Econ3101 - Section 006
Intermediate Microeconomics

Xavier Vinyals-Mirabent
Department of Economics
University of Minnesota.

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Solutions to Homework 3.
1. Consider a perfectly competitive industry where each (identical) rm has the following total cost function:

\[ C(q) = 1 + 225q^2 \]

The market demand facing the industry is as follows:

\[ Q^d(p) = (100 - p) \frac{34}{525} \]

where \( p \) is the market price. Circle your final answer!

(a) Define a competitive equilibrium in the Short Run.

A CE is a price and an allocation such that given price all agents act optimally and market clears.

(b) Following that definition, compute the equilibrium if the number of firms equals \( N = 153 \).

\[ p = 450 \frac{q}{q^*} \quad \Leftrightarrow \quad q^* = \frac{p}{450} \]

\[ Q^d(p) = (100 - p) \frac{34/525}{525} = 153 \cdot \frac{p}{450} = N \cdot \frac{q^*}{q^*} = Q^d(p) \quad \Rightarrow \quad \frac{100 \cdot 34/525}{525} = \left( \frac{153}{450} + \frac{34}{525} \right) p \]

\[ \Leftrightarrow p^* = 16 \quad \Rightarrow \quad q^* = \frac{16}{450} = \frac{8}{225} \]

(c) Compute the firm’s individual profit and predict whether the number of firms will increase, decrease or stay the same in the Long Run.

\[ \Pi(q^*) = 16 \cdot \frac{8}{225} - (1 + 225(\frac{8}{225})^2) = -0.7156 \quad \Rightarrow \quad N \text{ should decrease} \]

(d) Define a competitive equilibrium in the Long Run.

A CE in the long run is a price, an allocation and a number of firms in the market such that agents act optimally, firms don't want to enter/exit the market and the market clears.

(e) Following that definition, compute the equilibrium in the Long Run.

\[ p = 450 \frac{q}{q^*} \quad \Rightarrow \quad 450 q^2 - (1 + 225 q^2) = 0 \quad \Rightarrow \quad 225 q^2 = 1 \quad \Rightarrow \quad q^* = \frac{1}{15} \]

\[ p \cdot q - (1 + 225 q^2) = 0 \quad \Rightarrow \quad \left[ \frac{p^*}{p} = 450 \cdot \frac{1}{15} = 30 \right] \]

\[ (100 - p^*) \frac{34/525}{525} = N^* \cdot \frac{q^*}{q^*} \quad \Rightarrow \quad (100 - 30) \frac{34/525}{525} = N^* \cdot \frac{1}{15} \quad \Rightarrow \quad N^* = 68 \]
2. Now suppose that the government levies a lump sum tax on all rms in the market of the previous exercise (i.e. use the previous information on demand and costs).

(a) Write down the new cost function. Then compute and compare the marginal cost before and after the implementation of the lump sum tax. Do the same for the average total cost. Conclude whether the short run and/or the long run equilibria will change. Support your conclusion relating it to the changes in cost!

\[
\hat{C}(q) = C(q) + T = 1 + 225q^2 + T
\]

\[
\hat{MC}(q) = \frac{\partial \hat{C}(q)}{\partial q} = 450q
\]

\[
\hat{ATC}(q) = \frac{\hat{C}(q)}{q} = 1 + 225q^2 + \frac{T}{q}
\]

\[
\rightarrow \text{Since } \hat{MC}(q) = MC(q) \text{ the firm's behavior doesn't change in the short run, and will only be affected in the long run through } T \geq 0.
\]

(b) Suppose that the number of rms in the industry is equal to the long run \( N^* \) found in the previous exercise. Find the short run equilibrium and the equilibrium individual profit. Circle your final answers!

The answer here is identical to that in 1e) since we're using the long run number of firms we found there (\( N^* = 68 \)).

\[
\begin{align*}
\text{.1) } & p = \frac{450}{q^*} \\
\text{.2) } & (100 - p^*) \frac{34}{525} = 68 \cdot q^* \\
\end{align*}
\]

\[
\rightarrow q^* = \frac{45}{15} \quad p^* = 30
\]

However, individual profits change:

\[
\pi(q^*) = 30 \cdot \frac{45}{15} - (1 + 225\left(\frac{45}{15}\right)^2 + T) = -T
\]

(c) Compute the equilibrium in the Long Run. Circle your final answers!

We would expect \( N^* \) because of the tax.

\[
\begin{align*}
\text{.1) } & p = \frac{450}{q} \\
\text{.2) } & p \cdot q - (1 + 225q^2 + T) = 0 \\
\end{align*}
\]

\[
\rightarrow 450q^2 - (1 + 225q^2 + T) = 0 \\
\Rightarrow 225q^2 = 1 + T \\
\Rightarrow q^* = \frac{\sqrt{1 + T}}{15}
\]

\[
\Rightarrow p^* = 450 \cdot \frac{\sqrt{1 + T}}{15} = 30\sqrt{1 + T}
\]

\[
\text{.3) } G(p) = (100 - 30\sqrt{1 + T}) \frac{34}{525} = N^* \cdot \frac{\sqrt{1 + T}}{15} = N^* \cdot q^*(p) = G^*(p)
\]

\[
\Rightarrow N^* = \frac{100 - 30\sqrt{1 + T}}{525} \cdot \frac{45}{\sqrt{1 + T}} = \frac{1800 \cdot 34}{35} \frac{1}{\sqrt{1 + T}} - 30 \cdot \frac{34}{35}
\]

\[
= 680 \frac{\sqrt{1 + T}}{1 + T} - \frac{204}{35} < 68 \text{ as long as } T > 0
\]
3. Now suppose that the government levies an excise tax on all rms in the same market.

(a) Write down the new cost function. Then compute and compare the marginal cost before and after the implementation of the excise tax. Do the same for the average total cost. Conclude whether the short run and/or the long run equilibria will change with respect to . Support your conclusion relating it to the changes in cost!

\[
\tilde{C}(q) = C(q) + t \cdot q = 1 + 225q^2 + t \cdot q
\]
\[
\tilde{MC}(q) = MC(q) + t = 450q + t
\]
\[
\tilde{ATC}(q) = ATC(q) + t =
\]

\[
\Rightarrow \text{Since \( \tilde{MC}(q) \neq MC(q) \), the behavior of firms changes in the SR and so does the equilibrium. However notice if \( \hat{q} \Rightarrow MC(\hat{q}) = ATC(\hat{q}) \), then it also holds that \( \tilde{MC}(\hat{q}) = \tilde{ATC}(\hat{q}) \). Which means firms produce the same in LR, everything else changes.}
\]

(b) Suppose that the number of rms in the industry is equal to the long run \( N^* \) found in the first exercise. Find the short run equilibrium and the equilibrium individual profit. Circle your final answers!

\( (N^* = 68) \)

\[
\begin{align*}
(100 - p)\frac{34}{525} &= 68 \cdot \frac{p - t}{450} \\
\Rightarrow 100 - p &= \frac{68}{5} p - \frac{68}{5} t \\
\Rightarrow p^* &= \frac{30 + \frac{3}{4} t}{10} \\
\Rightarrow \frac{q^*}{450} &= \frac{30 + \frac{3}{4} t - t}{450} = \frac{1}{15} - \frac{3t}{450}
\end{align*}
\]

(c) Compute the equilibrium in the Long Run. Circle your final answers!

\[
\begin{align*}
P = 450q + t
\end{align*}
\]

\[
\begin{align*}
(100 - p^*)\frac{34}{525} = N^* \cdot q^* \Rightarrow (100 - 30 - t)\frac{34}{525} = N^* \cdot \frac{1}{15}
\Rightarrow N^* = 68 - \frac{3t}{35}
\end{align*}
\]
4. Consider a monopolist that faces a linear inverse demand of:

\[ P(q) = 100 - (1/4)q \]

and has a cost function of the form:

\[ C(q) = 2438 + 4q \]

(a) What are the monopolistic market price, quantity, and profits?

\[
\max_q \Pi^H(q) = \max_q (100 - \frac{1}{4}q)q - (2438 + 4q)
\]

\[ \text{FOC: } MR(q) - MC(q) = 0 \implies (100 - \frac{1}{2}q) - 4 = 0 \implies q^H = 192 \]

\[ P(q^H) = 100 - \frac{1}{4} \cdot 192 = 52 \]

\[ \Pi^H(q^H) = 52 \cdot 192 - (2438 + 4 \cdot 192) = 6778 \]

(b) If the government follows a price control regulation, setting effectively \( P = MC \), what will be the firm's reaction in the long run? Would this be fixed by setting \( P = ATC \)?

\[
P(q) = MC(q) \implies 100 - \frac{1}{4}q = 4 \implies q^* = 384; \quad P^* = 4 \implies \Pi(q^*) = -2438 < 0
\]

\[ \rightarrow \text{The monopolist stops producing in the long run.} \]

\[ P(q) = ATC(q) \implies \Pi(q) = 0 \] which allows the firm to stay in the market.

(c) What will be the subsidy required to induce the monopolist to choose \( q^* \) such that \( P(q^*) = MC(q^*) \)?

Set a per unit subsidy \( s \) such that the monopolist freely chooses to produce \( q^* = 384 \). The monopolist behaves according to:

\[ MR(q) = MC(q) \]

\[ \implies MC(q) = 4 - s \quad (\text{subsidy} = -\text{tax}) \]

\[ MR(q) = \frac{d[(100 - \frac{1}{4}q)q]}{dq} = 100 - \frac{1}{2}q \]

\[ \implies 100 - \frac{1}{2}q^* = 4 - s \]

\[ 100 - \frac{1}{2} \cdot 384 = 4 - s \implies s = \frac{284}{2} + 4 - 100 = 96 \]
5. A profit maximizing monopolist has marginal costs given by: \( MC(Q) = 2Q \), where \( Q \) is the monopolist's total output (thus, you may rewrite marginal cost as \( MC(q) = 2(q_1 + q_2) \)). The monopolist can sell her output in two geographically separate markets. Demand in the two markets is, respectively:

\[
\text{Market 1: } P_1(q_1) = 52 - 68q_1 \\
\text{Market 2: } P_2(q_2) = 58 - 24q_2
\]

Assume the monopolist can prevent all resale of its product between Markets 1 and 2. That is, there are no arbitrage opportunities for the consumers in Markets 1 and 2. Answer the following.

(a) How much does the firm sell in each market?

The monopolist solves

\[
\max \quad P_1(q_1) \cdot q_1 + P_2(q_2) \cdot q_2 - C(q_1 + q_2)
\]

FOC: 1) \( MR_1(q_1) = MC(q_1 + q_2) \)

\[
2) \quad MR_2(q_2) = MC(q_1 + q_2)
\]

1) \( 52 - 136q_1 = 2(q_1 + q_2) \) \( \Rightarrow q_1 = \frac{52 - 2q_2}{138} \)

2) \( 58 - 48q_2 = 2(q_1 + q_2) \)

\[
\Rightarrow q_2^* = \frac{58 - 2q_1}{50} = \frac{58 - 52}{50} = \frac{6}{50} = 1.15
\]

\[
q_1^* = \frac{52 - 2 \cdot 1.15}{138} = 0.36
\]

(b) What price does the monopolist charge in each market?

\[
P_1^*(q_1^*) = 52 - 68 \cdot 0.36 = 27.51
\]

\[
P_2^*(q_2^*) = 58 - 24 \cdot 1.15 = 30.51
\]

(c) In equilibrium, what is the price elasticity of demand in each market?

\[
|\varepsilon_1(q_1^*)| = \left| \frac{\partial P_1}{\partial q_1} \cdot \frac{q_1}{P_1} \right| = \left| \frac{-1}{68} \cdot \frac{52 - 68q_1}{q_1} \right| = \left| 1 - \frac{52}{68q_1} \right| = 1.12
\]

\[
|\varepsilon_2(q_2^*)| = \left| \frac{\partial P_2}{\partial q_2} \cdot \frac{q_2}{P_2} \right| = \left| \frac{-1}{24} \cdot \frac{58 - 24q_2}{q_2} \right| = \left| 1 - \frac{58}{24q_2} \right| = 1.11
\]

(d) Compare your answers for market 1 and 2, in b) and c). Does it make sense? Why? Why not?

It makes sense, the monopolist is charging a higher price in the relatively inelastic market.