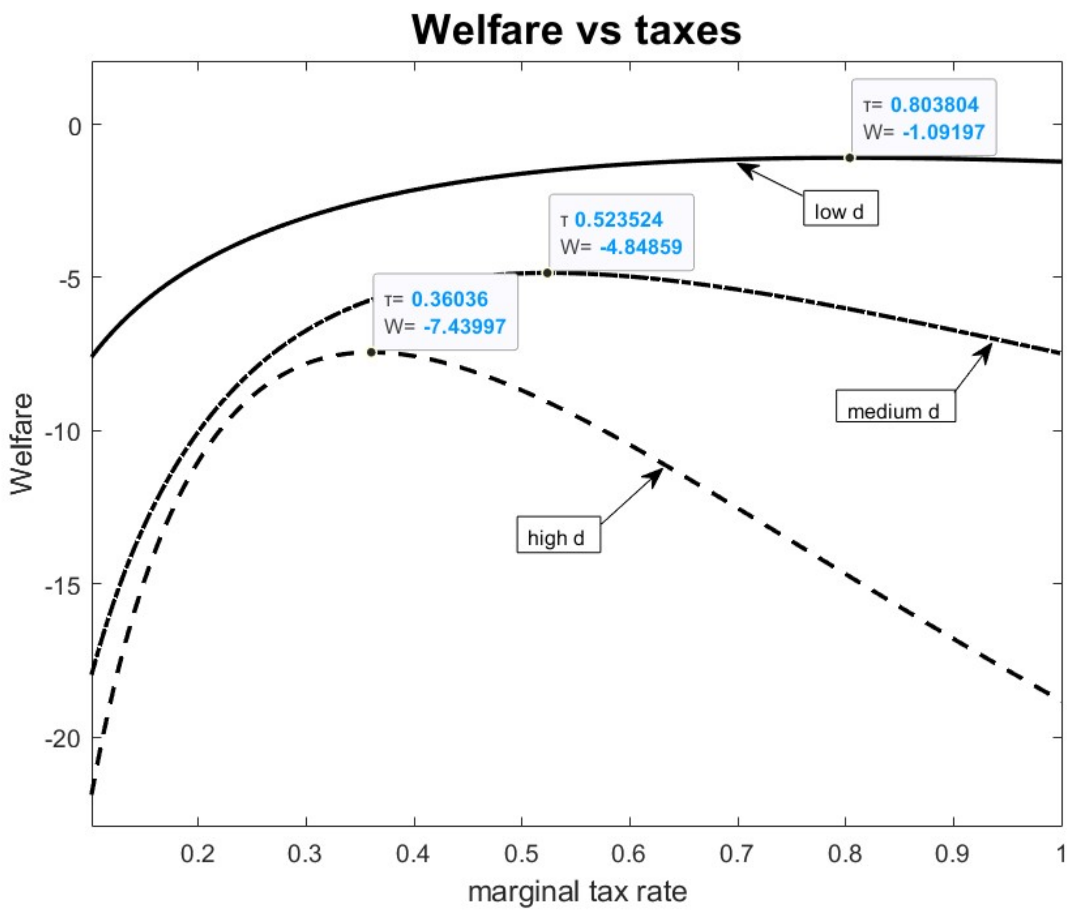

Figure 3: Social welfare function vs wealth taxes for different values of $d = 0.5, 0.75, 0.833$.

We can observe in Figure 3 that the optimal tax for $d = 0.5$ is 80.3%, and the growth rate of the economy is 1.018, which averages that the economy has a slightly positive growth. Moreover, if the social planner increases wealth tax to 1, the growth rate converges to $\delta(R_R/2 + R_S)/2 - 1 = 0.0075$, and in the absence of taxes, the growth rate is $\delta(\bar{R}_R/2) - 1 = 0.215$ (See Proposition 4).

If $d > \delta$, the optimal tax rate induces a larger growth rate. More concretely, if $d = 0.8$, the optimal marginal tax rate is approximately 52.4% and the growth rate is around 0.044. Moreover, if $d = 0.85$, the optimal marginal tax rate is approximately 36% and the growth rate is around 0.072.

Note that for all possible wealth taxes, the growth rate is positive. However, the growth rate increases as the social planner's discount factor increases. One explanation for this phenomenon is that the social planner is not too concerned about very distant consumption when the discount factor is low, which averages that low growth rates may be optimal. In this case, the social planner concern with inequality is more important than with consumption in the long run.

We also observe that, for all the numerical examples above, the optimal wealth tax is unique. Moreover, the social welfare function is strictly concave in the wealth tax rate for all the analyzed discount factors of the social planner.

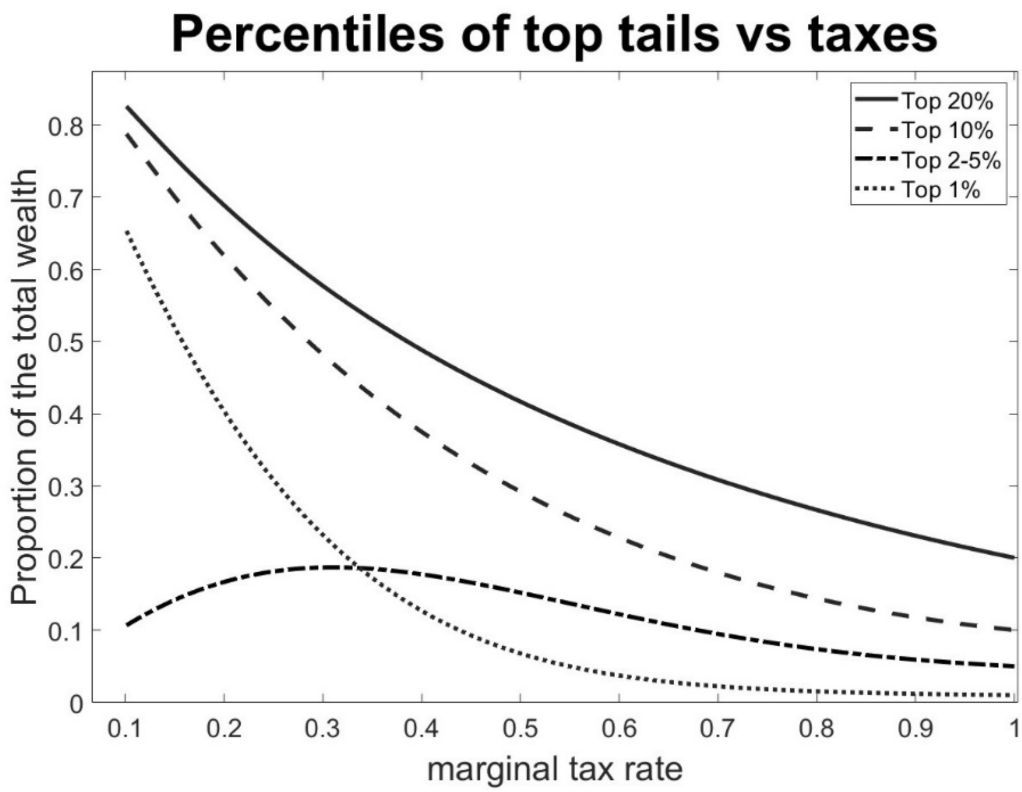

Figure 4: Income/wealth inequality for different wealth taxation rates.

We observe in Figures 3 and 4 that an increment in the discount factor of the social planner implies an increment in the growth rate and a more unequal distribution of wealth. Moreover, the level of inequality implemented by a social planner with a discount factor $d = 0.5$ is quite low ($\tau^W \approx 0.8$). In this case the top 1% has 2.85% of the total income, the top 10% has 20.7%, and the top 20% has 34.15%, which is more equal than Japan, a very equal country where the top 1% earns around 10% of the national income. For a social planner that with a discount factor equal to $d = 0.75$ (which implies $\tau^W \approx 0.52$), inequality is clearly larger with 9.69% for the top 1%, 34.85%

for the top 10%, and 47.6% for the top 20% which is closer to Japan. Finally, for a social planner with a discount factor equal to $d = 0.833$ (which implies $\tau^W \approx 0.36$), inequality is clearly larger than the other two cases with 21.44% for the top 1%, 48.37% for the top 10%, and 59.09% for the top 20% which is closer to the US where the top 1% earns around 20% of the national income, see Rodriguez, Díaz-Giménez, Quadrini, and Ríos-Rull (2002).

5.1.1. **Numerical properties**

We observe in Figure 5 that the welfare function is strictly concave in τ. This is supported by the decomposition of Theorem 2 since G and X are logarithm functions related to growth and the wealth distribution, respectively**(?)**. We also observe numerically that there is only one tax rate that maximizes the social welfare. Additionally, we observe that the optimal wealth tax is negatively related to the discount factor. In Subsection 5.3 we explore more this.

5.2.Examples with changes on productivity

In the second example of this section, we assume that $R_R \in [3.9, 5.9]$, $R_S = 1.6$, and $\delta = d = 0.5$.

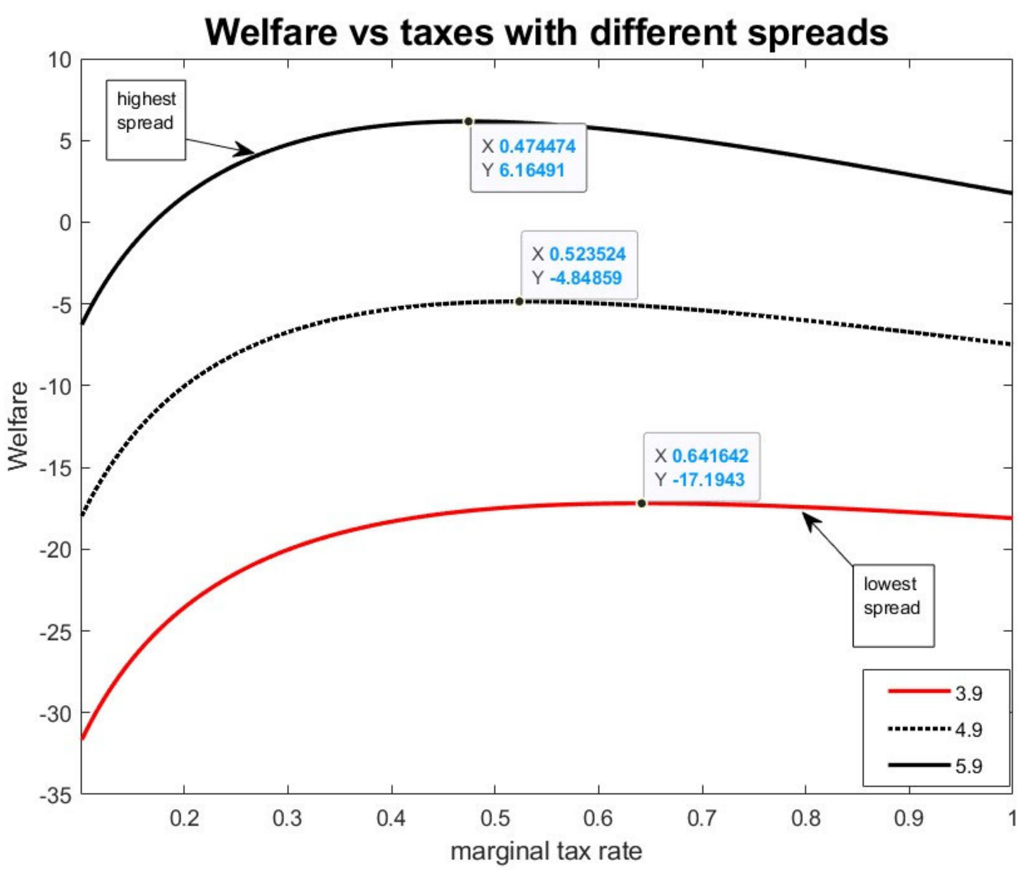

Figure 5: Welfare vs changes on productivity of R_R.

Note that, in this example, changes in the productivity of risky technology imply changes in the spread of the technology. Note that this phenomenon causes a change in optimal taxation. In this case, a more productive risky technology leads to a lower optimal tax rate. It is therefore optimal for a social planner to increase inequality due to the increase in productivity. This fact is also observed in a wide variety of economies around the world. The US has a higher level of productivity than the largest economies in Europe, such as Germany, France, and England, and also has considerably greater wealth and income inequality.

5.3. Different discount and bequest rates and their consequences on welfare, growth, and inequality

We assume that $R_R = 9$, $R_S = 2$, δ varies between 0.25 and 0.95, and d varies between 0.05 and 0.95.

$\delta = 0.25$	$d = 0.25$	$d = 0.5$	$d = 0.75$	$d = 0.9$
Optimal beq. tax	0	0	0	0
Op. wealth. tax	0.99	0.99	0.61	0.22
Welfare	0.40	0.61	0.10	-0.52
Growth rate	-0.19	-0.19	-0.12	0.01
Top 1%	0.01	0.01	0.045	0.424
$\delta = 0.375$				
Optimal beq. tax	0	0	0	0
Op. wealth tax	0.99	0.99	0.61	0.22
Welfare	0.44	1.07	2.62	9.07
Growth rate	0.22	0.22	0.32	0.52
Top 1%	0.01	0.01	0.046	0.425
$\delta = 0.5$				
Optimal beq. tax	0	0	0	0
Op. wealth tax	0.99	0.99	0.61	0.22
Welfare	0.48	1.45	4.49	15.97
Growth rate	0.62	0.63	0.76	1.03
Top 1%	0.01	0.01	0.046	0.425

Table 1: Optimal taxation plans for low bequest rates, δ, and discount factor of the social planner, d.

We see in Table 1 that if the social planner has a discount factor close to zero, the optimal wealth tax should be high to reduce inequality. Furthermore, when the social planner discounts the future less, he is less concerned with inequality and more with growth, which implies a lower optimal tax rate. Therefore, the growth rate is positively related to the demand rate and the discount factor. Note that, the bequest rate has no influence on optimal taxes (changes in the bequest rate have no impact on the optimal tax rate), but it does have a strong effect on the growth rate. Increases in the bequest rate or discount factor have a positive impact on growth. Moreover, for $\delta = 0.25$, the growth rate is negative if the discount factor is small, and it is positive if the discount factor is close to zero. This result is a direct consequence of Theorem 4 below since $\delta(R_R/2 + R_S)/2 - 1 = -0.1875 < 0$.

Note that in Table 1, the optimal bequest tax is always zero since the bequest rate is considerably low (see Proposition 3). A small bequest tax on investors can be seen as a way of discounting the wealth of future generations. Thus, when the bequest rate is low, it is optimal to avoid any tax policy that strongly reduces the growth rate, such as a positive bequest rate. Additionally, we observe zero bequest taxes independently of the discount factor if the bequest rate $\delta \leq 0.5$. Then, we observe numerically that $\underline{d} \approx 0$ (Proposition 3).

$\delta = 0.6$	$d = 0.1$	$d = 0.2$	$d = 0.6$	$d = 0.8$	$d = 0.9$
Optimal beq. tax	0.4	0.33	0	0	0
Op. wealth. tax	0.98	0.97	0.99	0.48	0.22
Welfare	0.18	0.41	2.71	7.99	20.46
Growth rate	0.4	0.63	0.95	1.20	1.43
Top 1%	0.01	0.01	0.01	0.103	0.425
$\delta = 0.8$					
Optimal beq. tax	0.97	0.85	0.5	0	0
Op. wealth. tax	0.66	0.93	0.98	0.48	0.22
Welfare	0.85	0.88	3.97	11.17	27.91
Growth rate	-0.32	0.18	1.17	1.93	2.24
Top 1%	0.01	0.01	0.01	0.103	0.425
$\delta = 0.9$					
Optimal beq. tax	0.97	0.95	0.8	0.49	0
Op. wealth. tax	0.64	0.77	0.95	0.05	0.22
Welfare	0.75	1.49	5.53	13.07	31.24

Growth rate	-0.35	-0.08	1.09	2.01	2.65
Top 1%	0.01	0.01	0.01	0.108	0.425

Table 2: Optimal taxation plans for high bequest rates, δ, and discount factor of the social planner, d.

We observe in Table 1 and 2 that wealth taxes are always extremely high, regardless of the bequest rate, when the social planner's discount factor is close to zero. When the social planner heavily discounts the future, he is almost no concerned about growth. When the social planner discounts the future less, wealth taxes decrease and the wealth share of the top 1% increases.

We see Table 2 that the optimal bequest taxes are positive when the bequest rate is large, $\delta \geq 0.6$, and it also increases as the bequest rate increases. However, it only occurs when the discount factor of the social planner is lower than the bequest rate. Moreover, it is observed that bequest taxes are positively related to the bequest rate. This may be due to the social planner's intention to reduce the proportion of wealth that is given to the next generation.

However, this fact is not present in all cases: for discount factors close to zero, function X is almost identical to the social planner's welfare function, and for discount factors close to one, the social planner is mostly concerned with growth in relative terms. The case where the discount factor is close to zero implies extremely high optimal wealth taxes, and the case where the discount factor is close to one implies extremely low wealth taxes.

For intermediate levels of the discount factor, the importance of inequality is not particularly great and the social planner's optimal bequest rate are quite low compared to the agent's bequest rate. Furthermore, it is observed numerically that the bequest taxes are positive if d/δ is less than 1.

5.3.1. **Additional properties of the optimal taxes**

When the discount factor is large, the social planner has a large concern about future consumption plans. To have a large consumption in the long run, the growth rate must be also large. This implies that the risky technology must be largely used, generating a more disperse wealth distribution. Thus, a social planner who is more concerned about the future chooses a lower tax rate than a social planner who is not. If the social planner is more concerned about distant consumptions, that is, the discount factor moves from d to $d + \epsilon$ with $\epsilon > 0$, functions X and D

do not change. However, the variation of function G is positive and proportionally to $(1+d)/(1-d)^2$. Then, the importance of the growth function in the welfare function increases with the discount factor. This suggest that the optimal taxes are decreasing in d due to the negative correlation between growth and taxes.

The sensibility of the optimal taxes with the discount factor does not imply that the growth rate is necessarily positive or negative. Moreover, Theorem 4 show that if there is a unique tax rate, the existence of a positive or a negative growth rate depends on the discount factor.

Proposition 5. *(Effect on growth and inequality of the social planner's discount factor)* If there is a unique optimal tax rate $\tau: (0,1) \to [0,1] \times [0,1]$ such that $\tau^B(d) = 0$, then

1. There exists $d_2 \in (0,1)$ such that, if $d > d_2$, then $g_{\tau(d)} > 0$. Moreover,
$$\lim_{d\to 1} \tau^W(d) = 0\,, \lim_{d\to 1} g_{\tau(d)} = \delta(R_R/2) - 1 > 0,$$
and
$$\lim_{d\to 1} \tilde{F}^W_{a,\tau(d)}(w) = \lim_{d\to 1} \tilde{F}^W_{l,\tau(d)}(w) = 1 \text{ for } w > 0$$
2. Also, if
$$\delta\left(\frac{R_R/2 + R_S}{2}\right) < 1, \qquad (5.1)$$
there exists $d_1 \in (0,1)$ such that, if $d < d_1$, then $g_{\tau(d)} < 0$. Therefore, the economy collapses. Moreover,
$$\lim_{d\to 0} \tau^W(d) = 1\,, \lim_{d\to 0} g_{\tau(d)} = (\delta/2)\left(\frac{R_R}{2} + R_S\right) - 1 < 0,$$
and
$$\lim_{d\to 0} \tilde{F}^W_{a,\tau(d)}(w) = \lim_{d\to 0} \tilde{F}^W_{l,\tau(d)}(w) = 1 \text{ for all } w.$$

Numerically, when the optimal bequest taxes are zero, Proposition 5 holds. Moreover, the discount factor is negatively related to the optimal wealth taxes and positively related to the growth rate (see Table 1). When the optimal bequest taxes are strictly positive, these properties do not always hold (see Table 2).

6. Extensions

6.1. Model without segmentation

Let us define a model based on Section 2 in which both agents have access to both technologies.

Assumption S2: Risk averse and risk neutral investors have access to the safe technology and to risky one, but not simultaneously.

Note that if an agent decides to invest in one of the technologies, he cannot invest in the other one. This may happen when each agent has a limited capacity to manage investments with quite different type of properties at the same time.

Under these assumptions, risk neutrals will continue investing only in the risky one. From now on, we assume Equation 5.1 and $\tau^B = 0$.

The taxation rate and the level of wealth of the agent affect the optimal production strategy of the risk averters. More precisely, we have that:

Proposition 6. Given a marginal wealth tax rate $\tau^W \in (0,1)$, there is a constant

$$\alpha^*_{\tau^I} = \frac{\tau^W (R_R - 2R_S)}{\delta R_S^2 (1 - \tau^W)} \tag{6.1}$$

such that:

1. if the after-tax wealth y_t^a is such that $y_t^a > \alpha^*_{\tau^W} \overline{y}_{t+1}$, the risk averter invests in the safe technology at date $t + 1$,

2. if $y_t^a \in \left(0, \alpha^*_{\tau^W} \overline{y}_{t+1}\right)$, the risk averter invests in the risky technology at date $t + 1$, and

3. if $y_t^a = \alpha^*_{\tau^W} \overline{y}_{t+1}$, the risk averter is indifferent between both type of investments at date $t + 1$.

Therefore, in the invariant distribution, taxation can have a positive impact on growth since it leads a portion of risk-averse agents to invest in the risky technology. In addition, low tax levels average that the wealth invested in risky technology by risk-averse investors is quite low, which implies a lower growth rate. However, if taxes increase, the gap mentioned in Proposition 5 becomes large and the proportion of wealth invested in risky technology increases.

6.2.Extension to a model with capital, labor, and innovation

Using the model exposed before, we can extend it to a capital, labor, and innovation model as follows. $K(b) = \left(R_R\theta_R(b) + R_S\theta_S(b)\right)\delta b$ for $t \geq 1$ and $K_0^i = 1$ is the amount of capital of the agent i's firm that depreciates completely, $\Theta_{(s^i,k)}^i = \Theta_{s^i}^i\left(\frac{K_{(s^i,k)}^i}{K_{s^i}^i}\right)^{1-\alpha}$ with $\alpha \in (0,1)$ for state s^i at date $t \geq 0$ and $k = 1,2$, and $\Theta_0^i = K_0^i$ is the innovation factor, $L_{s^i}^i \in [0,1]$ is leisure without any utility for it (which implies that $L_{s^i}^i = 1$ for all s^i), r_t is the price of the capital at date t, and w_t is the salary. The technology of the firm i at t is given by

$$y_{s^i}^i = \Theta_{s^i}^i\left(K_{s^i}^i\right)^{\alpha}\left(L_{s^i}^i\right)^{1-\alpha}.$$

The consumer constraint is

$$c_{(s^i,k)} + b_{(s^i,k)} + \frac{\tau^B}{1-\tau^B}\left(b_{(s^i,k)} - B_t\right) \leq w_t^i L_t^i + r_t^i K_t^i - \tau^W\left(w_t^i L_t^i + r_t^i K_t^i - W_t\right).$$

In equilibrium, since the firm has constant returns to scale, $w_{s^i}^i = (1-\alpha)\Theta_{s^i}^i\left(K_{s^i}^i\right)^{\alpha}\left(L_{s^i}^i\right)^{-\alpha}$, $w_{s^i}^i = \alpha\Theta_{s^i}^i\left(K_{s^i}^i\right)^{\alpha-1}\left(L_{s^i}^i\right)^{1-\alpha}$. Therefore, in equilibrium, the consumer problem of each agent is defined as before, and all the results related to the dynamics of wealth are still valid.

7. Conclusions

We developed an overlapping generation model with endogenous growth rate and heterogeneous technology productions. In this model, taxes and redistribution has a negative impact on growth if the more productive technologies involve larger amount of idiosyncratic risk. Moreover, in absence of taxes, the most productive technologies will dominate the economy in the long run and the long run inequality will depend mainly in the risk that it involves. In the presence of taxes, taxes ensure the existence of an invariant distribution of wealth among the agents and an invariant growth rate of the economy. We also showed that there is no poverty trap among the agents with the most productive. Among the agent that do not have access to the most productive technologies, their wealth may not reach the top in any future date.

Redistribute taxes has a negative effect in growth rate and inequality. To establish an optimal taxation, we introduced a central planner that considers the consumption of the agents at equilibrium. We showed that the social welfare function can be written as the sum of three independent functions, one depending on growth, one depending on inequality, and one depending on the difference of the discount factors of the agent and the social planner. The first function is comonotonic with the growth rate of the economy, implying that this function might be an increasing function on taxes. The second one is anticomonotonic with the inequality of the invariant distribution which implies that this function might be a decreasing function on taxes.

We also found that, for a fixed discount rate for every agent in the economy, the optimal taxation is strictly decreasing on how the social planner discounts the future. Moreover, the optimal tax will be such that the invariant wealth distribution tends to an equal one if the social planner strongly discounts the future, and, on the other hand, the optimal tax is zero when the social planner does not discount the future at all. The intuition behind these results is that, if a social planner discounts the future strongly, the weight of distant dates becomes almost irrelevant, and analogously with the growth rate of the economy. Therefore, the social welfare function will be dominated by the inequality effect. However, if a social planner almost does not discount the future, the weight of future consumption will dominate the inequality effect even when both effects are increasing. Additionally, our model suggests that changes in the tax policy may be based on changes on the form of the social planner discounts the future compared to how the other agents do so.

The optimal bequest taxes are positive when the bequest rate is quite large, and the ratio of the discount factor of the social planner with the bequest rate is lower than 1. This is due to the social planner's intention to decrease the proportion of the wealth that is given to the next generation. However, this concern does not occur in all cases, for discount factors close to zero, inequality tends to dominate social planner optimal tax policy, and, for discount factor close to one, the social planner is more concerned about growth. Then, for intermediate levels of the discount factor, the importance of inequality is not particularly dominant, and the social planner's optimal "levels of savings" is quite low compared to the agent actual bequest rate.

8. Bibliography

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Appendix A. Proofs

For any initial distribution of wealth of the risk neutral agents, a small proportion of them concentrates all the wealth in the long run in the absence of taxes. Then, there is only one invariant distribution of wealth. In this invariant distribution, all risk neutral agents have zero wealth.

A.1. Proof of Theorem 1

Let us prove a preliminary result that ensures that for any initial distribution $(w_0^i) \gg 0$, the aggregate wealth in hands of the risk neutral agents over the aggregate net wealth in hands of the risk averse agents, $\frac{\overline{y}_{\tau,t}^l}{\overline{y}_{\tau,t}^a}$, converge to a positive constant.

Lemma 1. For taxes defined by a nonnegative marginal tax rate $\tau = (\tau^W, \tau^B) > 0$ with technology returns such that satisfy Equations 3.1 and 3.2, $\lim_{t\to\infty} \overline{y}_{\tau,t}^l / \overline{y}_{\tau,t}^a = z_\tau$ where $z_\tau \in [1, \infty)$.

Proof. To simplify the proof, we assume that $\tau > 0$. We define z_τ^t as the proportion of the net wealth of the l agents and the a agents with the tax rate τ , $\overline{y}_{\tau,t}^l / \overline{y}_{\tau,t}^a$. Then, we have that

$$z_\tau^{t+1} = \frac{\bar{y}_{\tau,t+1}^{l}}{\bar{y}_{\tau,t+1}^{a}} = f(z_\tau^t) = \frac{\left(1 - \frac{\tau^W}{2} + \frac{\delta\tau^B}{2(1-\delta\tau^B)}\right)(R_R/2)z_\tau^t + \left(\frac{\tau^W}{2} + \frac{\delta\tau^B}{2(1-\delta\tau^B)}\right)R_S}{\left(\frac{\tau^W}{2} + \frac{\delta\tau^B}{2(1-\delta\tau^B)}\right)(R_R/2)z_\tau^t + \left(1 - \frac{\tau^W}{2} + \frac{\delta\tau^B}{2(1-\delta\tau^B)}\right)R_S}.$$

Note that $f(0) > 0$,

$$\lim_{z\to\infty} f(z) = \left(\frac{R_R/2}{R_S}\right)\left(\frac{(2-\tau^W)(1-\delta\tau^B) + \delta\tau^B}{\tau^W(1-\delta\tau^B) + \delta\tau^B}\right),$$

$f'(z) > 0\ \forall z \in (0,\infty)$, $f'(\infty) = 0$, f' is a decreasing function, and

$$f\big([0,\infty)\big) \subseteq \left[0, \left(\frac{R_R/2}{R_S}\right)\left(\frac{(2-\tau^W)(1-\delta\tau^B) + \delta\tau^B}{\tau^W(1-\delta\tau^B) + \delta\tau^B}\right)\right].$$

Then, using the intermediate value function Theorem, the function f has a fixed point. Moreover, there is only one fixed point z_τ such that $f(z_\tau) > 0$, and it is given by

$$z_\tau = \left(\frac{\delta\left(1 - \frac{\tau^W}{2}\right)(1-\delta\tau^B) + \delta\tau^B}{\delta\left(\frac{\tau^W}{2}\right)(1-\delta\tau^B) + \delta\tau^B}\right)\left(1 - \frac{R_S}{R_R/2}\right) + \left(\left(\frac{\delta\left(1 - \frac{\tau^W}{2}\right)(1-\delta\tau^B) + \delta\tau^B}{\delta\left(\frac{\tau^W}{2}\right)(1-\delta\tau^B) + \delta\tau^B}\right)^2\left(1 - \frac{R_S}{R_R/2}\right)^2 + \frac{R_S}{R_R/2}\right)^{1/2} \qquad (A.1)$$

and, for each $z^0 \in (0,\infty)$, $z_\tau^t = \bar{y}_{\tau,t}^{l}/\bar{y}_{\tau,t}^{a}$ converge to z_τ.

From Proof of Lemma 1, z_τ is a C^1 function for $\tau \in (0,1)\times(0,1)$. Additionally, we have that

$$\frac{\partial}{\partial \tau^W} z_\tau$$

$$= -\left(\frac{\delta^2(1-\tau^B\delta)(1+\tau^B\delta)\left(1 - \frac{R_S}{R_R/2}\right)}{2\left(\tau^B\delta + \frac{(\delta(1-\tau^B\delta)\tau^W)}{2}\right)^2}\right)$$

$$- \frac{\delta^2(1-\tau^B\delta)(1+\tau^B\delta)\left(1-\frac{R_S}{R_R/2}\right)^2\left(\tau^B\delta + \delta(1-\tau^B\delta)\left(1-\frac{\tau^W}{2}\right)\right)}{2\left(\tau^B\delta + \frac{(\delta(1-\tau^B\delta)\tau^W)}{2}\right)^3\left(\frac{R_S}{\frac{R_R}{2}} + \frac{\left(\left(1-\frac{R_S}{\frac{R_R}{2}}\right)^2\left(\tau^B\delta + \delta(1-\tau^B\delta)\left(1-\frac{\tau^W}{2}\right)\right)^2\right)}{\left(\tau^B\delta + \frac{(\delta(1-\tau^B\delta)\tau^W)}{2}\right)^2}\right)^{1/2}}$$

and

$$\frac{\partial}{\partial \tau^B} z_\tau$$

$$= -\left(\frac{\left(1 - \frac{R_S}{R_R/2}\right)\delta^2(1-\tau^W)}{\left(\tau^B\delta + \frac{\delta(1-\tau^B\delta)\tau^W}{2}\right)^2}\right)$$

$$- \frac{2\left(1 - \frac{R_S}{R_R/2}\right)^2\left(\tau^B\delta + \delta(1-\tau^B\delta)\left(1-\frac{\tau^W}{2}\right)\right)\delta^2(1-\tau^W)}{\left(\tau^B\delta + \frac{\delta(1-\tau^B\delta)\tau^W}{2}\right)^3 2\left(\frac{R_S}{\frac{R_R}{2}} + \frac{\left(1 - \frac{R_S}{\frac{R_R}{2}}\right)^2\left(\tau^B\delta + \delta(1-\tau^B\delta)\left(1-\frac{\tau^W}{2}\right)\right)^2}{\left(\tau^B\delta + \frac{\delta(1-\tau^B\delta)\tau^W}{2}\right)^2}\right)^{1/2}}.$$

Then, $z_.$ is strictly decreasing in τ^W for any $\tau^B \in [0,1]$, is strictly decreasing in τ^B for any $\tau^W \in [0,1)$, $z_\tau = 1$ for $\tau^W = 1$ and $\tau^B \in [0,1]$, and $z_\tau = \infty$, if and only if $\tau = (0,0)$. In this case, $\frac{\partial}{\partial \tau^W} z_\tau = \frac{\partial}{\partial \tau^B} z_\tau = \infty$.

Since the aggregate production depends on aggregate wealth of each group, the convergence of the ratio of the risk neutrals and risk averters aggregate wealth ensures the convergence of the growth path.

Lemma 2. For any fixed tax rate $\tau = (\tau^W, \tau^B) > 0$, the growth rate of the economy, $g_{\tau,t}$ converges to g_τ when t goes to infinity where

$$g_\tau = \frac{\left(\frac{(R_R/2)\delta(1-\tau^B)}{1-\delta\tau^B} - 1\right) z_\tau + \left(\frac{R_S\delta(1-\tau^B)}{1-\delta\tau^B} - 1\right)}{z_\tau + 1}.$$

Proof of Lemma 2. Since

$$g_{\tau,t} = \frac{\left(\frac{(R_R/2)\delta(1-\tau^B)}{1-\delta\tau^B} - 1\right) \bar{y}_t^l + \left(\frac{R_S\delta(1-\tau^B)}{1-\delta\tau^B} - 1\right) \bar{y}_t^a}{\bar{y}_t^l + \bar{y}_t^a}$$

$$= \frac{\left(\frac{(R_R/2)\delta(1-\tau^B)}{1-\delta\tau^B} - 1\right) z_\tau^t + \left(\frac{R_S\delta(1-\tau^B)}{1-\delta\tau^B} - 1\right)}{z_\tau^t + 1},$$

and z_τ^t converges to z_τ when $t \to \infty$, we obtain that $g_{\tau,t}$ converges to g_τ, which concludes the proof.

Due to the convergence of how each group invest in each technology, the growth rate of the economy will also converge. Then, the proportion of wealth of the poorest risk neutral agent converges, which implies that the proportion of wealth of a risk neutral agent that has received at least once the lower return $\underline{R}_R = 0$ also converges.

Proof of Theorem 1. Let us define after-tax wealth distributions $\tilde{F}_{a,\tau}^W(w)$ and $\tilde{F}_{l,\tau}^W(w)$. The distribution for the risk averters is a constant distribution with the level of wealth x_τ^a given by

$$x_\tau^a = \sum_{k=0}^{\infty} \left(\frac{\frac{1}{2}\left(\tau^W + \frac{\delta\tau^B}{(1-\delta\tau^B)}\right) R_S^k (1-\tau^W)^k \frac{\delta^k (1-\tau^B)^k}{(1-\delta\tau^B)^k}}{(1+g_\tau)^k} \right),$$

and the distribution for the risk averters is

$$x_\tau^{l,n} = \sum_{k=0}^{n-1} \frac{\frac{1}{2}\left(\tau^W + \frac{\delta\tau^B}{(1-\delta\tau^B)}\right) R_R^k (1-\tau^W)^k \frac{\delta^k (1-\tau^B)^k}{(1-\delta\tau^B)^k}}{(1+g_\tau)^k}$$

for the n^{th} poorest group of risk neutral agents with weight $1/2^{n+1}$ for $n \in \mathbb{N}$.

Since $\bar{x}_\tau^l / \bar{x}_\tau^a = z_\tau$ in this case, the distribution functions $\tilde{F}_{a,\tau}^W(w)$ and $\tilde{F}_{l,\tau}^W(w)$ are invariant distribution of after-tax wealth with the invariant growth rate g_τ.

Let us suppose that the initial distribution of after taxes wealth are $\tilde{F}_{a,\tau,0}^W(w)$ and $\tilde{F}_{l,\tau,0}^W(w)$. Given at date t, for the risk averse agents, the proportion of the wealth is given by

$$x_{\tau,t}^a = \sum_{k=0}^{t-1} \left(\frac{\frac{1}{2}\left(\tau^W + \frac{\delta\tau^B}{(1-\delta\tau^B)}\right) R_S^k (1-\tau^W)^k \frac{\delta^k (1-\tau^B)^k}{(1-\delta\tau^B)^k}}{\prod_{l=1}^k (1+g_{\tau,t-l})} \right)$$

$$+ (1-\tau^W)^t R_S^t \frac{\delta^t (1-\tau^B)^t}{(1-\delta\tau^B)^t} \frac{w_0^{a_i}}{\left(\prod_{k=1}^t (1+g_{\tau,t-k})\right)\bar{w}_0}. \qquad \text{(A.3)}$$

Since $R_S < R_R/2$, we have that

$$(1-\tau^W)\, R_S \frac{\delta\,(1-\tau^B)}{(1-\delta\tau^B)} < 1 + g_{\tau,t}$$

for all $t \geq 0$, which proves that $x_{\tau,t}^a \to x_\tau^{a_i}$ when $t \to \infty$.

The proportion of the wealth of the poorest l agents is $x_{\tau,t}^l = (\tau^W + \delta\tau^B/(1-\delta\tau^B))/2 = x_\tau^{l,1}$, and the weight of this group is 1/2. The wealth of the second poorest group of l agents only depends on the average wealth and the wealth of the poorest l agents in the previous period. Therefore, the proportion of the second poorest group of l agents is

$$x_{\tau,t}^l = \frac{\frac{1}{2}\left(\tau^W + \frac{\delta\tau^B}{1-\delta\tau^B}\right) R_R (1-\tau^W)\ \frac{\delta\,(1-\tau^B)}{(1-\delta\tau^B)}}{\left(1+g_{\tau,t-1}\right)} + \frac{1}{2}\left(\tau^W + \frac{\delta\tau^B}{1-\delta\tau^B}\right),$$

and its weight is 1/4. If we continue this process, we obtained that proportion of the n^{th} poorest group of l agents is

$$x_{\tau,s}^l = \sum_{k=0}^{n-1} \frac{\frac{1}{2}\left(\tau^W + \frac{\delta\tau^B}{(1-\delta\tau^B)}\right) R_R^k (1-\tau^W)^k \frac{\delta^k (1-\tau^B)^k}{(1-\delta\tau^B)^k}}{\prod_{j=1}^k \left(1+g_{\tau,t-k}\right)}, \qquad \text{(A.2)}$$

and the weight of this group is $1/2^{n+1}$ for $t \geq n$. Since $z_\tau^t \to z_\tau \in [1,\infty)$; when $t \to \infty$, $g_{\tau,t} \to g_\tau$ when $t \to \infty$. Then, $x_{\tau,t}^l \to x_\tau^l$ when $t \to \infty$, which concludes the proof.

A.2. Proof of Theorem 2

We denote the consumption and bequest in equilibrium in the invariant after-tax wealth distributions $\tilde{F}^W_{a,\tau}$ and $\tilde{F}^W_{l,\tau}$ at the tax rate τ as $\tilde{c}_{a,t}(w), \tilde{b}'_{a,t}(w)$. Using Theorem 1, the equilibrium consumption and bequest plan $\tilde{c}_{a,t}(w), \tilde{b}'_{a,t}(w)$ at date t and tax rate τ can be written as

$$\left(\tilde{c}_{a,t}(w), \tilde{b}'_{a,t}(w)\right) = \left(\left(\frac{(1-\delta)}{1-\delta\tau^B}\right) w(1+g_\tau)^t, \left(\frac{\delta(1-\tau^B)}{1-\delta\tau^B}\right) w(1+g_\tau)^t\right).$$

Using the properties of the logarithm and the form of the invariant distribution of wealth, the welfare function can be separated in three parts:

$$W(c,b') = W\left(\tilde{c},\tilde{b}'\right) = \int \log(w)\, d\tilde{F}^W_{a,\tau}(w) + \int_{x_\tau^{l,2}}^{\infty} \log\left(\frac{1}{2}\left(w + x_\tau^{l,1}\right)\right) d\tilde{F}^W_{l,\tau}(w)$$

$$= \log x_\tau^a + \sum_{n=2}^{\infty} \frac{1}{2^{n-1}}\left(\log\left(\frac{1}{2}\left(x_\tau^{l,n} + x_\tau^{l,1}\right)\right)\right) + \frac{d}{1-d}\log(1+g_\tau)$$

$$+ \log\left(\left(\frac{\delta^\delta(1-\delta)^{1-\delta}(1-\tau^B)}{1-\delta\tau^B}\right)\right). \qquad (4.2)$$

Proof of Theorem 2. Therefore, the welfare function can be rewritten as

$$
\begin{aligned}
W(c,b') &= W(\tilde{c},\tilde{b}') \\
&= (1 - d)\left(\sum_{t=0}^{\infty} d^t \left(\int U^a\left(\tilde{c}_{a,t}(w),\tilde{b}'_{a,t}(w)\right) d\tilde{F}^W_{a,\tau}(w) + \int \log U^a\left(\tilde{c}_{a,t}(w),\tilde{b}'_{a,t}(w)\right) dF^W_{l,\tau}(w)\right)\right) \\
&= (1-d)\sum_{t=0}^{\infty} d^t \sum_{n=2}^{\infty} \frac{\delta}{2^{n-1}} \log\left(\left(\frac{\delta(1-\tau^B)}{1-\delta\tau^B}\right)\frac{1}{2}\left(x_\tau^{l,n} + x_\tau^{l,0}\right)(1+g_\tau)^t\right) \\
&+ (1-d)\sum_{t=0}^{\infty} d^t \sum_{n=2}^{\infty} \frac{1-\delta}{2^{n-1}} \log\left(\left(\frac{(1-\delta)(1-\tau^B)}{1-\delta\tau^B}\right)\frac{1}{2}\left(x_\tau^{l,n} + x_\tau^{l,1}\right)(1+g_\tau)^t\right) \\
&+ (1-d)\sum_{t=0}^{\infty} d^t \delta \log\left(\left(\frac{\delta(1-\tau^B)}{1-\delta\tau^B}\right)(x_\tau^a)(1+g_\tau)^t\right) \\
&+ (1-d)\sum_{t=0}^{\infty} d^t (1-\delta) \log\left(\left(\frac{(1-\delta)}{1-\delta\tau^B}\right)(x_\tau^a)(1+g_\tau)^t\right).
\end{aligned}
$$

Then,

$$
\begin{aligned}
W(c,b') &= \log x_\tau^a + \sum_{n=1}^{\infty} \frac{1}{2^{n-1}}\left(\log\left(\frac{1}{2}\left(x_\tau^{l,n} + x_\tau^{l,1}\right)\right)\right) + (1-d)\sum_{t=0}^{\infty} d^t t \log(1+g_\tau) \\
&+ \log\left(\left(\frac{\delta^\delta(1-\delta)^{1-\delta}(1-\tau^B)}{1-\delta\tau^B}\right)\right) \\
&= \sum_{n=2}^{\infty} \frac{1}{2^{n-1}}\left(\log\left(\frac{1}{2}\left(x_\tau^{l,n} + x_\tau^{l,1}\right)\right)\right) + \log x_\tau^a + \frac{d}{1-d}\log(1+g_\tau) \\
&+ \log\left(\left(\frac{\delta^\delta(1-\delta)^{1-\delta}(1-\tau^B)}{1-\delta\tau^B}\right)\right)
\end{aligned}
$$

Due to Equation 4.2, we can define $X(\cdot)$ as

$$
X(\tau) := \log x_\tau^a + \sum_{n=2}^{\infty} \frac{1}{2^{n-1}}\left(\log\left(\frac{1}{2}\left(x_\tau^{l,n} + x_\tau^{l,1}\right)\right)\right),
$$

$G(\cdot,\cdot)$ as $G(d,\tau) \coloneqq d/(1-d)\log(1+g_\tau)$ which is clearly a decreasing function in τ, and $D(\cdot)$ as

$$D(\tau^B) \coloneqq \log\left(\left(\frac{\delta^\delta(1-\delta)^{1-\delta}(1-\tau^B)}{1-\delta\tau^B}\right)\right).$$

In this case, each function depends directly or indirectly on δ since the bequest rate affects the distribution of the invariant distribution and the growth rate of the economy by increasing inequality and the growth rate when δ increases.

The properties of G are a consequence of the properties of the invariant growth explained in the proof of Theorem 1.

Additionally, $X(\tau)$ can be written as

$$X(\tau) := \sum_{n=2}^{\infty} \frac{1}{2^{n-1}}\left(\log\left(\frac{1}{2}\left(\sum\nolimits_{k=0}^{n-1} \frac{\frac{1}{2}\left(\tau^W + \frac{\delta\tau^B}{(1-\delta\tau^B)}\right) R_R^k (1-\tau^W)^k \frac{\delta^k(1-\tau^B)^k}{(1-\delta\tau^B)^k}}{(1+g_\tau)^k} + \tau^W + \frac{\delta\tau^B}{(1-\delta\tau^B)}\right)\right)\right) + \log\left(\sum\nolimits_{k=0}^{\infty}\left(\frac{\frac{1}{2}\left(\tau^W + \frac{\delta\tau^B}{(1-\delta\tau^B)}\right) R_S^k (1-\tau^W)^k \frac{\delta^k(1-\tau^B)^k}{(1-\delta\tau^B)^k}}{(1+g_\tau)^k}\right)\right).$$

Since $\sum_{n=1}^{\infty}(1/2^n)x_\tau^{l,n} + x_\tau^a = 1$, Jensen's inequality and the dominated convergence theorem ensure that $X(\tau) < 0$ for $\tau^I \in (0,1)$ and $\tau^B \in [0,1)$, and function X is a continuous function in $\tau \in (0,1)^2$. For $\tau^W = 1$, we have that $X(\tau) = 0$. Then, function attains its maximum value when $\tau^W = 1$.

A.3. Proof of Proposition 4

Let us prove a preliminary result that establishes a relationship between different tax policies with small tax rates.

Proposition 7. Any taxation plan $\bar{\tau}$ with positive bequest taxes is dominated by any taxation $\bar{\tau}'$ such that $1 \geq \tau'^W > \tau^W + \delta\tau^B/(1-\delta\tau^B)$ and $\tau'^B = 0$.

Proof of Proposition 7. Consider a taxation plan $\tau' \in \mathcal{T}$ such that $1 > \tau'^W > \tau^W + \delta\tau^B/(1-\delta\tau^B)$. Since we analyze the welfare function in the invariant distribution, we can assume that $\bar{y}_0 = 1$ in both cases.

For the poorest *risk neutral agents*, we have that their level of after tax income with the taxation plan (τ^W, τ^B) is given by $\tau^W = \bar{y}_0\tau^W + \bar{y}_0(\tau^B/(1-\tau^B))\big(\delta(1-\tau^B)/(1-\delta\tau^B)\big) = \bar{y}_0\big(\tau^W + \delta\tau^B/(1-\delta\tau^B)\big) < \bar{y}_0\tau'^W = \tau'^W$ which is the after tax income with the new taxation plan.

Note that that since the function $f_k(x) = x/(1-x)^k$ is an increasing function for $x \in [0,1)$ for all $k, l \in \mathbb{N}$ and $\tau^W < \tau'^W$, we have that $\tau^W(1-\tau^W)^k(1-\tau^B)^l < \tau^W(1-\tau^W)^k < \tau'^W(1-\tau'^W)^k$. For the second poorest group of risk neutral agents, we have that their level of after tax income with the taxation plan (τ^W, τ^B) is given by

$$\begin{aligned}\big(R_R\tau^W\bar{y}_0\delta(1-\tau^B)\big)(1-\tau^W) + \bar{y}_0\tau^W + \left(\frac{\bar{y}_0\tau^B}{1-\tau^B}\right)\left(\frac{\delta(1-\tau^B)}{1-\delta\tau^B}\right) \\ = \bar{y}_0\left(R_R\delta\tau^W(1-\tau^B)(1-\tau^W) + \tau^W + \frac{\tau^B\delta}{1-\delta\tau^B}\right) \\ < \bar{y}_0(R_R\delta\tau'^W(1-\tau'^W) + \tau'^W).\end{aligned}$$

For the *n*-poorest group of risk neutral agents, we have

$$\sum_{k=0}^{n-1}\big(R_R^k\delta^k(1-\tau^B)^k\big)(1-\tau^W)^k\left(\tau^W\bar{y}_0 + \frac{\tau^B\delta}{1-\delta\tau^B}\bar{y}_0\right) = \bar{y}_0\left(\sum_{k=0}^{n-1}\left(R_R^k\delta^k(1-\tau^B)^k(1-\tau^W)^k\left(\tau^W + \frac{\tau^B\delta}{1-\delta\tau^B}\right)\right)\right) < \bar{y}_0\left(\sum_{k=0}^{n-1}\left(R_R^k\delta^k(1-\tau^W)^k\left(\tau^W + \frac{\tau^B\delta}{1-\delta\tau^B}\right)\right)\right) <$$

$$\bar{y}_0\left(\sum_{k=0}^{n-1}\left(R_R^k\delta^k(1-\tau^W)^k\left(\tau^W + \frac{\tau^B\delta}{1-\delta\tau^B}\right)\right)\right) < \bar{y}_0\left(\sum_{k=0}^{n-1}\big(R_R^k\delta^k(1-\tau'^W)^k\tau'^W\big)\right).$$

Then, the income of the invariant distribution in each period is always lower with the taxation plan (τ^W, τ^B) than with $(\tau'^W, 0)$. For the risk averse agents, the result is also true because of the convergence of an analogous series as the one described above. To conclude the proof, notice that the utility of a risk averse agent with an after-taxes wealth w is

$$u^a\left(\tilde{c}(w/\bar{y}_t),\tilde{b}'(w/\bar{y}_t)\right) = \log\left(\left(\tilde{c}(w/\bar{y}_t)\right)^{1-\delta}\left(\tilde{b}'(w/\bar{y}_t)\right)^{\delta}\right)$$
$$= \log\left(\left((1-\delta)w\right)^{1-\delta}(\delta(1-\tau^B)w)^{\delta}\right) = \log\left(\left((1-\delta)\right)^{1-\delta}\delta^{\delta}(1-\tau^B)^{\delta}w\right)$$
$$< \log\left(\left((1-\delta)\right)^{1-\delta}\delta^{\delta}w\right).$$

Proof of Proposition 4. From Proof of Theorem 1, we have that g_τ is a C^1 function for $\tau \in (0,1)\times(0,1)$, strictly decreasing in τ^W for any $\tau^B \in [0,1]$, strictly decreasing in τ^B for any $\tau^W \in [0,1)$, $g_\tau = 0$ for $\tau^B = 1$. Then, function $G(d,\cdot)$ attains its maximum value when $\tau = 0$.

Due to Equation 4.2 and the form of the invariant distribution, if d goes to 1, the welfare function converges to function G uniformly in τ when taxes are bounded away from zero and one. Then, optimal taxes d goes to 1 converges to zero, that is, $(0,0)$. Then, there is $\bar{\delta} \in (0,1/2)$, $\underline{d} \in (0,1)$ such that any optimal tax rate τ satisfies that $1 > \tau^W + \delta/(1-\delta) \geq \tau^W + \delta\tau^B/(1-\delta\tau^B)$ for $\delta \leq \bar{\delta}$ and $d \geq \underline{d}$. Finally, using Proposition 6 we conclude that $\tau^B = 0$, which concludes the proof.

A.4. Proof of Proposition 5

Proof of Proposition 5. Note that function D is constant when $\tau^B = 0$. The first part holds since when d goes to 1, the welfare function $(1-d)W$ converges to function $(1-d)G$ uniformly in τ^W when taxes are bounded away from zero. Then, the optimal tax rate converges to zero. Since $R_R/2 > 1/\delta$, for d large enough, the invariant growth rate is positive, which concludes the first part.

To prove the second part, it is enough to analyze asymptotic behavior of W when d goes to zero. When d goes to zero, W converges to X uniformly in τ^W when taxes are bounded away from zero. Then, optimal taxes converge to 1. Using Equation 5.1, we obtain that the growth rate is negative if d is small enough, which concludes the second part.

A.5. Other proofs

Proof of Proposition 6. We denote w_t^{*a} as the levels of wealth after-taxes for the risk averters such that the agent is indifferent between investing in the safe technology and investing in the risky one in period $t+1$, that is,

$$\log(\delta w_t^{*a} R_S(1-\tau^W) + \tau^W \bar{y}_{t+1}) = \frac{1}{2}(\log(\delta w_t^{*a} R_R(1-\tau^W) + \tau^W \bar{y}_{t+1}) + \log(\tau^W \bar{y}_{t+1})),$$

$$\log\left(\delta \frac{w_t^{*a}}{\bar{y}_{t+1}} R_S(1-\tau^W) + \tau^W\right) = \frac{1}{2}\left(\log\left(\delta \frac{w_t^{*a}}{\bar{y}_{t+1}} \bar{R}_R(1-\tau^W) + \tau^W\right) + \log(\tau^W)\right),$$

$$\left(\delta \frac{w_t^{*a}}{\bar{y}_{t+1}} R_S(1-\tau^W) + \tau^W\right)^2 = \tau^W \left(\delta \frac{w_t^{*a}}{\bar{y}_{t+1}} R_R(1-\tau^W) + \tau^W\right),$$

Then, we have that

$$\frac{w_t^{*a}}{\bar{y}_{t+1}} = \frac{\tau^W(R_R - 2R_S)}{\delta R_S^2(1-\tau^W)}$$

which concludes the proof.

Appendix B. Model without segmentation and an effort cost

Let us define a model based on Section 2 only with income taxes in which both agents have access to both technologies, but each agent that decides to invest in the risky technology will have an effort cost, $L \geq 0$, a fixed effort cost that the agent must take to have a positive probability of wining the highest return. In absence of this cost, the agent will have a null return in the next period. Therefore, if an agent decides to invest in the risky one, it is always optimal to pay the effort cost. In this section, $\tau^B = 0$ and $\tau^W = \tau$.

Assumption E1: Investments of the risk technology have constant effort costs L_α for the risk averse agents and L_β for the risk neutral agents. This effort costs are paid if R_R occurs.

B.1. Effort cost for risk-averse investors

Note that these costs reduce the return of the risky technology. If L_α is small, some risk averse agents will continue investing in the risky one, but the proportion of agent willing to invest will decrease. If L_β is small, risk neutral agents will continue investing as before. However, if L_β

or the income taxes are large, the poorest risk neutrals do not have incentives to invest in the risky technology. The following propositions analyze these cases.

Proposition 8. Given a marginal taxation rate $\tau \in (0,1)$, there is a constant

$$\alpha^*_{\pm,\tau} = \frac{\tau\left((R_R - 2R_S\, exp(L_\alpha)) \pm \sqrt{(R_R - 2R_S\, exp(L_\alpha))^2 - 4(exp(L_\alpha) - 1)R_S^2}\right)}{\delta R_S^2 (1-\tau)} \quad \text{(A.4)}$$

such that:

1. if the after-tax wealth y_t^a is such that $y_t^a > \alpha^*_{+,\tau}\overline{y}_{t+1}$ or $y_t^a < \alpha^*_{-,\tau}\overline{y}_{t+1}$, the risk averter invests in the safe technology at date $t+1$,

2. if $y_t^a \in \left(\alpha^*_{-,\tau}\overline{y}_{t+1}, \alpha^*_{+,\tau}\overline{y}_{t+1}\right)$, the risk averter invests in the risky technology at date $t+1$, and

3. if $y_t^a = \alpha^*_{\pm,\tau}\overline{y}_{t+1}$, the risk averter is indifferent between both type of technologies at date $t+1$.

Proof of Proposition 8. It is analogous to the proof of Proposition 6.

B.2. Effort cost for risk neutral agents

For risk neutral agents, we have the following result.

Proposition 9. Given a marginal taxation rate $\tau \in (0,1)$ and the aggregate wealth in $t+1$, $\overline{y}_{t+1}$, there is a constant $\beta^*_{t+1} = \frac{L_\beta}{(R_R - 2R_S)\bar{y}_{t+1}}$ such that:

1. if the after-tax wealth y_t^l is such that $y_t^l < \beta^*_{t+1}\overline{y}_{t+1} = \frac{L_\beta}{(R_R - 2R_S)}$, the risk neutral agent invests in the safe technology at date $t+1$,

2. if $y_t^l > \beta^*_{t+1}\overline{y}_{t+1}$, the risk neutral agent invests in the risky technology at date $t+1$, and

3. if $y_t^l = \beta^*_{t+1}\overline{y}_{t+1}$, the risk neutral agent is indifferent between both type of technologies at date $t+1$.

Proof of Proposition 9. It is analogous to the proof of Proposition 6.

Note that, if the economy has a negative growth rate for some periods in a row, it might cause that the economy collapses in the long run since all risk neutral agents invest in the safe technology eventually due to β_t^* going to infinity. On the other hand, the risk-averse agents might not invest in the risky technology due to Inada condition. In Subsection B.3, we will show this numerically.

If the risk neutral agents are risk lovers with a utility index given by $u(c,b) = \left(c^{1-\delta}b^{\delta}\right)^2$, all risk lover agents will consider not only the return of the technologies and the effort cost, but also consider taxes since they can reduce the amount of risk that they are taking.

Proposition 10. Given a marginal taxation rate $\tau \in (0,1)$ and the aggregate wealth in $t+1$, $\overline{y}_{t+1}$, there is a constant

$$\beta_{t+1,\tau}^{**} = \frac{-(R_R - 2R_S)\tau + \sqrt{(R_R - 2R_S)^2\tau^2 + \frac{L_\beta}{\overline{y}_{t+1}^2}(R_R^2 - 2R_S^2)}}{\delta(R_R^2 - 2R_S^2)(1-\tau)} \tag{A.5}$$

such that:

1. if the after-tax wealth y_t^l is such that $y_t^l < \beta_{t+1,\tau}^{**}\overline{y}_{t+1}$, the risk lover agent invests in the safe technology at date $t+1$,
2. if $y_t^l > \beta_{t+1,\tau}^{**}\overline{y}_{t+1}$, the risk lover agent invests in the risky technology at date $t+1$, and
3. if $y_t^l = \beta_{t+1,\tau}^{**}\overline{y}_{t+1}$, the risk lover agent is indifferent between both type of technologies at date $t+1$.

Proof of Proposition 10. It is analogous to the proof of Proposition 6.

B.3. Invariant Distribution

Due to Proposition 8, Proposition 9, and Proposition 10, we can ensure that a process similar to the one made in the other models can be done in this case since the wealth of each risk averter (more precisely, almost every risk averter) can be computed recursively.

Numerically, it requires only to compute a preliminary proportion of agents that invest in each technology in each period t by using an increasing function $\alpha_{\cdot}^{*}$ defined above. Then, we compute $\alpha_{\tau}^{*}\frac{\bar{y}_{t+1}}{\bar{y}_{t}}$ and the distribution of wealth step by step. Now, we restart the process with the proportion of investment induced by the distribution that we have just found.

By doing this, we can find the invariant distribution of wealth invested in each technology, and, therefore, the invariant growth rate of the economy, which are the only things that we need to know the invariant concentration of wealth among the agents.

Proposition 11. Given a positive marginal wealth tax rates τ, the growth rate $g_{\tau,t}$ converges to g_{τ} when $t \to \infty$. Moreover, there is an invariant distribution such that the growth rate is equal to g_{τ}.

This result implies the existence of the invariant concentration of wealth, and it also suggests that if the distribution is considerably close to the invariant one, it converges in the long run to the invariant distribution. In the following subsection, we explore numerically the invariant distribution.

B.2. Numerical examples

Based on the numerical examples defined before, we consider $R_R = 4.5$, $R_S = 1.6$, $\delta = 0.5$, $L = 0.04879$. We start with a distribution constant distribution of wealth among the agents.

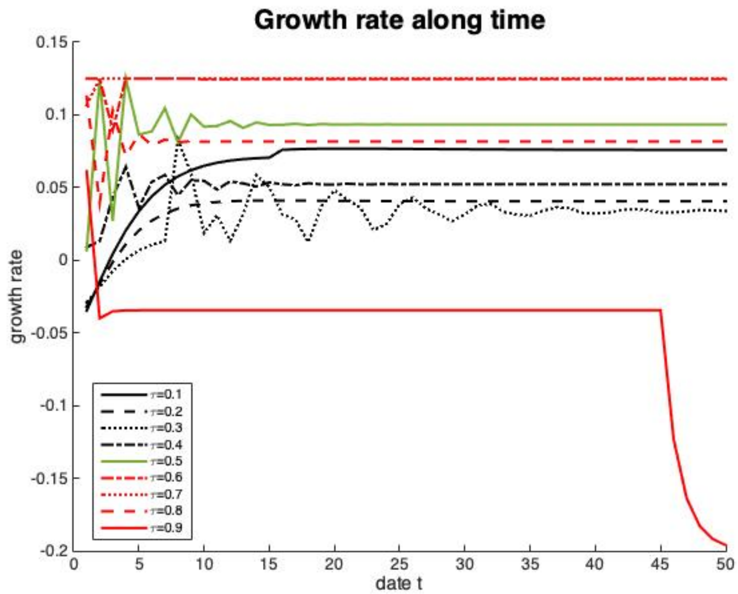

Figure 6: Growth rate over time for different income tax rates.

In this case, increments on taxes generate different effects on growth depending on the marginal tax rate that we start on. For very low levels of taxes, increment in the tax rates might decrease the growth rate due to the transfers from risk neutrals to risk averters. The risk lovers are investing completely in the risky and more productive type of investment, and the risk averters are investing part in the risky and part in the safe investment. Therefore, the increment in the marginal taxation rate implies less investments in the risky and more investments in the safe one.

For marginal tax rates between 0.3 and 0.5, the growth rate increases when the marginal tax rate increases. In this case, the risk averse agents are investing considerably more in the risky than in the safe one since they need a larger number of successful periods investing in the risky investment to reach the indifference threshold, $\alpha^*_{+,\tau}$. Intuitively, the threshold being attainable for the risk averters averages that the insurance effect caused by taxes is observed. Some risk averters decide to invest in a risky and more productive type of investment because the government ensures that the agent will receive minimum level of wealth if the investment does not give any return. In this case, given an increment of the marginal tax rate, the proportion of agents that decide to invest in the risky one compensates the transfers of wealth from the risk lovers to the risk averters who decide to invest in the safe one.

For marginal taxation rate slightly above 0.5, the wealth distribution is bounded. Moreover, there is always an upper bound depending on the aggregate wealth that is unattainable for any agent, including the ones who always invest in the risky investment and succeed. Therefore, there is $\tau \in (0.5,0.6)$ the lowest positive marginal taxation rate such that the growth rate in the long run is maximum, that is, $g_{\tau,t} \rightarrow R_R\delta/2 - 1 = 0.125 = 12.5$ when $t \rightarrow \infty$.

When the marginal taxation rate is larger than 0.75, the poorest risk averters satisfies that $y_t^{a_i} = \tau\overline{y}_t \approx \alpha^*_{-,\tau}\overline{y}_{t+1}$ therefore, an increment on the marginal taxation rate will imply that these agents decide to invest in the safe and less productive investment implying reductions in the growth rate. This phenomenon continues until the growth rate is positive. Once the marginal tax rate is such that the growth rate in the long run is slightly negative, we are under the conditions in which the poorest risk lovers start to invest in the safe one reducing even more the growth rate which makes that more agents (both, risk averters and risk lovers) switch to the safe one. Then, in the long run, almost all agents invest in the safe one instead of the risky one implying that the economy has the lowest growth rate possible, that is, $g_{\tau,t} \rightarrow R_S\delta - 1 = -0.2 = -20\%$ when $t \rightarrow \infty$.

Note that, based on Figure 6, the economy converges to a constant growth rate in the long run for all the marginal tax rates analyzed. Moreover, we observed that for almost all the marginal tax rates analyzed, the convergence of the growth rate holds.

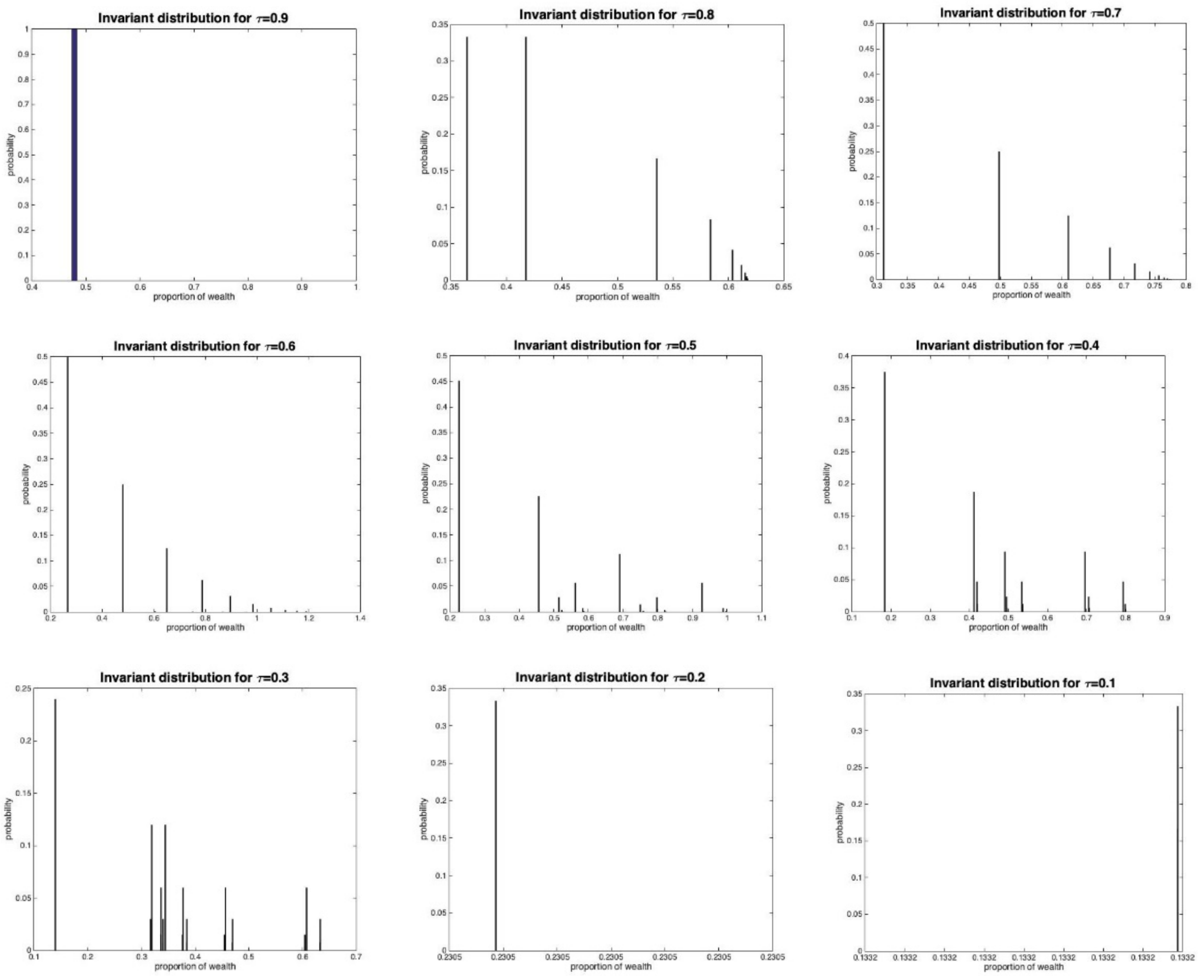

Figure 7: Invariant distribution of wealth for the risk averters for different income tax rates $\tau = 0.9, 0.8, 0.7, 0.6, 0.5, 0.4, 0.3, 0.2, 0.1$.

From Figure 7, we noticed that for low marginal tax rate, all risk lover agents invest in the safe one implying a constant distribution. When $\tau = 0.3$, 0.4, and 0.5 the averter agents invest in the risky one when they are poor and in the safe one when they are above the threshold $\alpha^*_{+,\tau}$. The biggest difference between these two distributions is that the threshold is considerably higher when $\tau = 0.4$ (moreover when $\tau = 0.5$) implying that a larger proportion of wealth is being invested in the risky in this case. This causes the phenomenon mentioned before, an increment on the growth rate. This happens because this effect overcome the transfers made from the risk lover agents to the risk averters that invest in the safe one a proportion of agents that is small in this case and decreases every time that the marginal tax rate increases.

When $\tau = 0.6$ and 0.7, all risk averters invest in the risky one all the time since there is no over-accumulation of wealth (fat tails) in this case. Therefore, a risk averse agent that is infinitely successful by investing in the risky one has a wealth in period t bounded by $2.6\overline{y}_{t+1}$ for $\tau = 0.6$, and $2.6\overline{y}_{t+1}$ for $\tau = 0.7$.

When $\tau = 0.8$, the poorest risk averse agents, the ones that their parents only received the transfers in the previous period, the invest in the safe one. However, once they invest in the safe one, the transfers made by taxes increases the wealth of their successors in such a way that they decide to invest in the risky one generating the invariant distribution observed. This phenomenon continues making that a larger proportion of risk averters ones decide to invest in the safe one instead of the risky one. However, once the growth rate is negative for a long number of periods, the risk lovers will switch too, implying that the invariant distribution is even lower. In this case ($\tau = 0.9$), all risk lovers and all risk averters will eventually invest only in the safe one implying that the invariant distribution is constant.